ACTUARIAL STUDIES

No. 6

DISTRIBUTION OF SURPLUS

JOSEPH B. MACLEAN
EDWARD W. MARSHALL

THE ACTUARIAL SOCIETY OF AMERICA

ACTUARIAL STUDIES NO. 6

DISTRIBUTION OF SURPLUS

PRINCIPAL CONTRIBUTOR

JOSEPH B. MACLEAN

SUPPLEMENTARY CONTRIBUTOR

EDWARD W. MARSHALL

PUBLISHED BY
THE ACTUARIAL SOCIETY OF AMERICA
NEW YORK
1937

THE MAPLE PRESS COMPANY, YORK, PA.

COMMITTEE ON ACTUARIAL STUDIES

Alden T. Bunyan	Edward W. Marshall
James S. Elston	Samuel Milligan
F. Bruce Gerhard	Hugh H. Wolfenden

Ray D. Murphy, *Chairman*

INTRODUCTION

In view of the fact that much valuable literature of actuarial science is contained in scattered original papers, the Actuarial Society of America has been issuing a series of small volumes upon important actuarial studies. Each volume is intended to bring together within a reasonable space the more important points of information on the subject discussed, thus assisting students of actuarial science and furnishing a means of ready reference for actuaries.

The present volume on Distribution of Surplus has been prepared to meet a growing demand. Inasmuch as the subject covered admits of a wide divergence of opinion and viewpoint, the Committee on Actuarial Studies felt considerable hesitation regarding the publication of such a work. However, in an attempt to meet the need and to present the subject from varied points of view, it was decided to have the work undertaken by two individuals—a principal contributor who would discuss the subject as a whole from one general viewpoint, and a supplementary contributor who would approach the subject from another angle. It was felt that this plan would make the book particularly useful to students. It will be understood, of course, that the two contributors are solely responsible for the opinions expressed.

A subcommittee of the Committee on Actuarial Studies, composed of Mr. Bunyan and Mr. Marshall, was appointed for the purpose of aiding and criticizing the work before publication and the chapters on group insurance, industrial insurance and annuities were given a further review by specialists in these fields.

Because of the nature of the subject matter contained in this book, it should be understood that the Actuarial Society of America is in no way responsible for either the material included or the views expressed.

CONTENTS

DISTRIBUTION OF SURPLUS

CHAPTER I

SURPLUS

Before considering the many theoretical and practical questions which arise in connection with the distribution of surplus among the policyholders of a life insurance company it is desirable to have a clear understanding of the different meanings which may be attached to the term *surplus*. In particular, a distinction must be made between "true surplus" which arises from *profits* and "valuation surplus" which is the excess of assets over liabilities and which depends on the particular values assigned to the assets and to the liabilities.

Surplus from Profits. Fundamentally, the sums refunded as "surplus" or "dividends" to policyholders under the mutual or participating system arise from the overpayments which result from charging premiums which prove to be greater than necessary. Strictly speaking, it is impossible to tell if the premiums on any particular group of policies have been greater than necessary until the last policy has terminated. If the premiums have been more than sufficient (and if no dividends have been paid) a balance will then remain on hand. If they have been too low a point will be reached at some earlier date when the company will be unable to meet its contract obligations except by drawing on the surplus funds of other groups. In the meantime, due to temporarily favorable conditions, surplus may be created but this may be extinguished because of a later adverse experience. From this point of view it might seem that, theoretically at least, no "profits" have been realized and therefore no refunds should be made until the complete history of the group is available, at which time any surplus which exists should be returned on an equitable basis to the representatives of all the original policyholders. Otherwise, some of those insured will have had the benefit of more favorable terms than others.

Such an extreme view is, however, impracticable and, in fact, in the majority of states (many of which now require annual distribution of surplus) could not legally be carried out. It emphasizes, however, a very important fact to which it will be necessary to refer later and at greater length, namely, that the true "profits" are determined solely by the premiums actually payable and the conditions actually experienced during the entire existence of the whole group. No other considerations, such as temporary fluctuations in the values of assets or changes which may be adopted in calculating the values of liabilities—in particular the policy reserve—can, in any way, affect these true profits. If surplus is distributed periodically such considerations may, however, affect the *incidence* of the appearance and therefore of the distribution of profits in point of time and in that way (because of the operation of interest) affect the aggregate amount of profit actually divided but the aggregate *value* of the profits will not be affected.

Provided the safety of the fund is not endangered, it is much more reasonable and, from a practical standpoint, it is necessary, to "take profits" from time to time, i.e., to distribute among the policyholders some part, at least, of the surplus formed by their overpayments to date. Thus, if it has been assumed that the funds will be invested at 3 per cent and if, in fact, the rate earned has been 4 per cent, it is legitimate to consider that an interest profit has been made and the company's position, if it returns the whole of the excess interest earnings to the policyholders, is thereafter as good as it was when the policies were issued. If, as a matter of conservatism, it is thought best to retain some part of the excess in case the interest earnings in some future year should fall below the required rate, the position of the company as a whole will thereby be strengthened at the expense of the individual policyholders, any one of whom may or may not later receive a corresponding benefit.

This is an important consideration which affects the whole question of equitable distribution. At present it is sufficient to point out that if surplus is held back without good reason the equities as between different generations of policyholders may be disturbed, since surplus created by the payments of certain policyholders may eventually be distributed to others.

In the same way, if fewer deaths have occurred than were actually provided for or if the expenses have been less than the

loadings, profits from mortality or from loading have actually been realized and may be dealt with in a similar manner.

Profits or losses may also arise under practical conditions (1) from terminated (i.e., lapsed or surrendered) policies, (2) by sale or revaluation of assets and (3) from miscellaneous sources.

The true profit arising from terminated policies is the excess, positive or negative, of the *asset share* of the policy over the amount paid or credited as a surrender value. The asset share is that part of the company's total assets which results from the accumulation at earned interest of the net payments made by the policyholder in question, taking into account his proper share of the expenses and the *actual* cost of insurance, as well as any dividends which have been paid. For reasons which will be discussed later, profit from lapse or surrender does not usually enter *directly* into the distribution formula, although it forms a part of the true surplus arising from the real profits of operation and affects the aggregate amount available for distribution.

Again, if an asset is sold for more or less than the book value or if the book value of assets is written up or down because of real (permanent) change in value there is, likewise, a profit or loss which will increase or decrease the true surplus. Such profits or losses are sometimes considered in connection with the interest profit and may be distributed on the basis of a modification of the interest rate. Otherwise, they do not enter directly into the formula of distribution but simply affect the total divisible surplus. The same is true with regard to any miscellaneous profits or losses realized.

Valuation Surplus. Up to this point we have considered the sources which give rise to surplus in the sense of profits from operation. Except under theoretical conditions the surplus shown by a financial statement will not, however, correspond to the realized profits as outlined above. There are various reasons for this. The values of assets shown in the financial statement as required by the various states may be, and usually are, different from the book values. For example, certain stock exchange securities must be shown at "market values" while certain other book assets are not admitted as having any value. Such modifications in a financial statement have, of course, no effect on the actual results of the company's operations and are quite different in

their nature from realized profits or losses from sales or revaluation which have been entered upon the company's books.

Moreover, the reserves set up from year to year and the reserve released or transferred on termination do not correspond to the asset share, the reserve being in the earlier years, as a rule, considerably more than the asset share, and in the later years either more or less, according to the conditions experienced and the amount of dividends previously allotted. The financial statement shows surplus based on the reserve rather than the asset share and the resulting profit from lapse or surrender likewise differs from the true profit based on the asset share.

Again, the reserve liability under the company's policies may be computed on a different basis from that upon which the premiums were calculated. This may be done from the inception of the policy or by changing the valuation basis at some later date. Neither procedure can alter the total value of the true surplus realized throughout the whole duration of the group of policies but such a change will affect the time and manner in which the apparent surplus will emerge. If, for example, a more stringent reserve basis is adopted after some years, as by assuming a lower interest rate, there is an immediate reduction in valuation surplus. Unless, in the case supposed, the interest rate has, in fact, fallen below the rate assumed in the premiums, this reduction in surplus is a "loss" only in a technical sense.

Whatever the basis of valuation the company will, under any given experience conditions, receive certain amounts of premiums and interest and will pay out certain amounts for claims and expenses. These are independent of the valuation basis. If in order to maintain a more stringent reserve basis, more of the net income is for a time placed in "reserve" and less considered as "surplus," the surplus must eventually receive correspondingly greater accretions since all reserve bases lead up to the same final result. In the meantime, however, the incidence of the nominal profits from excess interest, from mortality and from loading may be altered as compared with those which would appear under the premium basis. It is obvious that this will not only affect the total amount appearing to be available for distribution at any time, but that it will create problems in regard to the amounts of surplus to be returned to individual policyholders. Thus, if the valuation rate of inter-

est for reserves on existing business has been reduced while experience conditions remain the same, the excess interest is apparently increased and the profits from mortality and loading decreased. In such circumstances it may be a question whether dividends should not be computed with reference to the premium basis rather than the valuation basis.

Divisible Surplus. When a financial statement is prepared and before any allotment of surplus is made for distribution to the policyholders, the balance of admitted assets over all actual liabilities may be considered as the gross surplus. This corresponds to the accumulated profits as modified by the various considerations referred to above. Out of the gross surplus there must in practice be set aside such an amount as the company considers necessary as a fund for contingencies. The remainder constitutes the divisible surplus, which is the actual amount available for distribution. Assuming an annual distribution, as soon as action is taken by the directors of the company definitely providing for such distribution the divisible surplus is converted into a liability. Normally, where the dividend year corresponds to the calendar year, such action disposing of all of the gross surplus not required as a contingency fund will have been taken prior to the end of the year so that in the financial statement the contingency fund becomes the net surplus while the divisible surplus appears as a liability.

A reverse process may be followed by first determining the amount of the gross surplus required to pay the scale of dividends which the company decides to adopt for the following calendar year, the remainder of the gross surplus then being the amount available as a contingency fund. Under practical conditions the actual procedure will naturally be a combination of these alternatives, consideration being given both to the changes, if any, to be made in the existing dividend scale and to the effect thereof on the amount of the contingency fund.

The various factors which will be taken into account in determining the basis of the new dividend scale will be discussed in a later chapter. In general these will include (1) the rates of current surplus earnings from interest, mortality and loading; (2) the probable trend of these earnings in the near future; (3) the sufficiency of the existing contingency fund with particular regard to the investment situation and the company's objective in regard

to the ratio of net surplus to liabilities; (4) the desirability of making the minimum equitable departure from the existing scale. It will be necessary to make a careful estimate of the aggregate sum required to provide the proposed dividends whether these are to be on the existing scale or on some modification thereof. This can be done by preparing in advance an analysis of the cost of the dividend on the basis of insurance in force, separating each of the factors involved by age, plan or duration as required in accordance with the details of the dividend formula adopted. Such an analysis in conjunction with the subsequent terminations and new issues (on an estimated basis if not already determined) will provide the basis for estimating the total cost of the new dividend and the financial effect of modifications or changes in any of the factors.

If conditions generally have been favorable it will be reasonable to provide for an increase in the contingency fund at least sufficient to maintain the existing ratio to liabilities (assuming that ratio to be satisfactory) or to increase the ratio if it is below the figure which the company desires to maintain. If conditions have been unfavorable it becomes a matter of judgment whether the contingency fund should be drawn upon in order to avoid a eduction in the scale of dividends—as would be legitimate where ıe unfavorable elements are in the nature of a temporary fluctuaion—or whether dividends should be decreased and the contingency fund maintained, or increased as when there is either a marked decrease in surplus earnings or a definite downward trend or increased uncertainty in regard to the future financial position or otherwise.

Whatever the process by which the actual amount of the aggregate divisible surplus is arrived at, it is important to realize that the total sum to be divided is not simply the aggregate of the individual "contributions" to surplus arising from a favorable experience as to the rates of interest, mortality and expense and other elements, but is rather a *lump sum* which is, of course, related more or less closely to these primary sources of profit but which is also affected materially by considerations which are quite independent of the actual profits. Consequently, the general problem of the distribution of the divisible surplus among individual policyholders is, in actual practice, not merely a question of refunding "overpayments" although the system of dis-

tribution may be (and, in America, generally is) *based* on these overpayments.

Dividend Liability. While the aggregate divisible surplus is arrived at after a valuation of liabilities and assets at a fixed date, usually December 31st, dividends on individual policies are, under the contribution system, which is the usual system on this continent, computed in the *form* of refunds on a policy year basis. For example, part of the dividend allotted to any policy is the amount of excess interest earned on the reserve during the period from the previous policy anniversary to the anniversary in the year in which the dividend is paid—and similarly for the other factors of profit involved. This may seem to imply the assumption of surplus earnings at a predetermined rate from December 31st until the following policy anniversary or, in the case of some policies, for nearly the whole of the ensuing calendar year. If that is so, it would be logical (provided the rate of surplus earnings is reasonable) that the dividend liability at December 31st should be reduced by the amount of surplus which it is assumed (in the dividend formula) will be earned in the following calendar year and prior to the respective policy anniversaries or, say, roughly, during the first half of the calendar year. Alternatively the divisible surplus could be arrived at after allowing for the estimated surplus earnings up to the succeeding policy anniversaries.

The usual practice, however, is to ignore the fact that part of the cost of the new dividend will be met by surplus earnings after December 31st and the effect is therefore that, from this point of view, there is in the financial statement a substantial understatement of net surplus and a corresponding overstatement of the liability for dividends. There is, in fact, some inconsistency between the idea of determining the divisible surplus as the net surplus earnings of the calendar year adjusted by the amount of any change determined upon in the contingency fund and the system of distribution on the basis of policy years. However, it may be said that all methods of distributing surplus to individual policyholders are necessarily of an approximate character and that while divisible surplus is naturally determined in the aggregate on the basis of the calendar year the *basis of distribution* where dividends are allotted on policy anniversaries should be based on the theoretical "contributions to surplus" during policy years. In the State of New York the law relating to distribution

of surplus (Ins. Law, Sec. 83) specifically requires that each company domiciled in the State of New York shall, at December 31st, ascertain "the surplus earned by such corporation during said year" and after deducting the amounts required for dividends to stockholders, deferred dividends and contingency fund, "shall apportion the remaining surplus equitably to all other policies entitled to share therein" the share of each policy being payable either on the next policy anniversary or on a "day certain" in the following year. This definitely indicates the distribution of surplus which has already been earned at December 31st and would logically require a distribution to the individual policies based on the calendar year rather than the policy year. However, distribution, except on industrial insurance, is invariably made on a policy year basis.

A further question as to dividend liability at December 31st may arise where the dividend year differs from the calendar year and where no actual allotment of dividends has been made at December 31st beyond the end of the current dividend year. In that case no true liability exists except for the dividends actually allotted. Even in a company in which the dividend year coincides with the calendar year dividend action may, because of uncertainties in the general situation or for other reasons, be taken, prior to December 31st, only in respect of a portion of the ensuing calendar year. In that case, similarly, there is no actual liability at December 31st except for dividends payable in the part of the year covered. This situation is provided for in the Convention Blank (Liabilities, Line 33) which calls only for dividends "declared or apportioned" up to and including a date specified by the company and not necessarily for the whole year.

In practice the majority of those companies which have not allotted dividends for the whole calendar year provide nevertheless, in their financial statements, in one way or another, for dividends for a whole year. This may be done by estimating the approximate cost for the whole year on the current basis or, where a change is contemplated, taking into account the modified basis for the balance of the year, and entering the total in Line 33. In some cases a separate liability is shown under some such title as "Reserve for Dividends Payable from (*month and day*) to December 31, 19—" this, of course, being in addition to the amount entered as declared or apportioned. In either case, the

effect is to reduce the net surplus or contingency fund as compared with a company which enters the actual liability, i.e., the dividends for the balance of the dividend year only. If a separate entry is made for dividends not apportioned the true position can be seen, but if these are included with dividends apportioned the company's position is made to appear less favorable than it actually is. For that reason a uniform practice in entering these items seems desirable. This should indicate both the actual liability for dividends apportioned and also any additional amount held for dividends not yet apportioned and which, until apportioned, is, in fact, surplus.

A dividend year different from the calendar year is adopted (1) in order that the actual experience of the full calendar year, instead of only a part thereof, or an estimate, may be used in determining the new dividend scale, and (2) because of the practical difficulties involved in the actual calculation and notification of dividends on policies dated in the first month or two of the year if a change in the scale is to take effect immediately after December 31st. With regard to the first of these it would seem, in view of the considerations referred to above, that any additional *accuracy* obtained by using the full year's experience rather than that of a part of the year is more apparent than real, particularly if, as is usually claimed to be the case, *trends* rather than actual rates of profit determine the basis of the dividend scale. Under normal circumstances it can make little difference to the company as a whole whether the new scale is determined, say, in October or November with a dividend year commencing January 1st (as is, in fact, usually done) or in January with a dividend year commencing a month or two later. Under abnormal conditions as, for example, where a considerable variation in market values of securities may be expected during the last few months of the year it will, however, be more difficult to determine divisible surplus in advance and the scale adopted may be too high or too low in view of subsequent developments.

In New York, as already mentioned, the law pertaining to domestic companies refers specifically to the distribution of surplus earned during the calendar year and a literal fulfilment of the requirements of the law would, for practical reasons, make it impossible to change the scale on January 1st. In 1920 the law was amended with the object of permitting the adoption of a

dividend year commencing not later than four months after the end of the calendar year. Some of the principal companies have adopted the special dividend year basis, the interval from the end of the calendar year being usually three or four months. The majority of the companies, however, have a dividend year corresponding to the calendar year and must therefore determine the amount and basis of distribution of the divisible surplus before the results of the whole year are available.

Incidentally the Superintendent of Insurance of New York has ruled that if a dividend scale is being increased the announcement cannot be made more than sixty days before the time when the scale becomes effective; whereas if the scale is being decreased the announcement shall be made as soon as the decrease has been decided upon.

CHAPTER II

GENERAL PRINCIPLES OF DISTRIBUTION

In this chapter we shall discuss some of the general principles which should be followed in distributing the divisible surplus.

Interval of Distribution. A preliminary question which arises is how frequently a distribution of surplus should be made. In the very early days of life insurance, both in America and in Europe, distributions were made only at intervals of several years. Some of the companies organized in the United States during the first half of the nineteenth century distributed surplus annually from the start. Others adopted for a time a period of three or five years. The trend has been toward a more frequent distribution and in the United States and to a considerable degree in Canada annual distribution is now the rule. As already mentioned, annual distribution is, in fact, now required by law in many states. In other countries annual distribution is still the exception. In Great Britain distribution is usually made at intervals of five years, with, however, provision for "interim bonuses" for policies becoming death claims between distributions.

Where, as in America, dividends are declared in cash and where it is the custom of the majority of the policyholders to apply the dividend each year in part payment of premium, an annual distribution is the normal and most suitable basis. Where dividends are declared in the form of additional insurance (although such insurance may be surrendered for cash) and where it is not customary to apply dividends in payment of premium, an annual distribution is not necessarily the best arrangement.

The advantages usually claimed for a distribution less frequent than annual are (1) equalization of the surplus earnings, thus eliminating undue fluctuations, and (2) reduction of expense. The first of these is not of much importance except in very small companies. So far as the rates of interest, mortality and expense are concerned, dividends may properly follow the general trend

of earnings from year to year rather than the actual factors of experience, while larger factors of profit and loss such as may occur from the investment operations can, as a rule, be stabilized by absorbing them, in whole or in part, in the net surplus or contingency fund. In fact, some companies specifically earmark a part of their net surplus as a "dividend fluctuation fund." Some go further and establish separate funds for fluctuations in mortality and in investments. This practice is, of course, largely a matter of nomenclature. There is nothing to prevent a company applying part of a "mortality fluctuation fund," for example, against investment losses. All such funds are part of the net surplus and are available for any purpose for which the net surplus is available.

There is, therefore, little force from a practical standpoint in the argument that a quinquennial distribution (say) will result in more stable dividends than an annual distribution. In fact, as the trend of gains and losses is apt to show itself over longer periods than a year the result might prove to be in the opposite direction if the actual gains of each quinquennium were distributed.

The expense involved is a better argument against an annual distribution. No doubt such expense may be considerable and from that standpoint the policyholders would be better off on a different basis. Apart from legal requirements, however, the question of expense must be weighed against the convenience and the wishes of the policyholders. In the United States at least, law and custom both require an annual distribution of surplus. Moreover, it might be argued that, under the mutual system, where the object is to furnish insurance at cost, it is reasonable to make the proper adjustments of premium payments without unnecessary delay.

It may be admitted that the British system of quinquennial distribution in the form of additional insurance which is almost always left undisturbed until the maturity of the policy has a great deal to be said for it in that it results in greater protection and larger payments to beneficiaries. While the same results can be and frequently are obtained under the American system, the fact of annual declaration in cash at the time each annual premium falls due usually results in the immediate disposal of their dividends by the majority of policyholders.

Reference will be made in a later chapter to the systems of "tontine" and "deferred" dividends which were largely used by certain companies up to about thirty years ago and which usually involved a dividend distribution period of ten, fifteen or twenty years. These systems were based on entirely different considerations from those just discussed.

Criteria of a Good Distribution System. There are, in practice, only two essential qualifications for a good system of distribution. It should be *equitable* and it must be *practical.* Rigid equity, which is the only theoretical requirement, is, of course, impossible of attainment in practice and by equity we must therefore understand such a degree of fairness as is reasonably attainable. Any additional qualifications such as have been specified by other writers on the subject may be placed under one or another of these heads. For our present purpose it is convenient to consider in more detail the various criteria which have been laid down. Stated in approximate order of importance, these require that the system should be (1) equitable, i.e., fair; (2) flexible, or adaptable to changing circumstances; (3) reasonably simple in operation; (4) capable of being understood by the policyholders; (5) popular with the policyholders and the public; (6) in harmony with current usage.

In addition to these general criteria of a good *system* there are other considerations relating to the choice of dividend *scales* to which reference will be made in a later chapter.

Equity. Equity clearly requires a system of distribution which does not favor any class of policies at the expense of any other class. Within that definition it is not easy to say exactly what is "equitable." It is usually considered that, from a theoretical point of view at least, equity requires a return of surplus to individual policyholders in the proportions in which they have contributed to it. This principle, although at first sight axiomatic, involves both theoretical and practical difficulties. What is to be the basis of a "class" of policyholders and how are individual contributions to surplus to be determined? Mortality experience, for example, varies widely with duration, occupation, residence (particularly in large countries such as the United States or Canada), amount of insurance, and even with plan of insurance and year of issue. It is evidently impracticable to distinguish all these factors in computing the individual contribu-

tions to surplus from favorable mortality. That would involve a multiplicity of different dividend scales every year.

Again the rate of expense depends very largely on amount of insurance, being lower for a policy of large amount than for a policy of small amount. The expense rate is also affected by other factors such as different premium tax rates in different localities. It might be claimed that strict equity would require an allocation of expenses which took such factors into account but, in practice, that cannot be done. The interest factor of the individual contribution to surplus actually depends *inter alia* on the different times and interest rates at which the premiums were invested, but it would be impossible to trace the actual interest contribution of any individual policy or even of any (except, perhaps, a very large) class of policyholders.

The situation is, of course, that the whole plan of operation assumes and depends on large groups, not individuals, so that for the most part contributions to surplus must necessarily be determined on the basis of the experience of the company as a whole. Refinements may be made as in the assessment of expenses between policies for different plans of insurance, ages at issue or durations, or in a classification for mortality experience in a few broad groups but the idea of *individual* contributions reduces very largely to the pro rata contributions based on the aggregates of large groups, if not of the entire company.

It is necessary to remember also that, as pointed out in the previous chapter, the divisible surplus will not, except by accident, equal the total "profits" or aggregate "contributions" to surplus but is affected by considerations which are quite unrelated to the elements of profit or which are incapable of being related to individual policies or even to classes of policies. Hence, a method of distribution which calls for the computation of individual contributions to surplus would require modification in order to reproduce in the aggregate the actual sum available for distribution. Such modification may disturb the relative amounts of the assumed contributions from different sources.

The most that can be said is that from a practical point of view *equity* requires a reasonable recognition of profits and the sources from which they have arisen. No system which does not, in some degree, take account of the sources of profits can properly

claim to be equitable. But it is clear that "equity" must be understood in a very broad sense.

Another conception of equity which has been suggested[1] is that *any* system is "fair" provided the present value of the dividends for the entire duration on the basis of experience factors of interest, mortality, expense, termination rates and surrender values is equal to the present value on the same basis of the excess of the actual premium payable over the premium required on the experience basis. This would imply that any one of an indefinite number of possible systems of distribution—allotting surplus in entirely different ways—may be equitable. Apart from the difficulty involved in the apparent necessity of determining the rates of surplus earnings in advance, a system which complied with this condition could not necessarily be regarded as *fair* unless the policyholders understood and agreed to it in advance.

An extreme case would be one where the entire accumulated surplus was paid to the last survivor. In the absence of a definite understanding and agreement this would be distinctly unfair although at the outset every policyholder would have an equal chance of being the last survivor, so that on the basis just stated the plan would be equitable. Similarly, any distribution at intervals among the survivors might, on this basis, be theoretically equitable but practically inequitable unless agreed to, since any such scheme involves the possibility of loss by individual policyholders of surplus which they have contributed and its consequent allotment to other policyholders who have not provided any part of it, which would be contrary to the general principle enunciated above. In any case, equity would not be maintained between persisting and withdrawing policyholders unless the dividends were such as to make the asset share approximately equal to the surrender value.

A practical application of the conception of equity as involving simply an equation of premiums and benefits is found in the British system of uniform reversionary "bonuses" (dividends). The theory underlying and justifying that system is that the premiums payable include a "bonus loading" which provides for dividends in the *form*, if not of the exact *amounts*, required.

---

[1] W. P. Elderton, "Fair Distribution of Bonus," T.I.C.A., IX, I, 158.

Naturally, no such system can predetermine the actual surplus. The dividends provided for in this way by a bonus loading are more in the nature of guaranteed benefits which change the character of the policy from a level to an increasing insurance. Further real surplus (positive or negative) which is not anticipated in the bonus loading will arise, in regard to which the problems of equitable distribution would still remain.

It may be that such additional surplus being of relatively small amount can reasonably be applied, as a practical matter, to change proportionately the main scheme of dividends without any serious disturbance of the basic equity of the plan. However, the equity of such a scheme depends not only on the gross premiums payable but also on the stability of the incidence and relative amounts of surplus earnings from various sources over a considerable period. Consequently, it would seem to be necessary, under this plan, to test the fairness of the premiums more or less frequently and, if necessary, to adjust them. But such adjustments of premium could apply only to new policies, not to those already in force. For these, the only course would, in such circumstances, be to change the basis of the dividend system—a procedure not contemplated in the original plan. In considering the foregoing, it should be remembered that in Great Britain the non-forfeiture values are relatively low, so that the problem of overpayment to the withdrawing policyholder does not complicate the situation.

Flexibility. To say that the method of distribution must be adaptable to changing conditions is merely an enlargement of the statement that the method must be equitable. Flexibility is secured (1) by separate recognition in the dividend formula of as many as possible of the elements of profit; (2) by the use of a type of formula which permits of proper adjustments to be made in any of the factors involved; (3) by the avoidance of arbitrary expedients in adjusting scales of dividends as between one year and another.

It will not, as a rule, be possible or necessary to adopt a dividend formula which involves separate recognition of more than the three primary factors involved in the premium rate. This degree of flexibility is usually regarded both as necessary and sufficient, the distribution of miscellaneous profits often being,

to a large extent, arbitrary. The type of formula used should be such, for example, that it can be modified so as to give effect to changes in the mortality experience as between different ages or plans or changes in the expense rate as between one plan, age at issue or duration and another. Otherwise, there will be a tendency to continue a basis of distribution after it ceases to reflect the incidence of surplus earnings or to make changes which disturb the true equities involved. Temporary expedients such as constant percentage changes in the total dividend or elimination of increase for duration, while possibly justifiable under certain circumstances, tend to destroy the simplicity and rationale of the dividend formula and its relation to the actual factors experienced and consequently its flexibility.

Simplicity. It is desirable for practical reasons that the method of distribution should be reasonably simple in application. A complicated rule is more troublesome, more expensive and less easy to explain to policyholders and others, while the increase in accuracy may be more apparent than real. This does not mean that some rough and ready rule may be justified on grounds of practical expediency. Such methods as were formerly sometimes used as, for example, distribution in proportion to the amount of insurance or in proportion to the premium, would be inadmissible by present day standards. Simplicity in this connection means the elimination of refinements which, because of the fundamental limitations on the degree of equity attainable, can have little real meaning. For example, extensive subdivisions or classifications in determining profit from mortality or excessive elaborations in the method of computing expense charges are not necessary or desirable. Equity, however, is always the first consideration and such equity as is practicable must not be sacrificed for ease in application.

Comprehensibility by Policyholders. It is sometimes said that the system should be capable of being understood by the policyholders. This is a minor consideration. Intelligent policyholders will have no difficulty in understanding the general basis of any reasonable plan. This, however, furnishes an additional reason for eliminating unnecessary complications.

Popularity. More than one writer on this subject has stated that the system of distribution should be popular with the insuring public. Such a consideration has little force in this country

where one system is practically universal. Where there is a choice as between one general system and another the influence, if any, on new business written would naturally be taken into account. One very desirable characteristic of a dividend scale from this standpoint is that it shall produce dividends which increase by duration.

Conformity to Usage. In the United States all companies use the same general system of surplus distribution. In Canada probably the majority of the companies employ to some extent the quinquennial distribution plan under new business. In other countries there is more variety of systems as between different companies. Where the same general system is used by all companies there is likely to be a better understanding of it by the public and less likelihood of misunderstanding or misrepresentation. Special or unusual features in the system are, from this point of view, to be avoided. They are apt to increase the problems of competition by rendering comparisons more difficult and usually are of little significance.

"Weeks' Axiom." A general rule in regard to the principles of surplus distribution which has been laid down by such authorities as Weeks in this country[1] and G. F. Hardy in England[2] is that if a company experiences each year uniform rates of initial expense, renewal expense, mortality and interest then the dividends ought not to change merely because of changes either in the volume of new business or in its distribution by plan or age. This rule is generally regarded as an axiom of surplus distribution and follows from the principle that equitable distribution requires that each year's issues and each class of business should, in general, depend for its dividends on its own surplus earnings. As a practical matter, however, under modern conditions in this country, if the net level premium method of reserve valuation is used, conditions might be such as to make a strict adherence to this rule inadvisable. For example, a very large increase in new business might temporarily deplete the surplus to a point where, unless all conditions were very favorable, it would be advisable to reduce dividends although the rates of interest, mortality and expense being experienced had not changed.

---

[1] T.A.S.A., IX, 310.
[2] J.I.A., XXXII, 112.

Amount of Surplus to Be Distributed

The primary practical question which the actuary has to determine is what part of the gross surplus should be reserved for contingencies and what part may be set aside as divisible surplus for distribution among the policyholders in the following year. In considering this question the chief factors, as already noted, are the amount of existing contingency funds, the amounts and trend of surplus earnings from the principal sources and the general situation, financial or otherwise, in relation to probable future effect on the total amount of surplus funds.

The amount of the "free surplus" or general contingency fund which should be maintained is generally related to the amount of the policy liabilities, i.e., the reserve. In New York the amount of such funds is limited to 10 per cent of the aggregate policy reserve in the case of companies having a total reserve of not less than $100,000,000, a higher percentage being allowed for smaller companies. This law was enacted with the object of preventing the companies from unduly withholding surplus from the policyholders and not directly with reference to the question of the degree of security desirable. The opinion has been expressed that a more rational law would prescribe the minimum rather than the maximum basis of the contingency funds.

The proper percentage of contingency fund to be held in any particular case under normal conditions depends on many factors including the size of the company, the character of the assets, contract provisions, scale of premiums payable, etc., and is largely a matter of opinion and judgment. The percentage of contingency funds to reserves varies in different companies. Each company must decide on the basis of its own circumstances what is a suitable and sufficient fund for contingencies and what variation in relation to reserves and amount of insurance in force is permissible.

Under normal conditions where no unusually large gains or losses in surplus (as from changes in security values) have taken place or are to be anticipated, the amount of surplus which may be distributed is likely to be determined chiefly by the trend of surplus earnings, that is, by a consideration of what, if any, changes are appropriate in the various factors of the existing dividend formula.

As a general rule, changes in the basis of distribution which would result only in relatively small increases or decreases in individual dividends are not desirable. Apart from the trouble and expense of changing the scale, a very small increase in dividends has little practical value, particularly in view of the normal increase in dividends with duration, while a small decrease may cause harm through disturbance of policyholders and agents and by adverse publicity without giving the company any real compensating advantage. It is better to absorb small fluctuations in profits in the contingency fund which, to that extent, properly has the function of a dividend fluctuation fund.

Under the conditions supposed, i.e., in the absence of any abnormal gains or losses in surplus, changes in the basis of dividends will, therefore, as a rule, be made only in accordance with definite trends, either upward or downward, in the rates of interest or mortality. Changes may also properly be made as a means of correcting any inequities which may appear from a detailed study of the general equity of the existing basis (as described later) or where the rate of growth of the contingency funds indicates that the existing dividend scale is either more or less favorable than conditions warrant.

Where conditions are exceptional or abnormal in any way, and are such as have resulted in or may lead to more substantial gains or losses in surplus, attention will be given primarily to the amount which should be retained in the general contingency fund rather than to the cost of any modified dividend scale. It is evident, of course, that these are closely related and that under any circumstances both the cost of the dividend and the amount of the contingency fund must be considered together.

Some of the conditions which may require special consideration of the amounts to be allotted in dividends or reserved for contingencies are the following: (1) Greatly increased new business; (2) greatly reduced new business; (3) exceptionally unfavorable mortality experience; (4) large profits or losses from investments; (5) rapid change in net interest rate; (6) uncertainty in regard to the general financial situation.

Changes in New Business. Under the net level premium system of reserve valuation, a substantial and sustained increase in new business will, if other conditions remain the same, result in a drain on surplus which will continue until new business

becomes stabilized. Theoretically, as already pointed out, dividends should not rise or fall according as new business decreases or increases but in practice, because of the use of a valuation system which ignores the incidence of expense, it may not be possible to adhere rigidly to this principle. If the company regards excess initial expense as an "investment" in new business which will eventually be repaid out of gains, not otherwise distributed, from select mortality and from excess of reserves over surrender values on surrender and lapse, or, as Bowerman suggests,[1] as a "New Business Fund" in the nature of a non-ledger asset, then it may regard the true surplus as including the amount of such new business fund. It is evident, however, that new business may increase to the point where some modification in dividends will become necessary to prevent the contingency fund from temporarily falling lower than is desirable.

The opposite condition, namely, a substantial falling off in new business, will, in general, have the opposite effect and will result in a more rapid accumulation of surplus than is contemplated by the current dividend scale. If dividends are increased under such circumstances the company may not be able to maintain the higher scale when business again increases although the rates of surplus earnings remain the same. On the other hand, under extreme conditions legal limitations on the amount of undivided surplus may require some disposition to be made of part, at least, of any additional surplus so released. These difficulties arise, as we have said, from the use of an artificial valuation system which makes it difficult to follow the reasonable rule that dividends should not depend on the amount of new business written.

It is likely, however, that in a period of rapidly increasing new business the surplus earnings from general operation will show an increase due to more generally favorable conditions while the opposite will probably be true when business is falling off, so that the effect on surplus of changes in the volume of new business may be partly counteracted by changes in earnings.

Fluctuations in Surplus Earnings. So far as the surplus earnings from loading, interest, terminations and miscellaneous sources are concerned, important fluctuations are unlikely to

[1] T.I.C.A., IX, I, 92.

occur as between one year and the next. The rate of expense does not change suddenly, and the profit from terminations in most companies is (because of the scale of surrender values) a relatively small part of the total gain in surplus so that changes in its amount are usually of comparatively little importance.

Mortality profit, however, may show a quite serious and unforeseen decrease in amount in a single year. During the influenza epidemic of 1918–1919 the mortality profit, normally an important part of the total divisible surplus was, in the case of some companies, completely wiped out and in most other companies was reduced to a small fraction of its normal amount. Such a condition is extreme and is unlikely to happen except at long intervals. There may, however, be, at any time, an adverse fluctuation in mortality profit of sufficient magnitude to require special consideration in relation to dividends. Some part, at least, and possibly all of such an unusual loss may be charged against the contingency fund or mortality fluctuation fund where these are sufficient. In practice the amount of mortality profit divided is generally based on the experience of the company over a period of years. Mortality experience, even in large companies, is bound to vary somewhat as between one year and another and normal differences in surplus from favorable mortality should be absorbed in a mortality fluctuation fund or in the general contingency fund.

Except under very unusual conditions the rate of interest earned on the funds as a whole does not change rapidly and the general trend can be followed in the dividend scale by occasional slight increases or decreases in the dividend interest rate. Where conditions are such as to cause a rapid and substantial fall in the earned interest rate, such as may occur, for example, from extensive defaults during a period of severe financial depression, judgment is necessary as to the extent to which the divisible surplus should be reduced. It is evidently unwise to maintain, by drawing on surplus funds, a dividend scale based on a substantially higher rate than is being earned, when the reduction is more than an accidental fluctuation and when there is no reason to expect that the rate will shortly return to a higher level. The object of dividend fluctuation funds is to assist in smoothing out accidental irregularities, not to provide funds for maintaining dividends on a higher basis than is justified by earnings.

If substantial losses occur from investments it will probably be necessary to reduce the divisible surplus and to revise the dividend scale in order to avoid too great a reduction in the contingency fund. Whether and to what extent this will be done will depend on the extent of loss, and the amount of the contingency or investment fluctuation funds available to take up the loss. The nature of the modifications which should be made in the dividend scale in such circumstances will be discussed in the following chapter.

Uncertainty in the Financial Situation. The first consideration in the operation of a life insurance company is security. The conservative bases adopted for premiums and reserves together with the contingency fund provide under normal conditions an ample guarantee of the performance of all obligations and permit a reasonable latitude in the determination of divisible surplus. The company must, however, at all times, consider not only the present strength of assets and reserves but their prospective strength and where any uncertainty exists as to the maintenance of investment income or of security values preparation should be made to meet less favorable conditions. This may require an ultraconservative dividend policy. Reduction of the dividend scale accompanied by an increase or even by maintenance of surplus funds in such circumstances may be criticized by some policyholders and agents who feel that in bad times the company should use its surplus funds to maintain the same dividends as were declared under more favorable conditions. The contingency fund is, however, not primarily a dividend fluctuation fund, although within limits it is properly used for that purpose. Its principal object is as a safety fund and when the margin of safety appears likely to be reduced as by a reduction in asset values, the proper course is to increase the safety fund, not to reduce it, or to keep the safety fund level while marking down assets to a thoroughly conservative basis. Naturally, individual judgments differ as to the probable course of future events and it is not to be expected that there will be universal agreement as to general dividend policy in such circumstances.

CHAPTER III

THE CONTRIBUTION PLAN

The contribution plan, which is the method of surplus distribution usually employed in the United States and Canada, is based on the principle of returning to each policyholder that part of the divisible surplus which may be considered as having been contributed by him. In practice the individual contributions to surplus are, in general, determined by a comparison of the rates of interest, mortality and expense experienced with those expected and upon which the premium rates were based. If we consider surplus in its true sense of accumulated profits except that no elements of profit or loss other than those above mentioned are considered, then the contribution to surplus made by any policy in any policy year may be readily derived by comparing the usual reserve accumulation formula with the corresponding formula involving the actual rather than the assumed rates of interest, mortality and expense.

By the usual formula,

$$({}_{n-1}V + \pi)(1 + i) - q(1 - {}_{n}V) = {}_{n}V \qquad (1)$$

Denoting actual rates of interest and mortality by i' and q', the actual expenses for the given policy year by e, the loading by L and the contribution to surplus by C, the actual progress of the fund during the policy year is represented by the equation:

$$({}_{n-1}V + \pi + L - e)(1 + i') - q'(1 - {}_{n}V) = {}_{n}V + C \qquad (2)$$

Deducting formula (1) from formula (2) the contribution to surplus during the year from the three fundamental sources is

$$C = ({}_{n-1}V + \pi)(i' - i) + (L - e)(1 + i') + (q - q')(1 - {}_{n}V) \qquad (3)$$

This is the familiar and typical three factor contribution formula in which the total contribution or dividend consists of (1) excess interest on the reserve; (2) excess of loading over

expenses—with interest on the net amount; and (3) saving from favorable mortality experience.

As we shall see in a later chapter, the contribution plan in its original form called for (1) a calculation of the "contributions" of individual policies from the primary three sources as in the foregoing formula, omitting all other profits or losses, and (2) the distribution of any excess (positive or negative) of the divisible surplus over the total of the contributions either in proportion to the contributions, as was done in the early applications of the method, or in any other way deemed by the company to be convenient and suitable. This procedure soon developed into the calculation of dividends *directly* as the sum of three factors which are *nominally* the contributions from interest, mortality and loading but which are, in fact, usually modifications of these contributions such as will reproduce in the aggregate the approximate sum which it is desired to distribute and which must, of course, depend on *all* sources of profit or loss.

Such a modification may evidently be made in various ways as by adjusting each factor independently or by adjusting only one of the factors, in which case the other factors represent the assumed actual contributions to surplus. Whatever basis of modification of the contributions may be adopted the first step is to obtain the experience rates of interest, mortality and expense and to decide upon the *form* in which the contributions will be calculated. In this chapter we shall discuss the various methods used and some of the questions which arise in determining each of these rates and the forms in which they shall be applied.

Before doing so it may be noted that the dividend factors involved in the application of the contribution plan in practice are not, as a rule, directly based upon an analysis of surplus such as that furnished by the Gain and Loss Exhibit; in fact, in Canada, this exhibit is not made for Canadian business. In order to calculate the contribution from excess interest it is not necessary to know the amount of the aggregate interest profit. The total of the excess interest contributions actually earned on policy reserves will not equal the total "Gain from Interest" which includes interest earned on the contingency fund and other special funds and the amount of which depends, *inter alia*, on the treatment accorded in the company's accounts to such items

as interest in arrears or capitalized interest. Similarly the contributions from favorable mortality are, in practice, based on the rates of mortality experienced *age by age* as compared with those of the mortality table used, and cannot be derived from the aggregate gain from mortality shown in the Gain and Loss Exhibit. Almost invariably, as we shall see, the mortality factor of the dividend represents only the saving in the ultimate rate, extra profit from select, as distinguished from ultimate mortality being, as a rule, applied, in effect, in reduction of excess initial expense. The "Gain from Mortality" as shown in the Gain and Loss Exhibit includes select mortality profit and, of course, gives no indication of the incidence of mortality gain by age. Finally, the loading factor of the dividend could not be derived from the aggregate "Gain from Loading" because the latter makes no separation between initial and renewal expenses and loadings and also because it is necessary in a proper application of the contribution plan to assess expenses by plan, age and duration. In companies which use low rates of gross premium and, particularly, where a substantial new business is being written there may be a loss rather than a gain from loading in the Gain and Loss Exhibit. Obviously this does not mean that no policy has made any contribution from loading. Most policies have, in fact, done so, and therefore the form and amount of the loading factor have no relation to the aggregate gain or loss from loading as shown in the exhibit.

While the amounts of interest, loading and mortality profit appearing in the Gain and Loss Exhibit are neither absolutely nor relatively the amounts to be distributed in the dividend formula under these heads, an analysis of surplus in the form of the Gain and Loss Exhibit provides (when suitably adjusted) a general basis for determining or checking the aggregate amounts to be divided as interest, loading or mortality profit. For this purpose an analysis of the aggregate interest profit will be made, separating interest on special funds, etc., from excess interest on reserves. Excess initial expense and select mortality profit will also be computed and offset so as to show the effective amounts of divisible loading and mortality profits. Where any miscellaneous profits or losses are such as to affect the basis of distribution and the amount of divisible surplus the general extent of the appropriate increase or decrease in the respective

parts of the dividend scale may be determined from such an analysis in accordance with the judgment of the actuary.

Interest Factor

Some of the practical questions which arise in connection with the calculation of the interest factor are:

(1) How should the net earned interest rate be computed?
(2) Should the excess interest rate be applied to the initial reserve, the mean reserve or to some other function?
(3) Should any modification be made in the interest rate for particular classes of policies or for dividend accumulations, supplementary contracts or annuities?

Calculation of the Rate. The net earned rate of interest, i' depends on the definition of A and B and upon what is included in I in the formula

$$i' = \frac{2I}{A + B - I}$$

The quantities A and B may be either the admitted assets or the interest-bearing liabilities at the beginning and end of the year. In the Gain and Loss Exhibit the interest rate is based on the mean ledger assets which, under normal conditions (i.e., assuming no very great difference between book and market values of securities), will give a rate higher than that based on the admitted assets since the latter include a substantial amount of non-ledger assets. The rate based on ledger assets has no particular significance. It would, of course, be incorrect to use, for dividend purposes, the rate based on invested assets since we are not concerned for this purpose with the yield on invested funds but on the funds as a whole or, at any rate, on all funds which should be earning interest. A and B are usually defined as the interest-bearing liabilities. This excludes unpaid claims, unpaid expenses (including reserve for taxes) and unpaid dividends but includes all reserves, premiums paid in advance and the contingency fund and presumably also any other special funds of the same general character.[1] Provision is thus made for increasing these funds by the interest actually earned thereon.

[1] Weeks, T.A.S.A., IX, 310; Coburn, R.A.I.A., XII, 35.

If this interest or any part of it is not needed for that purpose it will be available to increase the dividend interest rate as will also *excess* interest earned on the reserves for non-participating contracts, if not needed to make such contracts self-supporting. The dividend interest rate may therefore properly be somewhat in excess of the actual net rate earned on total admitted assets. On the other hand, it may be considered desirable to transfer a part of excess interest earnings to an investment fluctuation fund.

It may be thought that A and B should include only liabilities upon which it is *necessary* to earn interest, thus excluding the contingency fund and other special non-interest-bearing reserves. In that case the rate would be higher and the entire increase in these funds would be provided by a transfer from surplus earnings as a whole. It is generally considered that the amount of the contingency fund should, unless under unfavorable conditions, increase as reserves increase, and it is reasonable, therefore, that part of the increase in the fund should be provided out of the interest earned thereon. At the same time the progress of the contingency fund is apt to be somewhat erratic since it is, to a large extent, a dividend equalization fund. On the whole it may be assumed that for dividend purposes the standard basis for computing the interest rate is the interest-bearing liabilities including contingency and other similar funds.

Of more importance and of greater effect on the net rate is the method of determining I, the net amount of earned interest. Special consideration must be given to the treatment of (1) accrued, overdue and capitalized interest; (2) investment profit and loss; (3) investment expenses.

Since excess interest in the dividend represents the refund of a realized profit it should evidently not include or be based upon interest the receipt of which is doubtful although such interest may be carried as a non-ledger asset. This applies chiefly to interest in arrears. The greater part of any overdue interest included as a non-ledger asset as well as any accrued interest the receipt of which is, for any reason, regarded as uncertain should be eliminated from the calculation of the dividend interest rate. Similarly, any mortgage interest which has been capitalized and which thus appears in the accounts as paid should be excluded. The general principle is that excess

interest included in the dividend must be an actual and not a paper profit.

With regard to investment profits and losses there appears to be some difference of opinion as to whether these should be applied (respectively) to increase or decrease the net interest earnings. Hunter[1] says that most companies do so. Such profits or losses may be substantial and, in that case, it would be undesirable and impracticable to make abrupt or drastic increases or decreases in the dividend interest rate, particularly if it is agreed that the rate should follow the general trend rather than the actual earnings. Presumably such a rule would apply only to the profits or losses, ordinarily relatively small, from sale or actual revaluation (i.e., adjustment of book values, other than by amortization) and not to the profits or losses resulting from changes in the difference between book and market values which are frequently large and which usually make up the greater part of the investment profit or loss as shown in the Gain and Loss Exhibit.

The better rule for use in normal times seems to be that indicated by Rhodes in the discussion to Hunter's paper. He suggests that investment profits and losses should be carried to a security fluctuation fund formed from part of all interest earnings. This is a more logical and practicable plan than to vary the net interest rate according to the accidental fluctuations of investment profit and loss. It is based on the insurance principle, part of the interest rate being regarded, as it should be, as a risk premium to cover capital losses. The same procedure may, of course, be followed where no specific security fluctuation fund is maintained, profits and losses being carried to the general contingency fund. However, when losses are large as in times of financial crisis, a reduction in dividend scale may be deemed necessary to absorb part of the losses. In such times, the companies are likely to experience not only losses from sales or revaluation, but also greater death losses and increased cash surrenders and policy loans, thus making it wise to conserve cash and to proceed carefully. Incidentally, at such a time the outstanding insurance and corresponding liabilities usually increase much more slowly than in normal times, so that the

[1] T.A.S.A., XXIV, 14.

amount of needed contingency reserves likewise increases more slowly and the interest earned on them often may be drawn on to meet investment losses.

A significant difference in the net interest rate may be caused by a change in the basis of computing investment expenses. These consist of (1) expenses chargeable specifically against gross interest earnings such as the present federal income tax, any other taxes on investments, and taxes and expenses on real estate, and (2) general overhead expense in connection with the making and management of investments and the custody of securities. While the expenses under the latter head (or the greater part thereof) are only a small percentage of the mean invested assets[1] they represent a substantial proportion of the entire investment expenses. A slight change in the basis of computation may, therefore, make a material difference in the assumed expenses.

Before leaving the question of the computation of the interest rate attention is again called to the fact that the dividend rate finally selected need not necessarily follow the fluctuations of the actual earned rate but under normal conditions should rather be based on the general trend of interest earnings so that where only slight changes occur the dividend interest rate may be kept constant for several years. Changes in the actual rate (unless the rate is modified by investment profits or losses) will not, except under most unusual circumstances, be violent or abrupt.

Function to Which the Excess Rate Should Be Applied. The next question is whether the excess interest payable should be based on the initial reserve, the mean reserve or on some other function.

In the basic contribution formula as above defined excess interest is computed on the *initial* reserve. The formula represents a debit and credit account by policy years, the contributions arising as the difference between the experience and assumed factors. Computation of excess interest for a full year on the initial reserve implies payment of claims at the *end* of the year of death whereas claims are now paid immediately upon completion of proofs of

---

[1] The New York Insurance Law (Sec. 97, Par. 2) specifies that, in computing the expense margins under the law, the total "actual investment expenses" shall not be taken as more than one-fourth of 1 per cent of the mean invested assets.

death. In practice, therefore, the use of the initial reserve overstates the excess interest earned. In his paper on Annual Dividends, Weeks[1] computed excess interest on the mean reserve instead of the initial reserve without, however, discussing the point and some other writers have done the same. Where death claims are paid immediately on proof, the cost of insurance is not deducted at the end of the policy year but is payable throughout the year so that interest is not earned on the initial reserve for the whole year. The proper adjustment of the interest factor for immediate payment of claims would seem to be to base the excess interest on the initial reserve less one half year's cost of insurance (as indicated by Bowerman[2]). The difference between this amount and the mean reserve would, however, be relatively small (being a half year's interest at the valuation rate on the initial reserve) and the excess interest on this difference would usually be negligible so that the use of the mean reserve, which is more convenient, is a satisfactory approximation. The difference between the initial reserve and the mean reserve is also small. In practice, the maximum difference per thousand on the basis of the American Experience Table with 3 per cent is about $40 so that the difference in excess interest as between initial and mean reserves would rarely be significant. While it is theoretically more correct, therefore, under modern conditions, to use the mean reserve it is not of much importance which is used. The majority of companies have retained the original basis, i.e., the initial reserve.

Modifications of the Interest Rates in Certain Cases. Modifications in the regular dividend interest rate are sometimes made as indicated in the following paragraphs. Such modifications are usually intended as a provision for expenses.

Fully Paid and Paid-up Insurance. If for single premium or limited payment policies the loading received during the premium-paying period is intended to cover the expenses for the entire duration of the contract, no special expense charge ought to be necessary after the policy is fully paid, assuming that some part of the loading has been reserved. If no provision has been made for expenses after premiums have ceased, i.e., if the dividends paid have included the whole balance of loading after

[1] T.A.S.A., IX, 310.
[2] T.I.C.A., IX, I, 85.

paying the assessed current expense only, it will be necessary to make a small reduction in the interest rate (or mortality factor) or a constant deduction per $1000 of insurance to cover expenses.

In the case of reduced paid-up insurance a somewhat larger expense charge would be justified because of the smaller average amount and the same is true for dividends on paid-up dividend additions.

A special case where some adjustment is called for is where conversion to paid-up life insurance at net rates is allowed as a settlement option on maturity of an endowment policy.

Many companies make no special adjustments of the interest factor for the various classes of paid-up insurance. Where an adjustment is made it is usually from 0.1 to 0.3 per cent.

Supplementary Contracts and Dividend Deposits. Some companies allow the same rate of interest on supplementary contracts and dividend deposits as is used in the regular dividend formula. The usual rule, however, is to allow a lower rate. The difference is small, as a rule, but differences of as much as 0.5 per cent are not uncommon and sometimes the difference is as much as 1 per cent. In addition to making provision for expenses there is a further reason for a reduction of the rate in this category in that such funds are very largely subject to withdrawal without notice and their existence, therefore, increases the need for liquid (i.e., low yielding) assets. For that reason a distinction may be made between the interest rate payable on those funds which are subject to withdrawal and those which are not, a rate 0.25 per cent higher often being allowed in the latter case.

In the case of dividend deposits the small average amount of the accounts provides a reason for a greater expense charge than on supplementary contracts and in some cases this distinction is made, although usually it is not.

Term Insurance. In the case of term insurance for periods up to ten years the reserves are so small that, as a practical matter, the interest factor may be omitted entirely.[1] For ages at issue up to fifty-five the amount of excess interest, on the most favorable assumptions, would, in most cases, amount to only a few cents. For longer term periods, particularly at the higher ages and middle durations, the interest factor may be a

[1] Flanigan, T.A.S.A., XXIV, 46.

significant part of the dividend and could not properly be omitted.

Retirement Annuities and Other "Investment" Contracts. When the general investment yield is low, business of this type increases —noticeably in the single premium forms. A large number of such contracts are taken simply to secure the benefit of the higher interest rate payable by the company as compared with that obtainable on other investments. This raises a general question of some importance in regard to the interest factor, namely, whether any distinction should be observed as between contracts effected during a period of low investment yield and those previously in force. It would probably be unnecessary and impracticable to make such a distinction as a permanent or general feature of the dividend system except, possibly, as regards very recent issues. In the case of the great majority of policies recently effected the interest portion of the dividend is relatively small so that for these it might not be worth while to make the distinction. The situation is different in regard to such contracts as single premium life or endowment policies and single premium participating annuities. Unless some adjustment is made, a considerable volume of such business is likely to be issued when current investment yields are low. This will be to the disadvantage of existing policyholders since the effect will be to reduce the company's average rate of interest by increasing the amount uninvested or invested at low rates. It is evident that the high rate of interest paid in such cases comes largely from the earnings of previously existing policies, which is clearly inequitable. Where the condition is extreme, as it has been in recent years, this would seem to be a point deserving serious consideration.

Mortality Factor

The contribution to surplus on account of favorable mortality experience is $(q - q')(1 - V)$ where q' is the "experience" rate of mortality and q the rate by the valuation table. As we shall see presently, the mortality factor of the dividend is not usually calculated in exactly this form but, before considering the other forms in which mortality profit may be expressed, we shall consider some of the primary questions which arise in regard to

the actual and expected rates of mortality. Among these are the following:

(1) Should the experience rate of mortality be on a select or an ultimate basis?
(2) Should the actual experience of the company be the basis and, if not, what is meant by the "experience" rate?
(3) Should the same table of experience rates apply to all classes of policies?

Select or Ultimate Basis. Theoretically the experience rate of mortality should be on the actual, i.e., the *select*, basis irrespective of whether premiums and reserves were computed by a select table or not. This would mean in practice a mortality factor in the dividend formula in the form $(q_{x+n-1} - q'_{[x]+n-1})(1 - {}_nV_x)$. This formula could be applied in practical operation in the form of a graded percentage of the ultimate cost of insurance, the percentage depending on both age at issue and duration. In practice such a form is almost never used. A recent tabulation of the dividend formulas of twenty-five important companies shows that only one of them based the experience rate of the mortality factor on select experience. A select formula will, of course, give comparatively large mortality profits to recently effected policies but with a sharp reduction in successive early policy years.

Weeks showed that select mortality profit in the early years together with profit from lapse and surrender was approximately equivalent to the excess initial expense on a normal basis. While there is no necessary relation between select mortality profit and initial expense, most companies have adopted the convenient rule that select mortality profit shall be applied toward payment of expenses and not included in the dividend. This, of course, affects the basis of the loading factor of the dividend as well as the mortality factor, as we shall see later. The elimination of select mortality profit under existing conditions does not mean that there is no mortality contribution in the dividend formula during the "select period" but that such contribution is limited to the difference between the assumed and experienced ultimate rates according to attained age only. Weeks assumed that there would be no such difference or that it would be very small and that the mortality factor could

therefore be omitted entirely from the dividend formula. Under present conditions the difference between the tabular and experienced ultimate rate is substantial and a mortality factor in the dividend formula is essential whether the experience rate is select or ultimate.

The usual basis of the mortality factor, irrespective of duration is, therefore $(q - q')(1 - V)$, where q' is the company's ultimate experience rate. This is generally computed as a percentage of the tabular cost of insurance, or say, $aq(1 - V)$, where

$$aq = (q - q').$$

The percentage, a, is normally a variable and under present conditions as to valuation bases and mortality experience decreases with attained age.

The effect on the mortality factor of the dividend of using ultimate instead of select experience rates is illustrated in the following table which shows mortality profit based on experience corresponding to the American Men Table with reserves on the American Experience Table at 3 per cent. The table does *not* illustrate current *amounts* of mortality profit—merely the approximate relation and incidence of select as compared with ultimate mortality gains.

ORDINARY LIFE—$1,000

End of Year	Age at Issue					
	30		40		50	
	A	*B*	*A*	*B*	*A*	*B*
1	5.36	3.93	5.99	3.89	6.83	2.15
2	4.48	3.94	4.66	3.72	4.52	1.90
3	4.35	3.96	4.33	3.53	3.69	1.64
4	4.30	3.95	4.06	3.34	3.14	1.40
5	4.14	3.92	3.60	3.12	2.20	1.15

$A = (q_{x+n-1}$ Am. Exp. $- q_{[x]+n-1}$ AM$)(1 - {}_nV_x)$.
$B = (q_{x+n-1}$ Am. Exp. $- q_{x+n-1}$ AM$^{(5)})(1 - {}_nV_x)$.

The differences between columns *A* and *B* represent the amounts of mortality profit which would (under the assumed experience) be applied to initial expense if the mortality factor is on the ultimate basis. The table illustrates also the more

rapid decreases in mortality profit in the very early years on the select basis. Such a rapid decrease is an undesirable feature from a practical standpoint since it tends to reduce, or even eliminate, the annual increase in the dividend in the first few years unless the true incidence of expense is also taken into account. There is thus an additional practical reason for basing the dividend on ultimate mortality rates. The difference between select and ultimate mortality savings will probably not provide more than a part of the excess initial expense, and the whole of such expense, or even any substantial proportion thereof, could not be charged against mortality profit without unduly decreasing the mortality factors.

Meaning of Experience Rate. The next question is whether q' should be the actual rate experienced by the company and, if not, how the "experience rate" should be determined. It is clear that even in the largest companies, the use of the actual experience of a single year would result in undesirable fluctuations of the mortality factor as between one year and another, while in moderate sized or small companies these fluctuations would be correspondingly accentuated. In practice it is necessary to compare the combined experience of several years (sufficient to give reasonably smooth results in small groups of ages) with the mortality rates by the valuation table. This will provide a basic scale of the ratio of q' to q which may then be modified as necessary according to the trend of current experience. In small companies where fluctuations are more considerable the basis of the experience mortality rate is necessarily more arbitrary. In some cases it may be advisable to use the most recent available combined experience of comparable companies as a general guide, modifying the ratios by any reasonable adjustment, the scale depending largely on the aggregate amount of divisible surplus available. This may seem to be getting away from the fundamental basis of the contribution plan and, to some extent, that is true. The fact is, of course, that the plan cannot be rigidly applied in such circumstances.

When the contribution formula was first introduced by the Mutual Life in 1863 it was not the intention that the actual experience of the company should be reproduced year by year in the dividend formula. Although the contribution plan was originated at a time when a new mortality experience of the

company had just been completed[1] which was, in fact, used as the basis for q', that table was based on the entire experience of the company since its establishment—not the five years of the dividend period—and the same values of q' were, in fact, used for several successive distributions. It must be understood that in the practical application of the system it is not possible (except, perhaps, in the case of the interest factor) to make more than an approximation to the ideal represented by the formula.

Mortality Rate by Plan and Amount. It is well known that mortality experience varies by plan of insurance, being in general more favorable as the rate of premium increases. With the exceptions noted below it is not, however, usual to make any distinction by plan in the experience mortality rate of the dividend formula. Because of differences in age distribution and proportion of new business, aggregate ratios of the relative mortality by classes would not be sufficient to distinguish the different experience scales which would require separate investigation. A further difficulty lies in the fact that there is no definite dividing line between one class of plans and another. The degree of accuracy attainable does not, in general, justify the attempt to make such distinctions.

The exceptions are "Preferred Risk" policies and term insurance. Where preferred risk policies are issued on a definitely higher standard of selection an adjustment of the mortality factor is appropriate, the extent of which may depend on the selection rules as well as on the mortality experience. Term insurance is a distinct class in which, as a rule, the mortality is definitely above normal. The higher basis of experience mortality usually adopted for term insurance is often subject to a further addition to cover the policy's share of the cost of the extra mortality on term policies changed at attained age, arising from anti-selection at the time of change. This is a legitimate charge against the dividends on the term policy, inasmuch as standard dividends are paid on the new life or endowment policy to which the term policy is converted. A still higher basis of experience mortality is often adopted for extended term insurance where that is granted on a participating basis.

---

[1] The American Experience Table.

The mortality rate is also usually higher for policies of large amount but no distinction is made in the dividend formula. There are obvious practical difficulties in the way of differentiating dividends by amounts of insurance while the lower rate of expense provides a compensating factor.

Mortality Factor Based on Amount at Risk. As indicated above, the usual basis for the experience mortality is the ultimate rate. When applied in conjunction with a valuation table not based on recent experience, such as the American Experience Table, the mortality saving is (because of the incidence of improvement) normally expressed as a decreasing percentage of the tabular cost of insurance.

Some companies express the mortality factor in the form $k(1 - V)$, i.e., as a constant multiplied by the amount at risk. Since $k(1 - V) = \frac{k}{q} \times q(1 - V)$, this is equivalent to a decreasing percentage of the tabular cost of insurance, the percentages being 100 multiplied by the successive values of k/q.

It is evident that such a form can (approximately) express the experience rates at all ages only by accident and that changes in the experience rates will usually not be capable of being reproduced throughout by a change in the value of k because for each value of k there is only one set of values of k/q. Where, therefore, it is desired that the mortality factor should be calculated as closely as possible in accordance with the experience rates this form is unsuitable. On the other hand, if it is remembered that the actual experience cannot, in any case, be followed very closely the results may be near enough. The formula, in general, gives values which are too great at the higher ages on the basis of current experience as compared with the American Experience Table.[1] The practical advantages of this type of formula over the usual basis (i.e., decreasing percentages of tabular cost) are not very apparent. There is possibly some small advantage in greater simplicity but this is much more than offset by the loss in accuracy and the absence of flexibility. Kattell proposed[2] a mortality factor in the form $k_1(1 - V) - k_2q(1 - V)$, i.e., a constant times the amount at

[1] See J.I.A., LXII, 256.
[2] T.A.S.A., XXII, 418.

risk less a constant times the tabular cost. Such a form is possible where the experience rates are such that the relation $\frac{q' + k_1}{q} = 1 + k_2$ holds good, approximately, for the greater part of the table. This form avoids the inflexibility of the "amount at risk" method but is evidently of limited applicability. It is also less simple to apply than the normal basis of reducing percentage of cost of insurance so that the reasons for adopting it are not clear.

At one time many companies omitted the mortality factor in the dividend formula because of the small difference between tabular and actual ultimate mortality rates. As mortality experience improved, the necessity for a separate mortality factor became generally recognized. Some companies at first used a *constant* percentage of the tabular cost of insurance but with the increasing importance of mortality profit a more accurate basis was required and the necessity of grading the percentage by age was gradually realized. It was then that the idea of adopting a mortality factor in the form of a constant times the amount at risk was proposed. This represented a considerable increase in accuracy as compared with the constant percentage basis but it is evident that a decreasing percentage of the cost of insurance is almost as simple and is certainly more accurate and flexible. Both methods are in use at the present time, the majority of companies using the decreasing percentage basis.

As shown in Appendix B, a rather wide variation exists in the numerical values in the tables of cost percentages of different companies—wider than might be expected from normal variations of mortality experience. This may be due partly to the different types of formula for the mortality factor as described above and also to the fact that in some cases part of the necessary final adjustment of the dividend formula (to reproduce the aggregate divisible surplus) may be made in the mortality factor. In some cases it may arise also from the fact that the contributions from mortality, loading and other sources apart from interest are often considered more or less together in determining the dividend formula, so that each factor in the formula is not entirely self-contained.

In Canada about one-quarter of the companies are using a two-factor scale combining mortality and loading factors and at least one large company in the United States does likewise.

Loading Factor

The loading factor of the dividend, consisting of the loading less the amount of expenses assessed against the policy (increased in some cases by one year's interest at the dividend rate) is of a different character from the other two factors. The interest and mortality factors may be considered, in a broad sense at least, as being capable of exact determination. The rate of interest and the reserve to which it applies are definitely ascertainable quantities as are the experience rate of ultimate mortality and the net amount at risk and although, as we have seen, considerable variation is possible in the details of calculation, these factors can reasonably be considered as representing amounts of surplus contributed by individual policies.

The contribution from loading, however, cannot be arrived at by any universally applicable or well-defined formula or rule and there is, in practice, a wide variety of *methods* of assessing expenses for dividend purposes as shown in Appendix B. This arises chiefly because of (1) the fact that many expenses are not related directly either to the amount of insurance or to the amount of the premium and can be assessed only by some rule depending largely on individual opinion or judgment; (2) the different views which are possible in regard to the treatment of initial expense; (3) different views as to the degree of accuracy which should be aimed at; and (4) the fact, already indicated, that contributions from loading and mortality are often considered more or less together.

An appreciation of some of the problems connected with the loading factor will best be obtained by considering first the different types of expense charge in common use.

Types of Formula for Expense Charge. An examination of the forms of expense charges in the dividend formulas of the principal companies shows a considerable diversity, the number of variations being almost equal to the number of companies. This lack of uniformity indicates the special character of this element of the dividend. The expense charge formulas may be

roughly classified according as they are based on (1) the premium payable; (2) the loading; or (3) the premium payable and the premium for an ordinary life policy. In each case the expense charge may or may not include a constant for each $1,000 of insurance and may or may not be modified in some way so as to provide an effective expense charge decreasing with duration.

Constant Percentage of Premium. A constant percentage of the premium payable results in a relatively greater assessment of general overhead and other "constant" expenses against policies issued at the higher ages or on the higher priced plans. For this reason the percentage is sometimes graded by plan. This modification, while an improvement, merely reduces the objections to the method, the only advantage or justification of which is simplicity. It is evident that even where the loading is a percentage of the gross premium, total expenses are not directly proportionate to the amount of the premium. Some expenses are independent of the premium while renewal commissions usually cease after a limited period. Serious inequity may therefore result from such a rule. Very few companies now compute the expense charge in this way.

Constant Percentage of Loading. A constant percentage of the loading is open to almost the same objection as a constant percentage of the premium with the additional complication caused by the necessity of a different constant for new issues in case of the adoption of a new loading basis. Where the loading is a constant percentage of the gross premium, as it frequently is for ordinary life policies, the two methods are, of course, the same, while in other cases there may be very little difference in result as between the two methods because of a system of loading which depends largely on percentage factors. It is sometimes said that the expense charge should follow the system of loading but, as a matter of fact, expenses have, as a rule, no direct relation to the amount of loading. In the case of participating policies the loading is not computed directly as the amount necessary to pay expenses but is frequently fixed at an amount sufficient not only to pay expenses but to provide also under normal conditions a certain amount of surplus for distribution. Loadings on participating policies may therefore have no relation to expenses such as would justify an expense charge related directly to the amount or method of loading.

The chief defect of both of these bases is that they do not distinguish between expenses which depend on the amount of the premium and those which do not. Even where the percentages are graded by plan the tendency is to charge too much against higher premium rate policies.

Percentage and Constant. An expense charge based on a percentage of the premium payable plus a constant for each $1,000 of insurance, although not the most common system, is suggested as probably the most satisfactory, both theoretically and practically. So far as percentage renewal expenses are concerned the use of a uniform percentage for all plans is probably sufficiently accurate but, if thought necessary, modifications can be introduced to allow for varying rates of renewal expense by plan or duration.

However, a constant expense charge for each $1,000 of insurance regardless of age, plan or duration cannot be justified on theoretical grounds. Most of the expenses other than commissions are, as already indicated, on a per policy basis. Consequently, the inclusion of a constant in the expense charge, while tending to correct the overcharge against the higher premium policies on the percentage basis, results in inequity as between policies of different amounts. This cannot be entirely avoided since it is impossible to vary either premium rates or dividend rates for the same plans and ages according to amount of insurance. It is, however, possible to minimize the inequity of a constant per $1,000 by grading the constant with reference to the average amount of insurance both by plan and age. In fact, there is no good reason why this should not always be done.

Moir[1] describes a simple method of arriving at the factors for a percentage and constant expense charge on renewal business. The first year is treated separately. Initial expenses, consisting chiefly of first year commissions, advertising and the greater part of agency expense are first eliminated. The remainder of the insurance expenses are then, in effect, divided into those depending on the premium, and the balance. The former are expressed as a percentage of renewal premiums and the latter as a constant per $1,000 of insurance in force (which should

[1] T.A.S.A., X, 200.

be the renewal insurance, i.e., excluding new business) thus giving the two factors of the expense charge.

Moir regarded this rule as essentially just and fair to all but, as we have seen, a simple percentage and constant applied without modification to all plans, ages and durations cannot be regarded as entirely equitable, particularly as modifications, such as are indicated above, can very readily be made.

Modifications in the percentage factor will be suitable to account for different rates of renewal expense by plan. Modifications in the "constant" factor may be used to allow for different average amounts of insurance either by plan or by age at issue or to effect a decrease in the expense charge by duration. Some reduction by duration is, in general, reasonable since renewal commissions cease after a limited period. Other renewal expenses do not decrease but as a practical matter a decreasing expense charge is desirable. In any case, no formula can be more than an approximation to the facts.

It may be noted at this point that the rules here discussed for expense charges relate entirely to premium-paying policies. As we have seen in discussing the other dividend factors the expenses of paid-up insurance, if not specially provided for in advance, must be taken care of by adjustments in the mortality or interest factors or by deduction of an expense charge as, for example, a constant per $1000 of insurance. Where that is done the amount so provided should be deducted from the renewal insurance expenses before calculating the renewal expense charges on premium-paying policies.

Weeks' System. In Weeks' paper "A Practical Rule for Calculating Annual Dividends,"[1] a new basis for computing expense charges was proposed. This method is important since the general principle involved has been very widely adopted and is employed by many companies at the present time. For the present, reference will be made only to renewal expenses.

Weeks expressed the renewal expense charge entirely as a percentage of premiums using, however, one rate for the "ordinary life" premium and a lower rate for the "higher premium element," i.e., the excess of the premium payable over the ordinary life premium. There is thus no "constant" in his

[1] T.A.S.A., IX, 310.

expense charge which may be expressed as

$$k_1 P' + k_2(\pi' - P')$$

where π' is the gross premium payable and P' the gross premium for an ordinary life policy at the same age. Weeks assumed, arbitrarily, that the higher premium element was subject to one-half the rate of expense chargeable on the ordinary life element, i.e., that $k_2 = \frac{1}{2}k_1$, in which case the expense charge becomes

$$k_1 \frac{\pi' + P'}{2}$$

k_1 being the rate of renewal expense applicable to the ordinary life element. Reference should be made to the paper for a description of the procedure in calculating the value of k_1 (there denoted by e'').

The general principle of this system of assessing expenses seems to be open to question. Apart from the arbitrary relationship between k_1 and k_2 there is no apparent reason for supposing that all expenses can be fairly assessed against premiums. Weeks does not discuss the point, the assessment of expenses being merely incidental to the general purpose of the paper. In a rough way the method may be said to follow the customary system of loading premiums but, as we have seen, there is no very good reason why the assessment of expenses should necessarily follow the method of loading.

By making $k_2 = \frac{1}{2}k_1$, the result is an expense charge which is the sum of equal percentages of the premium payable and the ordinary life premium. If $k_2 > \frac{1}{2}k_1$ (as suggested in the discussion to Weeks' paper) the expense charge is a percentage of the ordinary life premium and a larger percentage of the premium payable. For example, if $k_2 = \frac{2}{3}k_1$, the expense charge is $\frac{1}{3}k_1 P' + \frac{2}{3}k_1\pi'$.

Many companies have adopted an expense charge which is expressed as a percentage of the ordinary life premium and a percentage of the premium payable but in most cases there is no indication of any relationship between the two percentages such as is indicated above. Since

$$aP' + b\pi' = (a + b)P' + b(\pi' - P'),$$

all such methods are of the same general character as Weeks' method, i.e., a percentage of the ordinary life premium plus a *lower* percentage of the *excess* of the premium payable over the ordinary life premium whatever values are assigned to a and b. Since the expense factor is frequently the "balancing item" of the dividend formula it is to be expected that considerable differences will be found as between different companies even where the same general basis of expense charges is used. At any rate, there is very little apparent uniformity in regard to the values of a and b or in the relation between them.

Treatment of First Year Expenses. In the first year of insurance the expenses are normally much greater than the loading. The excess, under ordinary conditions, is greater than the combined profit from interest and mortality in the first year so that the "dividend" calculated by the regular contribution formula would be negative. However, there are two sources of surplus which are not usually included in the regular dividend formula, the profits from which are, in that case, available in reduction of initial expense. These are profit from select, as distinguished from ultimate, mortality and profit from lapse and surrender. The latter, as Moir points out, may reasonably be considered as an offset to initial expense because in order to maintain the company as a going concern it is necessary or, at any rate, desirable to replace a lapsing policyholder by a new one. The profit from select mortality is a direct result of the amount spent in the first year for medical examination and inspection and can, therefore, reasonably be considered also a legitimate offset to initial expense.

If these profits are considered as an offset to excess initial expense and if together they equal or exceed the *excess* initial expenses the same dividend formula can be applied to the first year as to subsequent years. If, as would usually be the case, the combined profit from these sources is *less* than the excess initial expense a lower first year dividend is indicated, or possibly no dividend.

From another point of view excess initial expense may be regarded as an expense of the company as a whole, not specifically chargeable against first year premiums. This view is based on the fact that new business is necessary for the continued existence of the company so that its cost is properly chargeable,

to a considerable extent at least, against existing as well as new policyholders. From this standpoint any part of the initial expense in excess of select mortality and lapse and surrender profit can properly be treated as a general expense and the regular dividend formula applied from the first year.

Weeks held that no dividend should be paid in the first year since the premiums of new policyholders, after paying expenses and claims, were insufficient to supply even the necessary reserves much less to create surplus from which dividends could be paid. This view is clearly right if the whole of the first year expense is to be charged against the new policyholders, but not otherwise. Moreover, Weeks does not apply the same reasoning to the second year. An examination of the detailed accumulations on pages 312–313 of his paper shows that the deficit at the end of the second year is greater than the dividends paid. In other words, a deficit would still have existed if no dividends had been paid in that year. The fact is that if it be held that dividends should not be paid until surplus over the reserve is created from a policy's own accumulations, i.e., until they are "earned," it would not normally be possible, even allowing for select mortality and lapse profit, to pay any dividends until the third or fourth year in many cases.

The two views as to the treatment of excess initial expense indicated above are represented respectively by the companies which do and which do not pay a first year dividend. Those which do, charge the balance of such expense (after crediting select mortality and lapse profit) against all policyholders. The first year dividend is usually on the same basis as in subsequent years but may be on a somewhat lower basis as a result of some adjustment of the expense factor. In most cases payment of the first dividend is contingent upon payment of the premium due at the beginning of the second year. This condition is necessary in practice (although not in theory) because in many cases the dividend payable exceeds the amount of the fund on hand which has arisen from the policyholder's own payments (i.e., charging all initial expense) so that payment of the dividend followed by lapse would result in an actual loss.

Where no dividend is paid at the end of the first year the assumption is that excess first year expense (reduced as above) is charged largely, if not entirely, against first year business. Logi-

cally this view would (as shown above) require modifications in some cases in the second year as well as the first, but from a practical standpoint the payment of dividends on the regular scale from the second year is a satisfactory compromise between the requirements of theory and practice.

Analysis of Expenses. Any method of *assessing* expenses, i.e., of charging expenses against different classes of policies in the dividend formula, must be based on an *analysis* of expenses. An analysis of expenses may be carried out with any desired degree of refinement, but, as we have already seen, any analysis, however elaborate, must be partly arbitrary and must depend to some extent on judgment. So far as the dividend formula is concerned, it is impracticable to adopt an elaborate system of expense charges involving all the elements into which expenses might be analysed. An elaborate analysis of expenses may, however, be of real value in connection with asset share studies (as described later in this chapter) for the purpose of checking the general equity of the dividend scale. The apparent accuracy of a complicated system of expense charges based on an elaborate analysis of expenses is open to question. This would be so because of the impossibility of distinguishing expense rates by amount in the dividend formula, if for no other reason.

Apart from asset share studies, therefore, analyses of expenses may be limited to an allocation (1) as between different classes of business such as ordinary, group, industrial, participating and non-participating insurance, etc.; (2) as between new business and renewal business; (3) as between expenses depending on the premium and those not depending on the premium; and (4) as between durations or ages at issue. The first of these is, of course, essential for any rational assessment of expenses. The others are needed only as a general guide in selecting appropriate expense factors. In all cases certain items of expense are, because of their nature, specifically chargeable and it is only in regard to items which are not specifically chargeable that any difficulty arises.

So far as a separation by different classes of business is concerned the most satisfactory system is to maintain separate expense accounts for all expenses which can be definitely allocated by class of business such, for example, as commissions and cost of medical examination and inspection. Items of expense charge-

able to any one class of business are then carried directly to the expense account of that class. All other items of expense must be distributed among the different classes of business by *rule*.

The rules for distributing such expenses will depend in each case largely on the character or purpose of the expense. For some items, such as taxes on premiums or general taxes, the best basis of allocation is fairly obvious while for others it must depend almost entirely on individual judgment. A good general idea of the problems involved in such a distribution of expenses and of the different methods of dealing with them will be obtained from a study of the paper by Turnbull on "Separation of Accounts,"[1] from the first part of Elston's paper "Expense Investigations, etc."[2] and from the report on comparative expenses of life insurance companies contained in the 1932 Proceedings of the Life Office Management Association. Particular attention is directed to Elston's warning against the indiscriminate apportionment of miscellaneous expenses on the basis of premium income. Because of the investment feature of life insurance premiums such a basis is likely to throw too great a proportion of expenses on the life insurance business.

The following extracts from the report of the New York insurance department on methods used by one large company for distributing expenses by classes of business will give some further indication of the general procedure followed:

"Separate accounts are kept for ordinary, industrial, group life including group annuities, group accident and health and personal accident and health. . . .

"The six main departments are charged with the respective parts of certain expenses which have been incurred directly on their behalf. A distribution sheet with supplementary schedules for salaries and general Home Office expense is prepared each month in the auditor's division in order to allocate the remainder and other items which must be pro-rated. The bases of the ratios which are used for this purpose depend upon the type of expense and the percentages applicable to the several departments are generally recomputed half-yearly or monthly. For certain items the allocation is made in accordance with the results of statistical studies and changes are adopted from time to time when revisions become advisable. . . .

---

[1] T.A.S.A., XXVIII, 216.
[2] T.A.S.A., XXIII, 83.

"Pro-rates are generally determined on bases consistent with the type of expense to which they apply and the ratios are changed at frequent intervals. Periodical surveys at the Home Office and in the field furnish new data regarding the incidence of expenses and serve as a basis for modification of prorates. The separation of group expenses follows the general plan for the allocation to departments."

A separation of expenses between new and renewal business or as between renewal expenses chargeable as a "percentage" of premiums or as a "constant" on sums insured involves the same general problems. Certain items are definitely allocable, others must be allocated by rule.

An elaborate expense analysis such as would be of value in accumulating asset shares for representative plans, ages and durations involves separate consideration and apportionment of each item of expense in such a way as to arrive at the costs of issue and maintenance, taking into account the different types of expense which depend on the premium, the sum insured and the number of policies issued and in force. A discussion of this method of analysis in relation to dividend distribution is contained in the supplement. (See page 137.)

Comparison with Previous Dividend Scale. As already indicated, one of the considerations in the determination of a new dividend scale is the desirability of making no radical departure from the previous scale. The general satisfaction of policyholders demands that on no group of policies shall dividend results fall materially short of the illustrations at the time of issue, if it can be avoided. Moreover, if the total distribution is to be increased, it is desirable that no policy receive a smaller dividend than was paid the previous year unless its dividend would have decreased under the former scale. These objectionable results are quite likely to occur in determining the factors of a new scale by the foregoing methods without regard to the previous scale, if previous dividend scales have not been determined by the same method or if any material change is made in the method of dividend analysis.

When it is obvious that no large leparture from the previous scale is necessary, it is frequently the practice to determine first how much the total distribution should be increased or decreased and then to make such changes in one or more factors of the pre-

vious scale as will produce this result. The factors to be changed will be those in which the old dividend scale is failing to reflect current experience.

The dividends for various plans, ages and durations produced by the proposed modification will be compared with those under the previous scale. The net costs (premiums less dividends) will be compared with those of other companies in the same class. Unless there are very compelling reasons, it is not desirable for a company to have its net costs on any plans or ages distinctly out of line with those of other companies, because of the effect on agency morale where net costs are distinctly higher and because of the excessive proportion of new business which is likely to be written where net costs are distinctly lower.

Post-mortem Dividends

The policy contract usually provides for a post-mortem dividend payable at death and covering the period between the preceding policy anniversary and the date of death.

The post-mortem dividend is computed in one of the following ways:

(1) Loading and mortality factors for a full year; pro-rata part of interest factor;
(2) Loading factor in full; pro-rata of balance of dividend;
(3) Pro-rata part of whole dividend;
(4) Whole of current dividend.

The first two of these methods are more correct, theoretically, than the other two but are not so simple in application and are, therefore, less practical. It is desirable, particularly where death claims are paid at branch offices, that the rule for ascertaining the post-mortem dividend be on a basis which can be easily applied. The first method, which allows the mortality factor in full, is more correct than the second which assumes only a fractional exposure in the year of death. Where the loading factor in the dividend formula is increased by one year's interest a small adjustment might be deemed necessary.

The third rule is a simple one which is easily understood and easily applied and involves no great injustice where all are treated alike. This is probably the most common method in practice.

Payment of the full dividend for the year of death can only be justified on the basis of practical convenience or on the theory that death claims are paid at the end of the year of death, the loss of interest which arises because they are not so paid being treated as a general loss in surplus. Where fractional premiums payable after the date of death are not deducted from the amount payable an adjustment of the post-mortem dividend would, of course, be required.

Where death takes place after the policy anniversary the post-mortem dividend will usually be based on the current scale until the scale for the dividend which would be payable on the next anniversary is determined. As a practical matter it might be reasonable to use the current scale for the whole dividend year. In this way all death claims during the same dividend year, which usually coincides with the calendar year, will be treated consistently. However, where a change in scale makes a material difference in the dividend which would be payable on the next anniversary the company might not feel justified in applying such a rule after the new scale had been definitely decided upon or announced.

General Equity of the Dividend Scale

Before the dividend scale is finally adopted it is desirable that a general test be made of its equity as between different plans of insurance, ages at issue and durations.

Asset Share Accumulations. The usual method of testing the general equity of the dividend scale is to compute tables of asset shares for each of the principal plans of insurance at quinquennial or decennial ages. The asset share is simply the net accumulated excess of actual receipts over payments based on experience rates of interest, mortality, expense, termination and surrender allowance. After deducting dividends on the proposed basis and terminal reserves on the valuation basis the balance (positive or negative) indicates at each duration the approximate total amount contributed to the general contingency funds or surplus. Assuming all the factors involved to be sufficiently accurate such tables will indicate the plans, ages or durations at which dividends are too large or too small.

Asset shares as applied to annual dividend calculations do not ordinarily constitute a method of dividend calculation but rather

a method of verifying the general fairness of a given dividend scale. An asset share system for annual dividends would be one in which the dividends were produced by an accumulative process deducting reserves and contingency fund rather than by computing the separate contributions from different sources. One approach to it is, perhaps, the method described by Gore,[1] based on separation of all receipts and disbursements by broad groups of business. Another angle of the asset share approach to the problem of surplus distribution is discussed in the supplement.

The most obvious way to construct a table of asset shares is by accumulating, for each group, on a radix of say, 10,000 policies, the actual receipts and payments (including dividends) on the experience basis thus showing for each duration the total fund or asset share on hand, i.e., the reserve plus surplus. Each year the fund is increased by gross premiums and interest and decreased by expenses, death claims, surrender values and dividends. This is the method described by Weeks[2] and which was used by him for computing the amounts of deferred dividends.[3] This method is laborious in that it requires a record both of numbers of policies and amounts of insurance and is cumbersome because of the large figures involved. A further objection when applied as a check of annual dividends is that no comparison is furnished of the respective annual contributions to surplus from different sources with the corresponding elements of the dividend scale.

The same process may be carried out on a unit basis showing the accumulation per $1,000 of insurance in force, thus greatly reducing the arithmetical work involved. The fund in that case will be increased by gross premium, by interest, by termination profit (based on experience non-renewal rate and amount of surrender charge) and by the share of contingency fund released (asset share less reserve) on account of terminated policies and decreased by expenses, by actual cost of insurance and by dividends. This method, as in the group method, shows the total (accumulated) amount of undivided surplus existing at each duration.

A better method is the one described by Bowerman in his paper "Contribution to Dividends."[4] The general basis of this

[1] T.A.S.A., XXIV, 42.
[2] T.A.S.A., IX, 93.
[3] See p. 76.
[4] T.I.C.A., IX, 1, 78.

method is to compute for the individual policy on a $1,000 basis the actual contributions to surplus from each source in each year. These are (1) excess interest; (2) loading less expenses, increased by one year's interest; (3) mortality saving; (4) surrender profit, i.e., surrender charge multiplied by rate of non-renewal; and (5) undivided surplus at end of previous year increased by one year's interest. Deduction from this total of the actual dividend apportioned shows the amount of undivided surplus remaining to be carried forward to the next year. By this method each element of the dividend scale can be compared with the corresponding "contribution" and the sources and amounts of the policy's contribution to the general contingency fund are also shown.

In considering the value of asset share calculations as a general check on dividends it is to be noted that some of the factors involved are necessarily of an approximate character. In smaller companies the rate of mortality by ages at issue and durations can hardly be based on the company's experience but, rather, roughly related to it, and termination rates graduated by plan and age at issue may be still less determinable. The expense factors, as we have seen, must depend to some extent on the opinion of the actuary. The amounts of the asset shares may therefore be varied, possibly to a material extent, according to the methods and judgment of the computer. It might be thought that, as the interest and mortality factors of the dividend presumably require no verification, there is not much point in undertaking such elaborate calculations. Such a view is, however, not justified. The dividend formula deals directly with only some of the elements involved, offsetting and eliminating others, and an independent calculation which takes direct account of the incidence and amount of all the sources of surplus, even in an admittedly imperfect way, is evidently of value. Moreover, as already emphasized, the dividend scale, while *based* on the rates of surplus earnings, must be modified in accordance with the total amount available for distribution. Such a general check as is furnished by the asset share process should therefore be regarded as essential.

Relation between Dividends and Cash Values. Asset share accumulations have a further and at least equally important function in that they provide a basis for coordinating the scale

of dividends with the scale of cash surrender values. It is generally admitted that a withdrawing policyholder should not receive as a cash surrender value more than the net accumulated balance of receipts over payments (including dividends) under his contract. In fact, there are good reasons why he should not receive as much as that. While this principle may be true it must be admitted that the definition of the "net balance" depends to some extent on personal judgment because of such factors as the method of assessing expenses, and more particularly first year expenses. Whatever may be the principle of dealing with such elements it is, however, evident that there should be an intimate relationship between cash values and dividends and that since the scale of cash values is determined prior to the issue of the policy the practical situation is that the dividend scale is, or should be, dependent on the scale of cash values. The desirability and, in fact, the necessity of checking the dividend scale by an asset share system is therefore apparent since adjustments, particularly at the earlier durations, may be required because of the scale of cash values, irrespective of the rates of surplus earnings.

Under ideal conditions surrender charges would be based, in general, on asset share accumulations and would vary by plan and even by age as well as by duration, but under practical conditions some more simple, if more arbitrary basis, is usually employed. Consequently it cannot be assumed that the dividend scale can be based solely on interest, mortality and expense rates without reference to the cash values payable.

Criticisms of the Contribution Plan

The contribution plan has been criticized chiefly on three grounds: (1) it is too laborious; (2) it can achieve only "partial equity" so that its accuracy is more apparent than real; (3) its application may be seriously affected by a change of valuation basis. Other criticisms will be mentioned later.

The student should consider these criticisms with an open mind realizing that the contribution plan, in its practical application, is far from perfect although the underlying principle is theoretically sound.

The contribution plan admittedly involves more work than most other systems of a more arbitrary or simpler character.

Whether the additional labor is or is not to be regarded as prohibitive depends on the degree of fairness obtainable and the minimum degree of fairness which it is considered essential to attain. It would seem that if a substantial increase in equity as compared with simpler systems can be attained at the expense of a *practicable* increase in work then the additional work should be undertaken. What constitutes a reasonable or practical amount of work is a matter of opinion. British actuaries seem to be united in the opinion that the contribution plan is "complicated and cumbersome" and although contribution plans of a much simplified character have been, and are being, used by a few British companies, no company there has, so far as we are aware, adopted the contribution plan as understood in America. One reason for this is that in Great Britain valuation methods are almost invariably based on a one-way classification, e.g., by attained age (whole life insurance) or by term of insurance (endowment policies), whereas the contribution plan requires classification by age at issue and duration so that its adoption in such circumstances would require almost revolutionary changes in long-established valuation procedure. Another related reason (to which further reference will be made in a later chapter) is that in Great Britain the valuation basis is seldom the same as the basis upon which the premiums were computed whereas in America these are usually identical.

The argument that the contribution plan can achieve only partial equity and that its *apparent* accuracy is therefore misleading is much more worthy of consideration. It must be admitted that, however perfect the plan may be in theory, the degree of equity attainable in any practicable application of the plan falls far short of complete equity. Dr. T. B. Sprague objected to the method[1] because it did not take into account the difference between select and ultimate mortality rates and also because of Homans' failure in the original description of the plan to provide a practical rule for the assessment of expenses. These are legitimate criticisms of the practical application of the method rather than of the method itself. A distinction between select and ultimate mortality rates can be and has, in fact, been observed in applying the plan, while it is not impossible to arrive

[1] J.I.A., XIV, 328.

of dividends with the scale of cash surrender values. It is generally admitted that a withdrawing policyholder should not receive as a cash surrender value more than the net accumulated balance of receipts over payments (including dividends) under his contract. In fact, there are good reasons why he should not receive as much as that. While this principle may be true it must be admitted that the definition of the "net balance" depends to some extent on personal judgment because of such factors as the method of assessing expenses, and more particularly first year expenses. Whatever may be the principle of dealing with such elements it is, however, evident that there should be an intimate relationship between cash values and dividends and that since the scale of cash values is determined prior to the issue of the policy the practical situation is that the dividend scale is, or should be, dependent on the scale of cash values. The desirability and, in fact, the necessity of checking the dividend scale by an asset share system is therefore apparent since adjustments, particularly at the earlier durations, may be required because of the scale of cash values, irrespective of the rates of surplus earnings.

Under ideal conditions surrender charges would be based, in general, on asset share accumulations and would vary by plan and even by age as well as by duration, but under practical conditions some more simple, if more arbitrary basis, is usually employed. Consequently it cannot be assumed that the dividend scale can be based solely on interest, mortality and expense rates without reference to the cash values payable.

Criticisms of the Contribution Plan

The contribution plan has been criticized chiefly on three grounds: (1) it is too laborious; (2) it can achieve only "partial equity" so that its accuracy is more apparent than real; (3) its application may be seriously affected by a change of valuation basis. Other criticisms will be mentioned later.

The student should consider these criticisms with an open mind realizing that the contribution plan, in its practical application, is far from perfect although the underlying principle is theoretically sound.

The contribution plan admittedly involves more work than most other systems of a more arbitrary or simpler character.

Whether the additional labor is or is not to be regarded as prohibitive depends on the degree of fairness obtainable and the minimum degree of fairness which it is considered essential to attain. It would seem that if a substantial increase in equity as compared with simpler systems can be attained at the expense of a *practicable* increase in work then the additional work should be undertaken. What constitutes a reasonable or practical amount of work is a matter of opinion. British actuaries seem to be united in the opinion that the contribution plan is "complicated and cumbersome" and although contribution plans of a much simplified character have been, and are being, used by a few British companies, no company there has, so far as we are aware, adopted the contribution plan as understood in America. One reason for this is that in Great Britain valuation methods are almost invariably based on a one-way classification, e.g., by attained age (whole life insurance) or by term of insurance (endowment policies), whereas the contribution plan requires classification by age at issue and duration so that its adoption in such circumstances would require almost revolutionary changes in long-established valuation procedure. Another related reason (to which further reference will be made in a later chapter) is that in Great Britain the valuation basis is seldom the same as the basis upon which the premiums were computed whereas in America these are usually identical.

The argument that the contribution plan can achieve only partial equity and that its *apparent* accuracy is therefore misleading is much more worthy of consideration. It must be admitted that, however perfect the plan may be in theory, the degree of equity attainable in any practicable application of the plan falls far short of complete equity. Dr. T. B. Sprague objected to the method[1] because it did not take into account the difference between select and ultimate mortality rates and also because of Homans' failure in the original description of the plan to provide a practical rule for the assessment of expenses. These are legitimate criticisms of the practical application of the method rather than of the method itself. A distinction between select and ultimate mortality rates can be and has, in fact, been observed in applying the plan, while it is not impossible to arrive

[1] J.I.A., XIV, 328.

at a reasonably fair and practical rule for assessing expenses. Reference has already been made to the impossibility of tracing the actual contributions to surplus made by individual policyholders and of distinguishing the expense rates of policies of different amount in dividend calculations.

Perhaps the best answer to criticisms of this nature is that broad groupings of policies are necessary in determining profit factors as for other purposes and that fine subdivisions of these factors (such as distinctions of mortality rates by numerous sub-groups or of expense rates by amount of insurance) do not necessarily mean greater equity. It might be urged, too, that although the method may, in practice, fall short of perfection, potentially at least it has a sound theoretical basis and admits of a greater degree of fairness and equity than any other method.

The third criticism referred to above is that under the contribution plan equities may be seriously affected by the use of a valuation basis differing from the premium basis or by a change of valuation basis. If, for example, the valuation rate of interest is reduced from 3½ to 3 per cent the individual contributions to surplus, if computed with reference to the valuation basis, are changed while the aggregate divisible surplus is reduced. A certain amount is at once transferred from divisible surplus to reserve. This reduces the amount available for immediate distribution but increases surplus earning power for the future—because of greater excess interest earnings. Individual "contributions" to surplus from interest will be increased on the new basis while those from loading and from mortality profit will be reduced because of the increased net premium and reserve. Some policyholders—those whose policies terminate soon—may thus get less in dividends than they would have received if no change had been made. Others—those who live long and whose policies remain in force—will eventually get more. The immediate effect, if the change of valuation basis is followed in the dividend formula (which is not necessarily the case), will be to cause a more or less marked break in the series of "contributions" of individual policyholders as between one year and the next. It may therefore be argued that a change of reserve basis will disturb the operation of the contribution plan and cause artificial changes in the amounts and incidence of profits allotted.

In considering this objection (regarded as a most serious one by critics of the contribution plan) we may first remark that the same criticism has been directed at practically every method of distribution ever adopted, including the reversionary bonus method (described in Chapter V), which is the method chiefly used by critics of the contribution plan.[1] It may be questioned whether under the circumstances supposed there is, in fact, any violence done to the individual equities. In the first applications of the contribution plan it was not assumed that there was any necessary relation between the reserve basis and the premium basis and in each of the first three applications of the plan a different reserve basis was, in fact, adopted. The principle of the plan simply was that a certain reserve was required at the beginning of the policy year and a certain different reserve (not necessarily on the same basis) at the end of the year and that certain gross premiums and interest were received and expenses and claims paid. The contribution was thus merely the balance of a debit and credit account representing the actual facts of operation. If the company deemed it necessary or wise to strengthen the reserve basis, that was done in the interests of the policyholders as a whole and, naturally, affected the divisible surplus. It is not clear that there is anything in such action which rendered the principle of the method in any way inapplicable or which could properly be considered as disturbing existing equities. This view is, in fact, indicated by Lochhead[2] where he speaks of the degree of unfairness (due to a change in reserve basis) and says that "it may amount to no more than a recognition that the first concern is the safety of the whole company." He, nevertheless, considers this criticism of the contribution plan as one which is "fatal to its continued claim to be regarded as the standard by which other methods should be judged," an opinion which is, apparently, not entirely shared by other British actuaries.[3]

It must be remembered, too, that American companies rarely change the valuation basis for existing policies and that in this country the valuation basis is usually the same as the premium

---

[1] Ryan, J.I.A., XXXVIII, 79.
[2] "Valuation and Surplus," 80.
[3] See, for example, Austin, J.I.A., XLII, 349; Elderton, T.I.C.A., IX, I, 163; Hustwitt, *idem*, 170.

basis so that the criticism is, in any case, largely academic. In Great Britain the valuation basis is changed quite freely, not however, as a rule, because of necessity but because the system of surplus distribution bears so little relation to the actual premiums payable that there is no necessity of or advantage in retaining the original valuation basis when it is thought desirable to adopt a new one. As we shall see later the British valuation system is very closely related to the distribution system in that provision for *maintenance* of existing dividend rates is usually involved in the valuation basis and in order to maintain this relation the margin between the earned interest rate and the valuation rate must be maintained, if necessary, by a change in valuation basis. Under the contribution plan there is a close relationship between the distribution system and the premium payable so that changes in valuation basis (with the consequences indicated above) are, naturally, not made unless as a matter of necessity due to substantial adverse changes in actual conditions as to mortality or interest rates. Adverse changes of that extent rarely occur.

Elderton states[1] that a serious objection to the contribution plan is the fact that if premiums were loaded to provide certain predetermined dividend rates and if all conditions assumed in the premium calculations were realized, the contribution formula would not (unless by chance) result in dividends in accordance with the predetermined scale. This argument involves a conception of dividends which is entirely foreign to American ideas of surplus distribution. Any benefits which are provided for in the premium rates, whether guaranteed in the contract or not, are as definitely "payable" as the face amount of the policy and require, as definitely, proper reserves to provide for them. The net premium (or so-called "bonus loading") for such predetermined "dividends" is thus really a part of the true net premium for the contract and the proper application of the contribution plan in such circumstances would have no relation to these "dividends" but only to *true* surplus.

---

[1] J.S.S., IV, 73.

CHAPTER IV

THE CONTRIBUTION PLAN—MISCELLANEOUS PRACTICAL QUESTIONS

This chapter deals with some general modifications of the contribution plan which are sometimes adopted either as temporary measures because of abnormal conditions, or as a part of the regular dividend system.

In the former category are two methods of effecting a general reduction in the dividend scale, namely, (1) a flat percentage reduction from a previous scale in lieu of separate adjustment of each of the dividend factors; and (2) the temporary elimination of the normal increase for duration, the dividend allotted to each policy being the same amount as in the previous year, a process sometimes called "pegging."

In the latter category are (1) the system of paying periodical extra dividends in addition to those computed by the regular formula; (2) payment of additional "maturity" or "special settlement" dividends under certain conditions at death, maturity or surrender.

Percentage Reductions. It has sometimes been suggested as a practical rule that the dividend factors based on experience should be computed only at intervals, such as five years, thus forming a temporary "standard dividend scale," and that in the intervening years dividends should be declared as percentages of the standard scale. While there is something to be said for such a rule the usual practice has been to reconsider each of the factors annually. The practice of effecting a general reduction in the dividend scale by taking a flat percentage of a previous scale, the percentage being the same for all plans, ages and durations, instead of adjusting one or more of the three factors in accordance with experience rates, was one of the most remarkable developments of the depression period which began in 1929. According to Hollenberg,[1] out of 32 changes in dividend scales made by 27 companies in the two years 1932 and 1933, 14 were

[1] T.A.S.A., XXXIV, 277.

flat percentage reductions. A tabulation of dividend scales used by 25 of the principal companies in 1935 indicates that 11 of them were obtained by percentage reductions from the previous scales.

The general argument for a procedure apparently quite inconsistent with the principles of the contribution plan is that where conditions are abnormal and where there is no recognizable trend in the rates of surplus earnings, and where, because of a reduction in divisible surplus, some reduction in the dividend scale must be made, a percentage reduction is a simple solution and one which is justifiable as a temporary measure. In times of depression, for example, when interest rates are falling and mortality and investment losses rising, first year expenses also fall and there are increased surrender gains. In other words, the whole picture is clouded for the time being. In such a period of violent transition in factors and uncertainty as to ultimate results there is something to be said for the percentage dividend reduction method if not continued in use too long. Also, where there is considerable uncertainty as to the probable surplus earnings of the year and where it is, therefore, desirable to postpone to as late a date as possible the final determination of the divisible surplus, a percentage adjustment, which can be made rapidly has obvious practical advantages. Another argument sometimes advanced is that the method is a convenient one where the former rates of dividend may be expected to be resumed at a later date.

From a theoretical standpoint it is clear that a uniform percentage reduction can be justified, strictly speaking, only where the changes in the amounts and incidence of surplus earnings are such that dividends by a new formula based on the modified rates of excess interest, mortality or expense would be the same, or at least approximately the same proportion of the dividends on the former scale for all plans, ages at issue and durations. This would be the case if the surplus from each source entering into the dividend formula were reduced to the same proportionate extent (and in the case of mortality profit, by the same proportion at all ages) or if the amounts of surplus were reduced in different proportions but in such a way as to produce in every case a uniform rate of reduction in the total contribution to surplus.

This is so unlikely to be the case that in making a percentage reduction the question is not whether the existing degree of

equity is or can be maintained but rather to what extent it is lost. The excess interest rate is, as a rule, subject to a much greater relative change over short periods than the rate of mortality saving, while the rate of expense is more stable than either. In recent years reductions in surplus earnings have been due largely to a reduction in the net interest rate or to investment losses and, to a lesser extent, to less favorable mortality experience while increased expenses have not been a material factor. Since dividends on different plans of insurance, or on the same plan but at different ages or durations, are made up of widely different proportions of interest, mortality and loading profit it is hardly necessary to argue that a uniform percentage reduction must, under such circumstances, disturb the equity of the dividend formula to a greater or lesser degree. The justification for adopting such a method of reduction must therefore be largely, if not entirely, a practical one and the main consideration will be whether the practical advantages and requirements are great enough to compensate or justify the loss in theoretical equity. It is evident that the disturbance of equity will increase as the percentage of reduction increases and that while a small percentage adjustment may not be open to serious objection on equitable grounds a large adjustment may cause too great a disturbance to be justified by any practical considerations.

Hollenberg thinks that because the interest factor is the most clearly defined of the three factors involved and the one which varies most widely in amount according to plan, age and duration and because the mortality and loading factors are more "flexible," i.e., more dependent on the actuary's judgment than the interest factor, no great inequity will be introduced by a percentage reduction (except possibly in the case of term insurance) provided (1) the percentage corresponds approximately to the proportion of reduction in the excess interest rate; (2) the percentage of reduction is not large; and (3) the dividend factors were on an equitable basis in the basic dividend scale. It is true that a small percentage reduction in either the loading or mortality factor cannot, in general, and in view of the practical limitations, be regarded as necessarily introducing any serious inequity. As between different plans and still more, perhaps, as between different durations, however, the importance of the different factors varies widely. A small reduction in the interest rate, for

example, causes only a trifling percentage reduction in the total dividend on ordinary life policies at the short durations but a reduction of possibly ten times as much at later durations. Any uniform percentage reduction, however small, is thus clearly inequitable where a lower interest rate is the main reason for reducing dividends.

Where policies have been issued on different interest bases or where different systems of loading have been employed at different times, further theoretical inequities will be introduced if the same percentage reduction is applied in all cases. Blackadar gives an illustration[1] of a consistent method of adjusting dividends as between different classes under such circumstances. This method involves the assumption that a reduction in the mortality factor of the same percentage as the reduction in the excess interest rate is appropriate. This could hardly be the case unless the percentage of reduction is small and the method is therefore applicable only to the case of such a uniform percentage reduction as that illustrated (which is 10 per cent). The method requires the adjustment of dividends on policies in the special classes by factors depending on the initial reserve or on the loading where premiums are based on a different interest rate or loaded by a different system. The simplicity of the percentage system is considerably reduced where such modifications are made, as they should be.

In view of the considerations indicated above, it would seem that the system of flat percentage reduction is open to considerable question. The validity of the method from a practical standpoint can be admitted only under exceptional circumstances and then only where the extent of the reduction is moderate. In theory, at least, the principles of the contribution plan are applicable under all circumstances. Modification by a uniform percentage reduction is a departure from that system. Such modifications, once adopted, may create new problems or may lead to further departures from the basic theory of distribution.

Elimination of Increase for Duration. This method of reducing dividends, under which the company pays the same *amount* of dividend to each policy as in the preceding year, is even more arbitrary than a flat percentage reduction. We have seen that a

---

[1] T.A.S.A., XXXV, 119.

small uniform percentage reduction corresponding to the reduction in the excess interest rate may not be open to serious objection on grounds of equity but it seems apparent that, whatever the causes or extent of decreased surplus earnings may be, the proper modifications in the dividend scale cannot be reproduced even approximately by the mere elimination of normal increase in the existing scale.

The relative reductions for different plans, ages and durations effected by this method naturally depend on the previously existing dividend formula but in most cases would, if converted into percentage reductions, show considerable variation, ranging, under typical dividend scales, from less than 1 per cent in some cases to 4 or 5 per cent or more in others for premium-paying policies. The method, therefore, cannot be justified on the same grounds as in the case of a small uniform percentage reduction.

The arguments in favor of such an adjustment are purely practical and may be the same as for a small percentage reduction. Where conditions generally and the trends of surplus earnings are uncertain it provides a practical means of making a small temporary reduction in the amount distributed which can be carried out with the minimum of trouble and expense and without the disturbance to policyholders or agents which is likely to result where there is an actual decrease in the amounts of dividends paid as compared with the previous year. Another explanation which has been given of this method of reduction is that it is an "economic adjustment" represented by a negative factor introduced temporarily into the regular dividend formula. As previously mentioned, very small changes in the dividend scale (whether increases or decreases) are usually undesirable. Some of the objections to a small decrease are removed or reduced by this method. It is also useful as a means of making a gradual transition from one dividend scale to another scale providing somewhat lower dividends, dividends being kept level until those on the new basis become greater on a given policy.

The "pegging" process, which is sometimes continued for more than one year, is, however, artificial and involves a departure from the principles of the contribution plan which it would be difficult, if not impossible, to defend on theoretical grounds. When the process is adopted in a period of declining interest rates it may result in marked inequity in extreme cases such as

single premium policies, especially if continued for more than one year.

Some indication of the general effect of pegging dividends in such circumstances is given in the following table, which shows for certain cases the percentage which the "pegged" dividend, i.e., the dividend actually paid (interest and mortality factors only) is of the "true" dividend, i.e., the dividend which would be paid on a recalculation of factors, under certain assumed conditions, which are (1) that the net interest rate falls by 0.1 per cent each year from a previously existing rate of 4.5 per cent and (2) that the mortality factor is $(70 - x)$ per cent of the cost of insurance. It is assumed, for simplicity, that the loading factor is a constant for a given plan and age at issue and is, therefore, not affected by the pegging process. This factor is not included since its inclusion would reduce the percentages and the object is to show the effect on those factors which are affected. The numerical results would, of course, be different under different assumptions. In considering these figures it must be remembered also that the total amount of surplus divided would not be the same under the two methods.

PERCENTAGE WHICH "PEGGED" DIVIDEND IS OF DIVIDEND ON ASSUMED EXPERIENCE BASIS
(Interest and Mortality Factors Only)
(Reserve Basis—American Experience Table, 3 Per Cent)

Dividend "Pegged," Years	*A*	*B*	*C*	*D*
1	93	93	100	105
2	88	87	101	110
3	85	84	103	117
4	83	81	107	124

A = ordinary life policy, age 50, durations 2 to 5.
B = twenty year endowment, age 35, durations 2 to 5.
C = twenty year endowment, age 35, durations 16 to 19.
D = single premium (or fully paid) life insurance; attained ages 57 to 60.

If the (constant) loading factor were included in both dividends (premium-paying policies) the differences indicated by these percentages would be reduced. The actual percentage reductions as compared with dividends for corresponding durations *on the existing scale* are, as stated above, quite small unless the process of pegging is continued beyond one year.

The foregoing table indicates that different plans and durations may be quite differently affected by pegging. Under the conditions assumed (moderate decrease in interest rate with no change in mortality) the dividends paid on premium-paying policies approaching maturity are too great and on those of short and medium durations too small. Dividends on fully paid insurance are too high and, if the process is continued for more than a year, may become much too high. Such a process of adjustment, therefore, may produce inequitable results.

Extra Dividends. The practice of allotting periodical "special" or "extra" dividends in addition to those provided by the regular dividend formula and as a definite feature of the dividend system is a not uncommon one. Normally, it might be supposed that special dividends would be payable only in the event that special profits, not anticipated when the dividend scale was determined, had unexpectedly been realized. Under such circumstances, or where, for some unusual reason, profits were temporarily and considerably increased, an extra dividend might be appropriate as a means of avoiding a change in the regular dividend scale. In that case the method of distribution should be consistent with the source from which the special profit was derived. Such a dividend would correspond to the extra dividends paid by commercial corporations when extra profits are realized which are not expected to continue and which, therefore, do not justify a change in the regular dividend basis. Extra dividends in life insurance are rarely, if ever, of this type.

The extra dividends met with in practice (excluding maturity or special settlement dividends discussed in the following section) are almost invariably a regular feature of the company's dividend system and do not depend on unusual profits. These extra dividends are of two kinds: (1) a single extra dividend paid a few years (usually five) after issue and intended to compensate for the non-payment of a first year dividend; and (2) periodical extra dividends, usually paid quinquennially. The reasons for such dividends are practical being, in general, (1) to effect an additional assessment of initial expense against policies lapsing in the first few years; (2) to avoid overpayment of dividends in the early policy years where the lapse rate is high; and (3) to encourage persistency.

A single extra dividend paid, say, at the end of the fifth year, and in lieu of a first year dividend, has, from a practical point of

view, distinct advantages and is justifiable to some extent on equitable grounds. The system reduces the strain of initial expense and may tend to reduce lapse in the early years. Extra dividends of this type are generally based on the accumulation of a hypothetical first year dividend. In one company the fifth year extra dividend is arrived at by computing hypothetical first year dividends on the basis of the scales used in the four preceding years and by applying the average of these amounts to purchase a pure endowment maturing at the end of the fifth year. Another company in describing such a rule says that "certain portions of the basic scale are withheld and paid with interest as a persistency dividend." The method is really a special system of assessing a larger part of the additional first year expense against policies terminating in the first few years.

Where an extra dividend is paid at regular intervals and is not dependent on plan or age but is of uniform amount for all plans the reasons which led to the adoption of that practice must, presumably, have been of a broad and practical character. One reason given for such a dividend[1] was that surplus was accumulated on the particular class of business in question and that "the flat extra dividend regardless of age has a tendency to counteract a recognized excessiveness of regular dividends at advanced ages."

In addition to the obvious theoretical questions which such a system raises, there are also some practical objections which apply to all extra dividend systems. Such dividends are likely to raise troublesome questions with policyholders who will want to know the reason for the unusual increase in one year or for the corresponding decrease in the following year. They are likely also to acquire the force of guarantees since agents will stress them when in competition with other companies which do not pay extra dividends so that dissatisfaction may be caused if they should be eliminated or markedly reduced on policies issued with such expectations.

In interpreting such a law as that of New York, requiring that dividends shall be distributed "annually and not otherwise," companies which have adopted this dividend practice evidently believe that annual apportionments of an increased amount of surplus to policies at certain durations, accompanied by normal amounts at all other durations, satisfy the legal requirements.

[1] Report N. Y. State Ins. Dept., 1926.

The Superintendent of Insurance of the State of New York made a ruling in 1925 that the New York insurance law "prohibits the payment of extra dividends at regular intervals other than annually." In 1928 the ruling was suspended and a subsequent ruling, in 1931, permits the practice, "unless there is good evidence to show that the ruling is being used as a subterfuge for the distribution of surplus earnings by means of deferred and *not* annual dividends."

Maturity, Mortuary or Settlement Dividends. A few companies are following the practice of paying a special extra dividend when a policy is terminated by maturity, death or surrender, after having been in force a specified period of years. This is done on the theory that definite portions of the surplus or contingency fund are approximately traceable to the contributions from policyholders who came into the company at various times. Thus when a given policy is terminated, the corresponding portion of the contingency fund is no longer needed for the protection of the policy and can be paid as an extra dividend. Under this theory each current generation of policyholders builds up its own surplus funds, which in turn are returned to it when the corresponding policies are terminated. At least one company doing business in New York now employs this plan and, in fact, includes in the annual dividend formula a negative factor to provide systematically for a reasonable growth in the contingency fund for each class of policies.

Such mortuary and maturity dividends are usually declared as a percentage of the reserve at time of termination, the percentage varying by plan and duration. Settlement dividends are usually paid as a percentage of the cash value on surrender, the percentage varying by plan. In a typical example, a mortuary dividend is payable on life policies in force at least seven years, a mortuary or maturity dividend on endowment policies in force at least ten years, or a settlement dividend on life or endowment policies in force at least twenty years.

Asset share accounts are used to trace the derivation of the contingency fund to individual classes of policies, and for fixing the proper amount of mortuary, maturity and settlement dividend regardless of whether a negative factor as above has been included in the dividend formula. The actual amount of such dividends, however, need not, and usually does not, correspond

exactly to the accumulation of the annual dividend deductions, if any, since other sources of profit not distributed by the dividend formula may operate to augment the contingency fund, and capital or other losses may operate to decrease it.

The theory advanced in support of the system is indicated in the foregoing paragraphs. Most companies, however, do not use the plan and have a different theoretical approach to the subject, somewhat as follows: The contingency fund is for the protection of the company as a whole, i.e., as a going concern. Each generation of policyholders does not have to build up the whole of the current contingency fund by annual contributions and draw it out again as its policies mature. The earlier generations of policyholders may, in fact, have actually built up a contingency fund in that way but after a certain point has been reached the contingency fund will normally be about the same relatively when a policy enters as when it terminates. Any increase in the fund in the intervening period thus would be provided, to a considerable extent at least, from the interest earned on the fund and any additional increase considered necessary might reasonably be regarded as analogous to expense, which should be charged against the policyholders in the same way as expense. Furthermore, the contingency fund is the resultant of decades of interplay in varying bases of expense, mortality, interest, lapse, surrender values, premiums, dividends, gain or loss on disposal of assets, valuation of assets and liabilities and miscellaneous profits and losses of many kinds. Thus, any exact allocation of this intricate fund to specific policies is impracticable.

Obviously, there are different points of view regarding maturity, mortuary and settlement dividends, and each has its advocates. As already stated, most companies are not using the plan.

The system of maturity, mortuary or settlement dividends has both advantages and disadvantages. The advantages are: (1) persistency is rewarded and encouraged; (2) the net cost (premiums less dividends) over a considerable period of years would appear more favorable, since the amounts withheld from distribution in the early policy years are accumulated with benefit of interest and persistency until termination of the policy; (3) inasmuch as dividends in the early policy years would be

lower and those in later years higher, an added discouragement to replacements or twisting would be afforded; (4) the use of lower guaranteed cash surrender values, when deemed desirable, would be facilitated since such values could be indirectly adjusted in accordance with current conditions through the amount of settlement dividends paid; (5) when a negative dividend factor to build up a contingency fund is employed, the other factors of the dividend formula can be closer to actual experience.

The disadvantages are as follows: (1) Assuming the total divisible surplus to be the same, the regular dividends would have to be somewhat reduced if maturity dividends are paid; (2) as in the case of other forms of extra dividends there is a danger, especially if the amount of the dividend is an apparently arbitrary or predetermined proportion of the reserve, that the dividends will come to be considered by agents and policyholders as guarantees; (3) the amount of the maturity dividend actually paid would be apt to depend partly on conditions which chanced to exist at the time of maturity (such as unduly low or high market values of certain assets with resulting abnormal surplus) and only partly on the amounts annually contributed to surplus on which the original illustration of the dividend was based.

During the years preceding the depression, when surplus earnings were high, several companies adopted the practice as a means of distributing additional surplus. However it did not receive very wide acceptance and some companies formerly using the plan have since discontinued it. This may be due in part to the unfavorable conditions of recent years during which few companies have been in a position to consider any such extension of the dividend system.

CHAPTER V

DEVELOPMENT OF DISTRIBUTION METHODS IN THE UNITED STATES AND OTHER COUNTRIES

(1) United States

In the early days of life insurance in the United States, the distribution of surplus was on an unscientific, rule-of-thumb basis. The correct principles underlying surplus distribution were apparently not yet understood. The first allotment of surplus to the policyholders of a life insurance company in America was made in 1844 by the Girard Life Insurance Annuity and Trust Company of Philadelphia. It consisted of an increase of 1¼ per cent in the face amount of each policy, i.e., a uniform reversionary addition payable at death, determined, apparently, somewhat arbitrarily.

In general, however, dividends allotted up to the year 1863 by most companies were computed as a uniform percentage of the premiums paid during the period covered by the dividend, regardless of plan. In some instances the dividends allotted to limited payment or endowment policies were of the same amount as for ordinary life policies issued at the same age. Other variations existed. Details of many of these early methods of determining dividends are given in a paper by D. P. Fackler.[1] In most cases, dividends at this time were not converted into additional insurance but were paid in cash or in scrip.

It is apparent that a distribution of surplus in proportion to gross premiums paid is unlikely to give equitable results, although the principal errors involved tend, in general, to counteract each other. Too much interest profit will be given where reserves are small, which is in general at the lower ages and shorter durations, while too much mortality profit will be given at the higher ages, where the margin in the mortality rate is small, and at the longer durations, where the smaller amount at risk results in lower profit.

---

[1] T.A.S.A., I (Part 2), 3.

The method of a constant percentage, however, is obviously inequitable between different plans of insurance.

Scrip. Many of the dividends allotted up to about 1870 were paid in *scrip*, i.e., in the form of conditional promises to pay at a future date. The date of cash payment was in some cases stated as a definite number of years from declaration—usually from one to five years—while in other cases the scrip was redeemable only "at the pleasure of the Board." In such cases the scrip was redeemed, in whole or in part, as cash surplus became available. Scrip dividends could, as a rule, be applied in part payment of premium and any unused or unredeemed scrip matured with the policy in event of death. In some companies scrip dividends bore interest until paid. In others no interest was allowed. Scrip was used chiefly by those companies which accepted notes in part payment of premium, the scrip dividend being available in payment of premium notes. In at least one company the scrip was subject to reduction in case of losses incurred subsequent to declaration and before the scrip was redeemed.

Dividends during this period were, in many cases, declared annually, although several important companies made quinquennial or triennial distributions for some time. Where dividends were paid in scrip an annual distribution was the rule and this eventually led to the general adoption of the annual distribution basis by all companies.

The Contribution Plan. The adoption of the contribution plan by the Mutual Life Insurance Company of New York in 1863 was the most important development in the history of surplus distribution in this country. By that change a scientific and at least partially equitable method was substituted for an arbitrary and inequitable one. The new method was soon adopted by other companies and eventually became practically the universal system in the United States and Canada.

There were several reasons for the change in method of distribution. That the percentage plan was unsound had been recognized for some time. References to the sources of surplus appear in the annual reports of the Mutual Life some years prior to 1863. Endowment insurance had been introduced and it had become evident that the uniform percentage plan gave too much to endowment policies, particularly of short duration. In addition the surplus to be divided had greatly increased. The current

net interest rate was 6½ per cent, as compared with a valuation rate of 4 per cent. The mortality experienced was about 70 per cent of the expected while "extraneous profits," chiefly from lapsed policies, were more than enough to pay all expenses thus leaving the entire loading to be added to surplus. On the existing basis of distribution the surplus in 1863 would have been sufficient to pay a dividend of 70 per cent of the total gross premiums paid during the previous quinquennium.

These circumstances, together with the fact that the Mutual's charter required an "equitable" distribution, caused Sheppard Homans, the actuary of the company, to propose a reconsideration of the dividend system. It was at first decided to adopt a plan proposed by an English actuary (Charles Jellicoe) whereby surplus was allotted in proportion to the accumulated excess of premiums paid over the "true premiums" based on the experienced rates of interest and mortality[1] but before this was carried out, that method was abandoned as theoretically defective and the contribution plan was substituted. Credit for devising the the contribution plan has usually been given to Homans alone but there seems to be no doubt that this honor should be shared at least equally by his assistant, D. P. Fackler.[2]

The basis and original method of application of the plan are described by Homans in a paper written in 1863 and published in the Journal of the Institute of Actuaries.[3] The plan is also described and compared with the percentage plan and other plans in letters written by Homans and Fackler to and at the request of the Insurance Commissioner of Massachusetts in 1868 and published in the report of the Commissioner for that year (see R.A.I.A., XI, 111–125).

The principle upon which the method is based is that "each participant should be benefited in proportion to the excess of his payments over and above the actual cost of insurance." This principle, Homans said, was "recognized by all actuaries." The words "in proportion to" are important, indicating that the dividends to be allotted by this method are not the actual overpayments based on the experienced rates of interest, mortality and expense, but the proportion of the total amount to be divided

---

[1] J.I.A., X, 328.
[2] J.I.A., LXII, 274.
[3] J.I.A., XI, 121.

(however obtained) which the individual overpayment bears to the total overpayments of all policyholders. Hence in applying the method in its original form it is necessary first to calculate the individual overpayments or "contributions" and then to increase or decrease them in the proportion which the divisible surplus bears to the total overpayments. The excess of the former over the latter, if positive, is the surplus from extraneous sources, such as lapses, and Homans maintained that the distribution of such surplus was not governed by the same principles of equity but might be divided by *any* method which seems suitable or convenient. This is a very questionable proposition. As a matter of fact, the bulk of this "extraneous profit" (after deduction of all expenses) was, in the first distribution on the contribution plan, divided in proportion to the calculated contributions, which included no deduction for expenses.

The application of the contribution plan during the early years of its use varied considerably, particularly in regard to the distinction between "contributions" and "dividends." Finally in 1873 the distinction was dropped and the dividend formula emerged in the now well-known three-factor form, dividends from that time on being computed as the sum of the assumed contributions from (1) excess interest, (2) mortality saving and (3) excess of loading over assessed expenses.

The method as now applied is therefore based on an adjustment of the difference between the three fundamental rates (interest, mortality and expense) involved in the premium calculations and the rates actually experienced. Theoretically other elements (of profit or loss) may be added to the formula but usually such other elements are not accorded separate treatment. This is considered reasonable because of the inherent limitations of the method.

Two-factor Systems. During the latter part of the last century and in the earlier years of the present century many companies used a contribution formula based on two instead of three factors, the mortality factor being eliminated.

The system of agency compensation had changed radically during the last half of the nineteenth century with an increasing tendency to pay high initial commissions for new business. Large mortality profits on new business (because of medical selection) were therefore offset by heavy acquisition costs while ultimate

mortality profit was small. Profits on lapsed policies had been materially reduced by the introduction of non-forfeiture provisions. This situation was discussed in an important paper by R. W. Weeks[1] who described a dividend formula in which the dividend (commencing at the end of the second year) consists of (1) excess interest earnings and (2) loading less renewal expenses.

Weeks justified this formula on the basis that, assuming certain reasonable rates (at that time) of mortality, interest, discontinuance, surrender allowance and expense (distinguishing initial and renewal expense), the excess initial expense could be shown to be approximately liquidated by mortality and discontinuance profit in about five or six years. Consequently, he proposed to anticipate this result, offsetting these elements and allotting only excess interest and renewal loading profit as a dividend. Any excess profit arising from mortality or discontinuance—after initial expenses had been liquidated—would, he assumed, be small and could reasonably be carried to the contingency fund rather than added to the dividend.

Weeks' two-factor formula, although based to some extent on assumptions of a rather arbitrary character, was quite closely in conformity with conditions as they were at that time. Since then mortality profit has increased to the extent where it demands separate recognition in a proper dividend formula and the three-factor basis has again become practically universal.

The question of the recognition of ultimate mortality profit as a separate factor in the dividend and the proper methods of its distribution were discussed by Moir[2] in 1913, about which time the three-factor basis was again becoming general. There was for a time some tendency to retain the more simple two-factor basis providing for interest and loading profits, and various arguments were advanced for doing so.[3] These arguments, while justified to some extent under the conditions then existing, are no longer valid.

As will be seen from Appendix B, a few companies employ a dividend formula with two factors as follows: (1) excess interest on the reserve and (2) a level dividend from other sources determined by subtracting from the gross premium an "experience"

---

[1] T.A.S.A., IX, 310.
[2] T.A.S.A., XIV, 17.
[3] See Moir's paper and the discussion thereto.

gross premium based on the valuation interest rate but on the company's experience as to expense and mortality. This second factor is sometimes increased after the renewal commission period has expired. Thus the usual three factors are taken into account but the distribution is made through a two-factor formula. The historical background of the formula can be traced to Jellicoe.[1]

The advocates of the method consider that it has certain advantages, among which is the fact that it must result in dividends increasing by duration under premium-paying policies on which the reserve increases, so long as the interest rate used in the dividend formula remains unchanged.

The disadvantages are that the method is somewhat inelastic and not so readily adapted to meet changes in experience as the usual three-factor formula. Also, certain precautions have to be observed in its use so that the distribution in the early policy years will not be greater than earned. This may be reflected in a reduced dividend from "other sources" for a period or perhaps no dividend whatever for several years after issue.

The change in commission basis created, in fact, as we have seen, a new problem in distribution, namely, whether *any* dividend should be paid before reserve and surplus have been created from the net receipts on any policy, proper allowance being made for the *actual* expenses and the *actual* cost of insurance. This question is still a matter of discussion and difference of opinion.

Deferred Dividends. The deferred dividend system which was adopted by some of the largest companies and under which most of the participating insurance issued between about 1880 and 1906 was written is now chiefly of historical interest. However, a considerable number of policies now in force still receive dividends on the deferred basis so that some knowledge of the application of the plan is desirable. The New York insurance law of 1906 and corresponding laws in other states which required annual distribution of surplus put an end to the system so far as new business was concerned.

The deferred dividend system was a development of the Tontine plan introduced by the Equitable in 1868 to meet competition

[1] J.I.A., I, 22*, 159.

of older companies which were paying dividends annually. Under the Tontine plan no surplus was distributed until the end of the period agreed upon, all surplus then being divided among the survivors. No cash or other surrender values were paid in event of lapse prior to the end of the distribution period, so that the dividends paid to survivors were much larger than the accumulation at interest of annual dividends on the usual basis. The element of total forfeiture in event of lapse was found to be an undesirable feature of the system and modifications were introduced under which provision was made for surrender values, this being known at first, as the "semi-tontine" plan. The general basis of dividend calculation was the same on both plans.

There were two general methods of calculating deferred dividends: (1) by accumulating hypothetical annual dividends with benefit of survivorship and (2) by the "asset share" plan.

Under the first system, annual dividends on the usual contribution basis were computed each year for each deferred dividend policy in force. The hypothetical annual dividends were accumulated at interest (at the net earned rate) to the end of the distribution period when the total was increased by a graded ratio based on the amount of the accumulated annual dividends on policies terminated prior to the end of the distribution period in each group.

The asset-share plan is described in a paper by R. W. Weeks.[1] The procedure indicated in that paper may be briefly summarized as follows:

(1) The *standard final asset shares* are the reserves plus surplus at the end of each policy year calculated for groups of similar policies on the basis of standard experience tables of mortality, discontinuance and average surrender allowance and using the average rate of expense and the net rate of interest earned.

Tables of standard final asset shares are first prepared on a unit basis and for quinquennial ages at issue for each "valuation group," i.e., for each group of policies identical except for amount, such as ordinary life policies issued at

[1] T.A.S.A., IX, 93.

age 35 in 1900 with twenty year dividend period. These tables represent, in effect, model accounts by policy years.

(2) From the tables of standard final asset shares at quinquennial ages a table of *standard interim asset shares*, i.e., by calendar years, is prepared.

(3) By interpolation of each of these tables two tables are formed complete for all ages showing respectively (*a*) the standard interim asset share at each duration and (*b*) the final asset shares—at the end of the dividend period only.

(4) The dividend payable to each of the survivors in any valuation group at the end of the distribution period may then be taken as:

$$\text{Standard final asset share} \times \frac{\text{total assets def. div. business prev. Dec. 31}}{\text{total interim asset shares Dec. 31}} \textit{ less } \text{reserve.}$$

Theoretically the ratio of assets to total interim asset shares should be unity. It will be noted that the asset share plan as here applied requires a separation of the assets of deferred dividend business whereas the hypothetical annual dividend system does not.

Alternatively, dividends may be based on what is called "approximative accounting by sections," a "section" consisting of *all* policies issued in the same year with the same dividend distribution period. On this basis tables of the *total* interim asset shares by *sections* are prepared each year which are constructed by an actual separation of income and disbursements instead of by the use of standard tables as in the first method. Weeks modified this alternative process by redistributing the net actual death losses among the sections in proportion to the expected net losses on the experience basis.

The totals of interim asset shares by sections thus arrived at (by separation of accounts and redistribution of net death losses) are then compared with consolidated totals for sections based on the tables of standard interim asset shares by valuation groups, which are thus also required in the alternative process, and a graduated scale is determined which is applied to the tables of standard final asset shares, the dividends then being obtained by deducting the reserve.

It is unlikely that the deferred dividend system will ever be revived. As explained elsewhere[1] it had, however, certain advantages both to the policyholder and to the company. The defects which led to its being prohibited by law could have been eliminated by proper regulation as was, in fact, done in Canada. The system was, however, contrary to the basic principles of insurance at cost in that those whose policies terminated within the distribution period selected paid more than the actual cost of insurance.

(2) Canada

The development of distribution methods in Canada has been, in recent years at least, pretty much along the same general lines as in the United States. There are, however, some features of special interest in regard to both historical development and current practice.

The oldest Canadian company[2] originally used the British uniform reversionary bonus system under which surplus is allotted, usually, at quinquennial intervals as a reversionary addition at he same rate for all policies.[3] One other old company also used iat system, which was retained by both companies until 1900. urplus under policies issued in these companies prior to 1900 is .till distributed quinquennially on the uniform reversionary bonus plan.

Other companies, organized later, adopted from the outset various modifications of the contribution plan and it appears that the "percentage plan" widely used both in the United States and Great Britain up to about 1865 or 1870 was never used in Canada. These earlier contribution plan distributions were all, or nearly all, made on a five year basis, and in fact for a period of about fifteen years after 1910 all the business written in Canada, with the exception of a small amount of deferred dividend business, was written on the quinquennial distribution plan. Since then there has been a gradual change in certain companies to the annual dividend basis and at the present time a considerable portion of the new business is written on the annual dividend plan.

---

[1] "Life Insurance," J. B. Maclean, 4th ed., p. 261.
[2] The Canada Life Assurance Company, founded 1847.
[3] The system is discussed later in this chapter.

The deferred dividend system was copied from the United States and for some time was almost exclusively used. It was never forbidden by law (as in the United States) and is, in fact, still used to a small extent and by a very few companies. Abuses of the system were checked by legislative enactment requiring an apportionment by classes at least every fifth policy year of the surplus earned on deferred dividend policies issued on or after January 1st, 1911.

A distinctive feature of Canadian surplus distribution is the "five year dividend system" which differs from five year "deferred dividends" chiefly in that "interim dividends" are paid in event of death during the dividend period but not in event of lapse or surrender. The five year dividends are obtained either by accumulating annual dividends at interest with an addition for termination gains or by an independent calculation of surplus earnings on the completion of the distribution period.

The application of the contribution method in Canada (now, as in the United States, practically universal) has been affected by the general use of a modern mortality table (the $O^{M(5)}$ Table and later the $AM^{(5)}$ Table) for valuation purposes. A two-factor system (with loading and mortality factors merged) has therefore been more common and has continued in use for a longer period than in the United States. At present about one-half of the companies use a three-factor formula, about one-quarter employing a two-factor formula and the remainder an asset share method.

(3) Great Britain

In Great Britain the uniform reversionary bonus plan occupies nearly the same position as that occupied by the contribution plan in America. According to Hustwitt[1] nearly 90 per cent of the participating insurance in force in British companies is on the reversionary bonus plan. Several other systems are, however, in use but none of these is used by more than a few companies. Most of the companies distribute surplus quinquennially, the remainder at more frequent intervals.

The history of the development of surplus distribution in Great Britain up to the year 1900 will be found in a paper by Ryan[2]

---

[1] T.I.C.A., IX, I, 166.
[2] J.I.A., XXXVIII, 69 and T.I.C.A., III, 252.

and of developments since that time in the paper by Hustwitt referred to above.

Ryan traces the origin of the reversionary bonus plan to the method adopted by the Equitable Assurance Society (after a single cash distribution in 1776) whereby surplus is allotted at each distribution as a uniform reversionary addition to the amount of insurance *for each year the policy has been in force.* Thus, if the rate of reversionary "bonus" is 1 per cent, a policy 10 years in force receives an addition of 10 per cent of the face amount while a policy 50 years in force receives an addition of 50 per cent. This plan, the origin of the whole system of uniform reversionary bonuses, was not, apparently, based on any considerations of equity.

The Equitable plan may have been followed by a very few other British companies about the beginning of the nineteenth century but there is little or no information on the point. For about half a century a variety of other methods was chiefly followed. These included a modification of the Equitable plan under which reversionary additions were allotted not for each year the policy had been in force but for each year of (or premium paid in) the current distribution period, usually five years. In some companies the rate of reversionary additions was applied to existing prior additions as well as to the face amount of the policy. This was called the "compound reversionary bonus" plan as distinguished from the "simple reversionary bonus" plan (addition based on face amount only). These modifications of the Equitable system subsequently became, as already indicated, the standard British distribution systems.

However, up to about 1860 or 1870 the most common system in Great Britain as in America was what we have called the "percentage plan," i.e., a uniform percentage of premiums paid. Some companies based the distribution on loadings rather than on gross premiums. Further comment on miscellaneous bonus systems used in Great Britain will be found in "Valuation and Surplus" (Lochhead), pages 84–87, to which the student should refer. It may be noted, however, that dividends under nearly all these systems were, apparently, declared in the form of additional insurance however the cash value might be computed.

"Contribution" Plans. Of more interest and importance than any of these miscellaneous methods are the two British "contribu-

tion plans" identified with the names of T. B. Sprague and T. G. C. Browne.

Sprague's method[1] was really no more than a modification of the percentage plan eliminating the most apparent injustice of that method, which was the allotment of interest profit in proportion to the premiums paid during the distribution period. Under Sprague's method a calculation of the excess interest earned is made for each policy. The total of these amounts is deducted from the total divisible surplus and the balance is then divided in proportion to the gross premiums, i.e., by the percentage method. Where the distribution is annual the dividend is, therefore, in the form $V(i' - i) + kP'$ where k is the ratio of the balance of divisible surplus (after deducting excess interest) to the total gross premiums.

In applying this system to a five year distribution period Sprague took the excess interest as

$$V_{-5}\,[(1 + i')^5 - (1 + i)^5]$$

where V_{-5} is the reserve at the end of the previous quinquennium. This omits adjustment on account of premiums paid during the five years and apparently, also, it was intended that no specific allotment of excess interest would be made to policies effected during the quinquennium, such excess interest being thus included in the general balance and divided in proportion to the gross premiums.

Sprague's method was, of course, a distinct advance over the then prevailing percentage method but went very much less than all the way toward an equitable distribution in accordance with the sources of surplus since it recognized the incidence neither of mortality profit nor of expense.

Browne's method[2] was an elaboration and modification of Sprague's and when first adopted was intended to charge more equitably any heavy or unusual loss from mortality. His idea

---

[1] J.I.A., XIV, 396. The method was probably due, in fact, not to Dr. Sprague but to his predecessor, Mr. Sylvester. It appears to have been used by the Equity and Law Life Assurance Society in 1854 about seven years before Dr. Sprague became associated with the society. It was not used again until 1864 when Sprague revived it. (I am indebted for this information to Mr. F. A. P. Lewty, the present Actuary of the Society. J. B. M.)

[2] J.I.A., XXXII, 194.

was (1) to keep in the loading group only loading profit and profit from lapse and surrender and (2) to place all other profits (or losses) which comprised all the *fluctuating* elements including mortality and investment profit in a third group which was then divided ratably between the other two (i.e., interest and loading). The dividend was then $k_1V + k_2L$ where V is the reserve at the *end* of the distribution period,[1] L is the total loading for the period and k_1 and k_2 are the "multipliers" obtained by comparing the adjusted total of interest profit with the total reserves and the adjusted total of loading and termination profit with total loadings. It will be noted that the factor k_1 is not in the form $(i' - i)$ since it includes a part of miscellaneous other profits in addition to interest profit.

While Sprague's and Browne's methods were an undoubted advance in equity over prevailing methods (including the uniform reversionary bonus plan) they were criticized as being too laborious and too much affected by changes in the valuation basis. For example, where a very low rate of interest was adopted for reserves (as was becoming common) most of the profit was, under both of these systems, thrown into the interest group to the detriment of more recently effected policies. It may seem strange that, although Homans' paper had been published in 1863, no attempt was made to modify these plans by introducing a specific mortality factor, especially as mortality profit was by no means negligible. The analysis of profits given by Browne shows that in two distribution periods mortality profit alone amounted to three-fourths of the combined loading and termination profit.

These two systems, although called "contribution" systems, were, as already indicated, really only improvements on the old "percentage" or "loading" systems rather than a recognition of the principle of returning surplus in proportion to the contributions made thereto. They go part of the way toward the American contribution plan but fail to go all the way and remain, therefore, to a considerable extent arbitrary and unscientific. As compared with modern contribution formulas as used on this continent they not only fail to give separate recognition to the incidence of mortality profit by age, now recognized as essential,

---

[1] Browne makes no comment on this change.

but divide the aggregate mortality profit on a quite arbitrary basis. They also make no attempt at assessment of expenses by plan, age or duration. They cannot, therefore, be regarded as of the same character as the American contribution plan. A tabulation given by Hustwitt indicates that the number of British companies using plans of this type is small and decreasing.

The Uniform Reversionary Bonus Plan. This plan developed, as already noted, out of the Equitable system rather than from theoretical considerations of equity. Its justification when not based on purely practical grounds has therefore been largely *a posteriori*.

In any consideration of the British system of uniform reversionary bonuses, it is important to note that in Great Britain the distribution system has become to a great extent interlocked with the valuation system. Uniform simple and, still more, uniform compound reversionary bonuses require increasing cash equivalents and this has been accomplished by the adoption of a valuation system which involves a substantial postponement in the release of surplus. This system is the use of an artificially low valuation interest rate.[1] The reserve held usually represents, therefore, not only the value of the policy itself but an implicit provision for maintenance of part at least of the future bonuses in the form of uniform reversionary additions. Incidentally, as may be seen by inspecting the dividend schedules of most American companies, a normal three-factor contribution formula produces (under the conditions which have prevailed in the past) cash dividends which increase with duration and which provide reversionary additions which, in general, increase slightly with duration.

The practice of providing in the valuation basis for maintenance of a more or less definite rate of bonus will seem to American actuaries to create a somewhat artificial situation in regard to surplus and its distribution. In America both the gross premium basis and the reserve basis are independent of the amount or type of dividends which it is intended or expected shall be paid. Dividends are paid only in accordance with current surplus earnings and there is no recognition of any neces-

[1] See C. H. Maltby, J.I.A., LIII, 175; Elderton, J.S.S., IV, 76; "Life Assurance Text Book," p. 166.

sity to "maintain" dividends if current surplus earnings do not provide such maintenance. On the contrary, both the general scale of dividends and the relative amounts allotted to policies of different plans, ages at issue and durations are freely increased, decreased or otherwise modified as conditions require. Consequently, there is no necessity in America as in England to increase the stringency of the valuation basis merely in order to maintain the interest margin and dividend rate. Changes in the valuation basis are therefore exceedingly unusual in America and would be made as a rule only because of a decidedly adverse trend in interest or mortality rates. Naturally the only basis upon which maintenance can be provided is to treat the dividends as if they were (partly or wholly) guaranteed. Such a basis seems to alter fundamentally both the character of the contract and the conception of "surplus" and to create difficulties which do not exist under the contribution plan. The "bonuses" so provided do not represent true surplus earnings since they are largely predetermined, but are rather additional benefits for the provision of which a premium is charged and a reserve held.

It seems clear, therefore, that the equity of the uniform reversionary bonus plan depends to a large extent on the scale of gross premiums and it would also appear that if conditions change in such a way as to disturb the relative equity of the bonus system as between different plans, ages or durations it will be necessary either to change the premium rates or to modify the *system* of distribution. It would, of course, be impossible to change the premium rates on existing policies, so that changes in the system of distribution would, in such circumstances, be necessary for existing policies if equity is to be preserved. This has been recognized in practice so that in certain instances it has resulted in the adoption of different rates of bonus for endowment policies than for life policies or even of variations in the rate of bonus according to age. Such modifications are, of course, merely a palliative. The fundamental objection to the plan, from a theoretical standpoint, is that, even if premium rates are loaded in such a way as to provide uniform bonuses at all ages and for all plans on the basis of then existing conditions, changes in the incidence and amount of surplus earnings cannot be taken into account except in the aggregate with the result that inequity must inevitably arise. It is true that inequity will necessarily

arise, to a greater or lesser degree, under any practicable plan of surplus distribution but under the contribution plan there is a much greater possibility of controlling it within reasonable limits.

From a practical point of view, however, it may be that the degree of equity attained by the uniform reversionary bonus plan may be deemed sufficient. There is also the fact that the policyholders may be assumed to have accepted this system or, as expressed by Lidstone,[1] the method may be justified practically because of "methods which may have been followed in the past and the representations which may have been made in regard to the future." The system has also undoubted and undisputed advantages over the contribution plan in simplicity and economy of application and in its ready comprehensibility by the public.

(4) Other Foreign Countries

Very little detailed information is available in regard to the methods of surplus distribution used in other foreign countries. At the Third International Congress, at which Distribution of Surplus was an assigned topic, only one paper by a continental European actuary was presented. Several papers on dividend methods in Europe will be found in the Transactions of the Ninth Congress. These deal, however, more with theory than with practice. The impression gathered from these papers is that little uniformity exists and that considerations of equity have in general received less weight than those of practical expediency. The general view seems to be that held by Dumas,[2] who says that as the determination of the amount of divisible surplus is to a great extent arbitrary there will consequently be great liberty in the choice of the method of distribution. This is a different view from that generally held in America, Canada and Great Britain.

In France, according to Bernus,[3] the most common system of distribution is in proportion to the total of premiums paid in.

Information as to methods of surplus distribution in Germany is contained in the *Versicherungslexicon* of Dr. A. Manes (1924, page 607). A large number of companies there still compute dividends as a proportion of the premium payable. Others base the dividend on total premiums paid since issue or on the policy

---

[1] J.I.A., XXXII, 73.
[2] T.I.C.A., IX, I, 287.
[3] T.I.C.A., IX, I, 124.

reserve. Still others (including one of the largest companies) use a "compound" method under which the dividend consists of a percentage of the annual premium plus a percentage of the policy reserve, this being a rough recognition of the fact that some elements of surplus are approximately constant while others are increasing. Manes describes the American contribution plan and states that it is used by one company in Germany to ascertain the "basic dividend" which is subject to subsequent modification.

In the Scandinavian countries a common method of distribution is to pay at death or maturity (and sometimes on surrender) an additional amount based on the insurance which could have been purchased by the premiums paid on the basis of the experienced rates of interest, mortality and expense.[1] This is similar to the methods proposed by Jellicoe[2] and Wilbraham[3] and referred to by Henderson in his presidential address on "Equity in Surplus Distribution."[4]

A paper by Onnen and Peek[5] indicates that in the Netherlands a two-factor contribution plan (interest and mortality) had been used by one Dutch company about 1890. Other companies here have evidently based dividends in general on total premiums paid.

In South America methods of distribution are largely empirical, participating insurance being, in many companies, of much less volume than non-participating. At least one company uses the uniform reversionary bonus plan.

In Japan participation does not begin, as a rule, until after the policy has been in force for from three to five years. A variety of methods of distribution is in use. The two principal methods, which are widely used, are (1) a percentage of the gross premium payable for *each year* in force in excess of the qualifying period, and (2) a constant percentage of the reserve. Dividends are declared as a reduction in premium or, in case of fully paid policies, paid in cash or, sometimes, applied to purchase a dividend addition. The contribution plan is not used.

---

[1] T.I.C.A., IX, I, 72 and 201.
[2] J.I.A., I, 22*, 159.
[3] J.I.A., VI, 278.
[4] T.A.S.A., XXIV, 291.
[5] T.I.C.A., III, 278.

CHAPTER VI

ANNUITIES

It is only within recent years that immediate life annuities have been issued on a participating basis. The fact that, in the past, such annuities have always been non-participating has probably been due chiefly to the nature of the contract and also to the continuously improving rate of mortality among annuitants resulting in losses which reduced or extinguished any profits from excess interest earnings. Other reasons may have been the difficulty of finding a satisfactory method of distributing surplus and also the relative unimportance, financially, of the annuity business as compared with the life insurance business.

In general, two reasons have been given for issuing annuities on a participating basis. These are (1) to increase the yield to the annuitant if earnings are favorable and (2) to reduce or eliminate the possibility of loss to the company if the experience proves unfavorable.

The former reason has been advanced when current interest rates were high and when the yield obtainable from an annuity at the lower ages was little, if any, more than that on high class securities. Under such circumstances it has been argued that the companies should allow excess interest dividends in order to make annuities more attractive. In that way more annuities would be issued, particularly on young lives, and adverse selection would be reduced.

In 1914, D. P. Fackler[1] advocated participating annuities on these grounds. At that time annuity rates and reserves were generally on a 4 per cent basis while the current net interest rate was from 4¾ to 5 per cent. In view of subsequent events it is interesting to note the following extract from Fackler's paper:

"Ten or fifteen years ago when the market rate of interest seemed to be falling lower and lower but little could be said in

[1] T.A.S.A., XV, 40.

favor of mutualizing the annuity contract, but now that the trend of interest rates has been steadily upward and there is little reason to expect much, if any, reduction for many years to come, it may be well to consider what should be done in the changed situation."

In recent years the fact that "the market rate of interest seemed to be falling lower and lower" was the very reason which caused annuities to be placed on a participating basis.

Fackler's argument took no account of the element of mortality. He does not seem to have given much consideration to the possibility of mortality loss, much less to have attempted to measure or compare such loss with interest profits. In fact, he says: "It is proposed that the dividends . . . should be made almost entirely, if not entirely from the excess of interest," thus indicating an expectation of profit rather than loss from mortality, and also, after suggesting that the mortality table might have to be changed to suit existing conditions, "if annuity mortality hereafter should exceed the table, the division of the surplus . . . would furnish an interesting problem but it is not worth while dealing with it at present."

It is not very clear whether Fackler's plan of participation was intended to be applied in connection with new and higher premium rates or whether he thought that dividends from excess interest could be paid on the existing contracts. It was pointed out by Ferguson[1] that the system of distribution proposed by Fackler (which is described later in this chapter) was applicable only where the mortality rate involved was the minimum rate which might be expected. It was further shown[2] that in spite of an interest margin of nearly 1 per cent the actual accumulated annuity fund in certain companies was only just sufficient to provide the required reserves and would have been insolvent if dividends of any substantial amounts had been paid. This was, no doubt, also the case in most other companies and clearly demonstrated the fact that, on the basis of the rates which had been charged, the companies, in spite of high interest rates, could not have paid dividends. This did not mean that annuities could

[1] T.A.S.A., XV, 354.

[2] See discussion of Fackler's paper by E. B. Morris and of Ferguson's paper by E. W. Marshall.

not have been issued on a participating basis but simply that on that basis premium rates would have been substantially higher. In other words, the guaranteed yields would have been substantially lower.

Where annuity rates have not been computed on an ultra-conservative basis in all respects, experience has shown that interest profit is likely to be reduced by mortality loss. The extent to which mortality loss can offset interest profit is illustrated by the following table. The figures in this table are illustrative only and are not intended to indicate either the rates which are charged or those which should be charged for annuities under present conditions.

VALUE OF $a_{[x]}$—AMERICAN ANNUITANTS TABLE (MALES)

Age	$a_{[x]}$ 3%	$a_{[x-1]}$ 3½%	$a_{[x-1]}$ 4%	$a_{[x-2]}$ 3½%	$a_{[x-2]}$ 4%
30	21.072	19.680	18.234	19.889	18.409
40	18.236	17.307	16.202	17.568	16.428
50	14.944	14.443	13.673	14.748	13.945
60	11.417	11.256	10.778	11.581	11.076
70	8.012	8.063	7.806	8.372	8.096
80	5.116	5.251	5.132	5.506	5.377

The table shows the prices required on a certain basis (American Annuitants Select Table with interest at 3 per cent) and the prices which are necessary on the basis of assumed actual conditions under which profit is made from interest and loss is made from mortality. Thus, if the company earns 4 per cent (net) instead of 3 per cent and if the mortality experienced corresponds to the tabular mortality for two years younger the rates for ages at issue up to nearly 70 will be more than sufficient so that dividends could be paid whereas the rates charged from about age 70 will result in a loss. If the interest rate earned is only 3½ per cent with the same assumed experience as to mortality, dividends would be earned only for contracts issued at ages under 60. If the mortality "differential" is reduced to one year a 1 per cent margin in interest would provide dividends at ages at issue up to 80 and a 0.5 per cent margin, at ages at issue up to nearly 70. In all cases interest profit will appear from the outset in amounts greater than the amount of mortality

loss but the accumulated surplus will in certain cases, as indicated, eventually become negative. Even where it does not, the "surplus" earned in later years may be negative.

These considerations indicate that as a practical matter participating annuities are likely to be unsatisfactory unless issued on a basis which will probably provide a clear margin of profit from both interest and mortality indefinitely. This means that guaranteed yields on the participating basis must be substantially less than on a non-participating basis.

It is doubtful whether participating annuities will result in higher effective yields to annuitants than non-participating annuities. Theoretically, of course, they should do so since on a non-participating basis some safety margin is necessary whereas under a participating contract the annuitant presumably gets his annuity "at cost." The fact that the mortality among annuitants has always been progressively less favorable (from the companies' viewpoint) renders it difficult to determine practicable non-participating rates which will certainly prove to be safe and sufficient over a long period of years. The result may quite possibly be that the guaranteed returns under non-participating contracts, even on a supposedly safe basis, will prove to be greater than those on participating contracts since in the latter case the company can eliminate loss provided the basis of the rates is sufficiently conservative. If interest rates decline materially this result will, of course, be more likely to occur.

Systems of Distribution

There are two general systems of distributing surplus on annuities: (1) the contribution plan and (2) the equalized dividend plan. The equalized dividend plan may, however, be regarded as a modification of the contribution plan.

Dividends computed on the contribution plan may be paid in cash or may be applied to purchase additional annuities. On the equalized dividend plan the uniform cash dividends are obtained by ascertaining the total effective annuity purchased on the basis of assumed true rates of interest, mortality and expense as distinguished from the more conservative rates involved in the premium charged.

Contribution Plan—Cash Dividends

Loading Factor. Since the rate of initial expense is known, any additional loading in excess of that required for initial expense may be intended either as a provision for future expenses or for contingencies or to provide surplus for distribution. Fackler proposed a provision for future expenses to be maintained as a percentage of the reserve. In such case the contribution from loading would be $e - e'$, where e is the level annuity equivalent of the loading for renewal expenses and contingencies in the original premium rate and e' is the actual overhead expense in any year (including an allowance for contingencies). Theoretically the contribution from loading should be increased by interest and benefit of survivorship for the year.

An alternative mode of providing for future expenses is by making a small reduction each year in computing the dividend interest rate. Any balance of loading intended to provide distributable surplus may be converted into a uniform (unguaranteed) dividend annuity on a conservative basis which will then form the "loading factor" of the entire dividend each year. It may be desirable to modify the amount of this factor from time to time as conditions change. This would be done by applying the current "reserve" (surplus) held for the loading dividend to purchase an increased or reduced annuity on the revised basis. Where a dividend from loading is provided for in this way as part of the regular annual dividend it must be treated in effect as a liability although the present value of such future loading dividends will be included in surplus.

Interest and Mortality Factors. The interest factor of the dividend is

$$\frac{1}{p'_x}(i' - i)a_x$$

and the mortality factor

$$\frac{1}{p'_x}(q'_x - q_x)(1 + a_{x+1})$$

where x is the age at the beginning of the policy year and accented symbols denote experienced as distinguished from assumed rates. These expressions follow from the relation

$$a_x = vp_x a_{x+1}$$

or

$$(1 + i)a_x = -q_x a_{x+1} + a_{x+1}.$$

The factor $1/p'_x$ implies that surplus is distributed only to survivors and its inclusion follows from the normal accumulation formula. There is thus no provision, by this formula, for post-mortem dividends. In life insurance the practice is to pay claims immediately, not at the end of the year of death, and to allow a dividend for the portion of the year completed. The "contributions," therefore, do not, in that case, depend on survival to the end of the policy year.

Where provision is made for renewal expenses by setting up a special reserve as indicated above, interest and mortality profit will also arise from this "secondary expense annuity" and will, in theory at least, increase the annual contribution and dividend.

Unless the value of i' should increase, the interest factor of the dividend will decrease proportionately to the decreasing reserve. Unless, therefore, there is profit from mortality which increases sufficiently to offset the decrease in interest profit the entire dividend will decrease (the loading factor, if any, being presumably a constant). This shows the necessity of a conservative mortality basis. In fact the contribution plan is not suitable unless there is an ample margin for profit in the mortality rate. If that is not the case the total dividend may not only decrease but may, as we have already seen, actually become negative.

Illustrations of the effect on dividends of a mortality loss combined with a constant interest rate are given by Kilgour in the discussion of Fackler's paper. The tables there given demonstrate very clearly the fact, already noted, that even where accumulated surplus for the entire period is positive throughout, the net contribution in later years may become negative. They also show the increasing importance of the mortality factor (whether positive or negative) with advancing age.

As a further illustration the following table is given comparing the mortality loss per $100 annuity caused by an experience corresponding to one or two years lower than the rated age with the interest gain due to an excess interest rate of 1 per cent. The figures are based on the American Annuitants Male Table (Ult.) with 3 per cent interest.

Age	Mortality Loss		Excess Interest Gain $(i' - i) = 1\%$
	Experience 1 year lower than expected	Experience 2 years lower than expected	
20	\$0.17	\$0.36	\$23.39
40	0.66	1.27	18.24
60	2.13	4.05	11.41
80	4.78	9.19	5.15

The probable relation between interest and mortality profits, where an adequate provision is made for mortality profit, may be illustrated by comparing the annual contributions to surplus which would result in an average case (age 65 male) on the basis of the participating rates adopted in 1934 by certain companies,[1] assuming that the rates of interest and mortality actually experienced are those then adopted for non-participating annuities. These bases are:

Interest: Participating, 3 per cent; non-participating, 3½ per cent

Mortality: American Annuitants Select Table (Males). Participating rate—age set back two years; non-participating rate—age set back one year.

AGE 65, MALE, ANNUITY \$100 PER ANNUM

Year	Profit from		Total
	Interest	Mortality	
(1)	(2)	(3)	(4)
1	\$5.29	\$1.61	\$6.90
5	4.54	3.15	7.69
10	3.75	3.91	7.66
15	3.03	4.74	7.77
20	2.41	5.61	8.02

The figures in column (2) are the values of

$$100 \cdot \frac{1}{p_{[64]+n-1}} - 0.005 a_{[63]+n-1} \ (3\%)$$

[1] Camp, T.A.S.A., XXXVI, 214.

and in column (3) of

$$100 \cdot \frac{1}{p_{[64]+n-1}} \cdot (q_{[64]+n-1} - q_{[63]+n-1})a_{[63]+n} \ (3\%)$$

The total dividends on this basis (excluding any loading dividend) are approximately constant, showing, in fact, a slight increase with duration. It is evident, however, that if either the interest profit decreased more rapidly or the mortality profit increased less rapidly (or decreased) dividends might show a decrease with duration. Assuming a constant interest rate the increase in dividends depends entirely on a sustained mortality "differential" and while the assumptions as to mortality may be conservative and may prove to be within the experience throughout, it is not at all certain, in view of past experience, that such a margin will be maintained indefinitely as will result in the increasing mortality profits needed to counteract the decrease in interest profit. It is, of course, possible that the interest rate may increase and so prevent any decrease in dividends. Under the contribution plan, however, decreasing dividends are quite likely to occur and for that reason the cash dividend method is not a satisfactory one.

Contribution Plan—Annuity Dividends

Under the dividend-annuity plan the cash surplus available for distribution each year is applied in the purchase of an additional annuity-due. In making the conversion a net rate on a conservative basis as to interest and mortality would be appropriate and the additional annuity may or may not be itself participating.

The advantage of this plan is that no matter how surplus may increase or decrease from year to year the total annual payment can never decrease. This is true, however, only if the dividend is declared as an annuity for which the company recognizes a definite liability and for which it will maintain the regular reserve. It is not true if the dividend is declared each year simply as a cash dividend of an amount equal to the total dividend annuities purchased. In that case the value of future dividend annuities is not a liability and may therefore be treated in the company's accounts as surplus and the amount of the total

annuity may be reduced from time to time if conditions become unfavorable.

The chief disadvantage of the annuity plan is that the total annuity payment provided is small in the early years as compared with that obtainable on a non-participating basis, rendering the participating annuity less attractive. The system provides, in effect, large dividends to those who live long as compared with those who do not. In fact, as shown below, if surplus arising only from excess interest is taken into account the total annuity payments provided by this system will, on the basis of a constant interest rate, and on the assumption that the dividend-annuities are participating, increase in geometrical progression. The fact that the survivors thus get increasingly larger dividends may be criticized on the ground that those who live longest cause loss rather than profit. The validity of such a criticism is open to question. It might be argued that, on this plan, each annuitant shares in surplus as earned during his lifetime and that it makes no difference, theoretically, whether the equitable share of surplus is paid in cash or in some other equivalent form. Where all stand the same chance of profiting there is no inequity provided the system of distribution is understood and agreed to at the inception of the contract.

As an illustration of how the dividend annuity plan might work out under average conditions we may make the same assumptions as before (see page 93), i.e., participating rate based on American Annuitants Select Table set back two years (male lives) with 3 per cent interest; experience, American Annuitants Select Table set back one year with interest at 3½ per cent. The following table shows for certain durations the cash surplus (as in the previous table), the additional annuity purchased each year and the total additional payment. For simplicity in computation, conversion has been made by a select annuity on the non-participating basis, no dividend being computed on the additional dividend annuities. If the dividend annuities were on a participating basis the conversion rate would be less favorable but the results would not differ greatly.

The table shows that on this plan the total payments are at first only slightly greater than the guaranteed annuity but eventually become substantially greater. The effective annual "dividend" is at first only about 10 per cent of the cash surplus,

increasing later to more than 200 per cent. These figures, of course, depend on the assumptions made and are merely illustrative.

AGE 65, MALE, ANNUITY $100 PER ANNUM
EXAMPLE OF DIVIDENDS AS ADDITIONAL ANNUITY

End of Year (1)	Cash Surplus (2)	Additional Annuity Purchased (3)	Total Additional Annuity (4)
1	$6.90	$0.67	$ 0.67
2	7.08	0.71	1.38
3	7.37	0.76	2.14
4	7.56	0.81	2.95
5	7.69	0.85	3.80
10	7.66	1.01	8.51
15	7.77	1.24	14.21
20	8.02	1.58	21.38

Where the additional dividend annuities depend only on excess interest earnings, the mortality rate being the same as that assumed, the relation between successive dividends may be expressed very simply as follows, the *total* annuity being on a participating basis.

Assuming first a constant earned interest rate of i' we have:

First Year:

$$\text{Cash dividend } \frac{1}{p_x}(i' - i)a_x = \frac{i' - i}{1 + i} \cdot a_{x+1}$$

$$\text{First dividend annuity} = \frac{i' - i}{1 + i} = D$$

$$\text{Total payment} = (1 + D)$$

Second Year:

$$\text{Cash dividend } \frac{1}{p_{x+1}}(i' - i)(1 + D)a_{x+1} = \frac{i' - i}{1 + i} \cdot (1 + D) \cdot a_{x+2}$$

$$\text{Second dividend annuity} = \frac{i' - i}{1 + i} \cdot (1 + D)$$

$$= D(1 + D)$$

$$\text{Total payment} = (1 + D) + D(1 + D)$$

$$= (1 + D)^2$$

etc.

nth Year:

$$\text{New dividend annuity} = D(1 + D)^{n-1}$$
$$\text{Total dividend annuity} = (1 + D)^n - 1$$
$$\text{Total payment} = (1 + D)^n$$

Ferguson calls this the "compound bonus" plan and points out that an excess interest rate of 1 per cent will approximately provide an annual additional annuity of 1 per cent. It will be noted that the amount of the additional annuities is independent of age, depending only on the amount of the guaranteed annuity.

Where the earned interest rate is not assumed to be constant the expression for the total annuity may be derived as follows, where i_n is the dividend interest rate in the nth year and i_0 is the assumed rate.

First Year:

$$\text{Cash dividend} = \frac{1}{p_x}(i_1 - i_0)a_x = \frac{i_1 - i_0}{1 + i_0} \cdot a_{x+1}$$
$$\text{Dividend annuity} = \frac{i_1 - i_0}{1 + i_0} = D_1$$
$$\text{Total payment} = 1 + D_1$$

Second Year:

$$\text{Cash dividend} = \frac{1}{p_{x+1}}(i_2 - i_0)(1 + D_1)a_{x+1}$$
$$= \frac{i_2 - i_0}{1 + i_0} \cdot (1 + D_1)a_{x+2}$$
$$\text{Dividend annuity} = \frac{i_2 - i_0}{1 + i_0} \cdot (1 + D_1)$$
$$\text{Total payment} = \frac{1 + i_2}{1 + i_0} \cdot (1 + D_1)$$
$$= 1 + D_2$$
$$\text{etc.}$$

nth Year:

$$\text{Dividend annuity} = \frac{i_n - i_0}{1 + i_0} \cdot (1 + D_{n-1})$$
$$\text{Total payment} = \frac{1 + i_n}{1 + i_0} \cdot (1 + D_{n-1})$$
$$= 1 + D_n$$

also

$$1 + D_n = \frac{(1 + i_1)(1 + i_2) \cdot \cdot \cdot (1 + i_n)}{(1 + i_0)^n}$$

In the above, values of i_n are not necessarily all greater than i_0. The total payment may therefore decrease provided the total "dividend," D_n is declared each year as a cash dividend. If the new "dividend-annuities" are declared each year as additional (guaranteed) annuities the total payment cannot be decreased so that in that case if, in any year, $i_n < i_0$ the dividend for that year would be zero.

Equalized Cash Dividends

This method, also called the "uniform cash method" which, in effect, is a modified form of the contribution plan, forms the basis of the methods used by the principal companies now issuing participating annuities. It has its limitations, however, because any system of equalized dividends is applicable only where the total annual contributions to surplus are on a decreasing (or constant) scale, not where they are increasing. The total annual contributions may (as shown above) increase where there is a substantial mortality margin at all durations especially if the actual interest rate is not much higher than that assumed (in which case contributions from interest will not decrease so rapidly). In these circumstances an equalized dividend, based on the assumed actual conditions, would result in overpayments of dividends in the earlier years. In what follows it is assumed, therefore, that the experience rates of interest and mortality are such that the total annual contribution to surplus does not increase.

If a is the rate per dollar, on a participating basis, involving (as we have seen is necessary) ultraconservative rates of interest and mortality and a loading in excess of actual requirements, and if a' is the rate on a reasonably conservative basis which is only slightly more stringent than required by current conditions, the company could afford to pay, so long as conditions remained about the same, an annuity of a/a' for each dollar of guaranteed annuity. This provides a uniform cash dividend of $[(a/a') - 1]$. The full benefit of current conditions, if more favorable than those involved in a', may, if desired, be allowed by providing

additional dividends on the contribution basis. These would, normally, be declared as cash dividends (whether calculated as such or as an additional dividend annuity), being added to the "basic dividend" as above. They would, of course, be computed on the basis of an annuity of a/a' per annum and would depend on the surplus provided in excess of the requirements on that basis.

This plan, since it involves the equalization of surplus earnings, requires additional "surplus reserves" as well as the regular reserves in order to maintain the full annuity rate. On the basis of the assumed actual conditions the purchase price a_x may be divided, as at date of issue, into

(1) Reserve for guaranteed annuity: a'_x

(2) Reserve for surplus:

$$\frac{a_x}{a'_x} \cdot a'_x - a'_x = a_x - a'_x$$

If the reserve held for the guaranteed annuity (which is not necessarily on the same basis as that upon which premiums were computed or on the basis denoted by a') be denoted by a'', the "reserve for surplus" at the end of n years will be

$$\frac{a_x}{a'_x} \cdot a'_{x+n} - a''_{x+n}$$

i.e., the total fund required to maintain the annuity on the basis denoted by a' less the reserve actually set up for the guaranteed annuity. Camp proposes to treat this amount as "contingency surplus." In whatever way the amount of this "surplus reserve" may be shown in the accounts, however, it must be separately computed and reserved as a necessary part of the annuity fund.

If the basic dividend $(a/a' - 1)$ is not computed on too liberal a basis and if additional dividends are added to it, as indicated above, depending on current fluctuations of interest and, perhaps, also of mortality profit, there should be no occasion to change the basic dividend except at long intervals and only when material changes have occurred in the interest or mortality rates. If the basic dividend is calculated on more liberal assumptions more frequent revision will be necessary. Where conditions have changed (either favorably or adversely) so that a reconsideration of the basic dividend seems desirable the new "total annuity,"

i.e., the new value of a/a', will be obtained from the total fund (reserve plus "surplus reserve") by use of a new current-experience annuity at the attained age. As Camp points out, the total fund varies according to age at issue and duration so that the new basic annuities must be computed separately for each duration at each attained age.

In order to reduce the probability of having to decrease the basic uniform dividend $[(a/a') - 1]$ because of adverse changes in the interest or mortality rate, Ferguson suggested as an alternative method that the initial total payment (i.e., from the end of the first year) be determined by using a more conservative value of a', say a'' (where $a > a'' > a'$) and that the initial difference of $\left(a - \frac{a}{a''} \cdot a'\right)$ be applied in the purchase of an additional deferred (dividend) annuity which, in event of adverse experience, could be reduced without affecting the actual payments being made, provided, of course, the reduction takes place within the deferred period. The existence of this deferred annuity is not communicated to the annuitant, being merely a form of dividend reserve. The deferred period is arbitrary. This might be a practicable plan although it appears to be unnecessarily complicated. It has the same defect as any plan which provides, in effect, for *increasing* dividends, namely, that it results in smaller total payments than necessary in the early years, which is a practical disadvantage. The value of the method in preventing any decrease in the annual payment depends (as Marshall points out) on how much of the total fund is set aside for the deferred annuity. The more that is set aside the smaller will be the early payments.

As already indicated, methods used in practice are sometimes based on combinations of the different systems indicated above. Thus the method described by Camp is a combination of the uniform (equalized) dividend plan and the dividend annuity plan. In another company interest profit is distributed as a (decreasing) cash dividend on the contribution formula while loading and mortality profit are distributed by the uniform dividend plan. This provides a decreasing cash dividend so long as all conditions remain the same.

Dividends on participating annuities will presumably be payable annually even where the annuity payments are made more

frequently. This may be regarded as a disadvantage of the participating plan since it involves comparatively low periodical payments with an additional payment once a year as compared with uniform payments on the non-participating basis. It would, of course, be possible to spread dividend payments over the ensuing policy year.

Although reference has been made throughout this chapter only to immediate single life annuities the same methods apply to other types of annuities. The use of the equalized dividend plan simplifies the treatment of refund, cash refund, and joint and survivor annuities since they may then be treated in the same way as life annuities, although with dividend results that may sometimes perplex the policyholder, since dividends on the higher priced plans are likely to be permanently less than on the others because of the insurance element involved.[1] The apportionment of surplus on these contracts seems never to have been discussed and appears to require analysis of the reserve into its components.

Retirement Annuities (Deferred)

These annuities are, prior to their maturity, practically accumulations of net premiums at interest and are nearly always issued on a participating basis. Dividends consist primarily of interest in excess of the rate assumed. The element of mortality loss or profit does not enter if, as is nearly always the case, the amount payable at death is based on the accumulation to that date of the net premiums. Theoretically, it should be allowed for if, as is frequently the case, the death claim amount at any time within a contract year is the reserve accumulation at the end of that year. At least one large company does make adjustment in its retirement annuity dividends for the average loss of interest arising from deaths within the year.

The dividend in case of annual premium contracts usually includes a refund of loading less expenses, and sometimes part of the loading in single premium contracts is returned in the form of a loading dividend. Expenses in the latter case are, probably, more suitably handled by making a small reduction in the interest rate. In computing these dividends the same general principles will apply as to life insurance policies. On

[1] For discussion and a suggested alternative treatment, see T.A.S.A., XXXVI, 221.

account of the nature of the contract, however, and because of the fact that more of them will be issued when the general interest rate obtainable elsewhere is low while surrenders are likely to increase if the general rate of interest increases, there is justification for allowing a generally lower interest rate than for other types of participating contracts on the ground that they call for a greater degree of liquidity of assets.

At maturity most of these contracts call for an annuity with a minimum number of guaranteed payments with the right, sometimes, to select a different type such as a regular life or refund annuity. Participation, if any, will be on the same basis as for other life annuities or supplementary contracts, as the case may be.

Supplementary Contracts

Dividends on supplementary contracts consist of excess interest on the fund held. In the case of "supplementary contracts—life," i.e., life income settlements with a minimum number of guaranteed payments certain, participation is usually, if not invariably, restricted to that portion of the contract providing for the payments certain. The general considerations involved in the determination of the interest rate have been discussed in a previous chapter. In order to provide for expense most companies allow a somewhat lower interest rate on supplementary contracts than the rate used in the regular dividend formula and in some cases a further distinction is made in regard to funds subject to withdrawal. An expense charge in the form of a deduction from the interest rate seems to be both proper and necessary and the payment of a lower rate on funds subject to withdrawal is justified by the greater degree of liquidity required.

Some practical questions arise in regard to "dividends" on supplementary contracts when these are payable, as is usually the case, with the same frequency as the guaranteed interest or instalments. The dividend for any contract year unless paid annually may, if contract payments are other than annual, be paid partly in one calendar (or dividend) year and partly in another. In event of a change in the interest rate, dividend payments may be computed at the new rate from the beginning of the calendar or dividend year, producing different rates of

dividends in the same contract year, or may be maintained on the same basis for the balance of the contract year.

Under the interest option where, as is usually the case, either the contract provides for, or it is desired to pay, excess interest with the first fractional payment, the most practical system may appear to be to pay the same dividend rate with all payments falling due in the same calendar or dividend year. Under instalment settlements if dividends are paid during the first year and dividends (and total payments) are equalized during contract years, a revision of the amount of the periodical payment is necessary if the interest rate is changed during the contract year. This system involves the apparent objection that when the rate is changed different contracts receive different effective interest dividends during the same contract year, according to the frequency of payment.

Where the contract is drawn so as to permit it, the more logical basis would be to pay no dividends until the end of the first contract year and, in the case of payments other than annual, to spread the dividend over the payments of the ensuing contract year. This procedure, as in the case of dividends applied to reduce each instalment of the year's premium on an insurance policy, involves the practical objections (1) that payments during the first year (other than annual) would differ substantially from those in the second year and (2) that in the event of a change in rate the company would apparently be continuing payment at the former rate for nearly a year longer.

The calculation of the dividends in the case of the interest option presents no difficulties. If the total rate payable, on an annual basis, is i' (consisting of the guaranteed rate i plus excess interest of $(i' - i)$) each total payment where interest is payable m times a year should be $j'_{(m)}/m$ where $j'_{(m)}$ is the nominal rate corresponding to i'. For practical purposes the "dividend" may conveniently be taken as $\frac{1}{m}(j'_{(m)} - i)$ and the guaranteed interest as i/m, without any material error.

In the case of instalment settlements (payments including both principal and interest) dividends are normally on a decreasing scale. This is apt to be unsatisfactory from a practical point of view and sometimes leads to misunderstanding on the part of the payee. Consequently it is sometimes suggested

that payments be equalized in somewhat the same manner as has been described above for dividends on life annuities. Where settlement is by an annuity certain this would be done by fixing an equalized payment of $a_{\overline{n}|}/a'_{\overline{n}|}$ where $a_{\overline{n}|}$ is at the guaranteed rate and $a'_{\overline{n}|}$ at a higher rate slightly lower than the full rate currently payable. The same general principles and the same necessity for periodical revision would apply to such a system as have been discussed for life annuities.

Where the settlement involves an annuity certain plus a deferred life annuity the equalization plan may be applied either to the annuity certain only or to the entire annuity. Where, as is usually the case, participation applies only to the payments certain, excess interest during the period certain may be spread over the guaranteed payments only or over the entire life of the contract.

The calculation of excess interest on annuities certain is discussed by Fassel in a paper[1] to which reference should be made. As a matter of convenience it is probably desirable to equalize the dividend payments during each supplementary contract year. One practical method where dividend payments are to commence in the first year is to compute the amount of excess interest for one year at rate $(i' - i)$ on the mean amount of principal outstanding during the supplementary contract year, namely,

$$P_{n-1} - \frac{m-1}{2} I^{(m)}$$

where P_{n-1} is the amount of principal outstanding after payment of the instalment due at the beginning of the nth year, $I^{(m)}$ is the amount of each guaranteed instalment and m denotes the frequency of payment. The total excess interest is then converted into the equivalent $m$thly dividend payment by dividing by $ms^{(m)}_{\overline{1}|}$ calculated at the effective rate i'.

[1] R.A.I.A., XIV, 21.

CHAPTER VII

GROUP INSURANCE

The methods of surplus distribution used in connection with group insurance are quite different from those applicable to ordinary business. As a preliminary to a description of these methods it is desirable first to review briefly the general basis upon which the group business is conducted and to examine some of the differences between group and ordinary insurance.

Premium and Reserve Basis—Sources of Surplus

Since 1919 group insurance has been written on the basis of the *T* rate as required by the law of the State of New York. The *T* rate for groups in standard occupational classifications is the net premium on the basis of the AM(5) Table with 3½ per cent interest loaded $1.70 per $1,000, plus 6½ per cent of the gross premium. This is a minimum initial rate for both mutual and stock companies.[1] In mutual companies, under normal conditions, dividends are paid at the end of the first and subsequent years. In stock companies rate reductions take the place of dividends and as rate reductions may be made retroactive for one year there is very little effective difference between the two systems so far as the policyholder is concerned, the insurance being, in either case, in effect, participating.

There is, however, some difference from the company's point of view, the mutual company regarding the dividend mainly, if not entirely, as a share of surplus earned during the year while the stock company regards the rate reduction more in the nature of a correction based not only on the surplus earned but on the probabilities of the future. Thus, the methods used in computing dividends and rate reductions are not necessarily the same.

---

[1] Mutual companies, for a few years, added from 5 to 7 per cent to the *T* rate but since about 1926 both stock and mutual companies have used the same rate.

In considering the sources of surplus of the business as a whole and as arising out of the premium assumptions there are, as in ordinary insurance, three elements involved, namely, interest, mortality and expense.

Interest. Group insurance is written on the one year renewable term plan. In some companies the reserve held is one-half of the periodical net premium. In others the unearned premium reserve is based on the actual premium due date. Frequently the premium is payable monthly but in any case excess interest on the reserve may be regarded as negligible insofar as it affects dividends payable and no account is usually taken of interest in the dividend formulas, the entire amount of excess interest being thus carried to surplus and contributing a part of the necessary contribution to the contingency fund.

Mortality. The New York insurance law[1] requires the use of the American Men Table of Mortality. Because of the fact that a large proportion of the group business hitherto issued included a substantial total and permanent disability benefit[2] the AM(5) Table is not a suitable representation of standard group claim experience in which are included claims by disability as well as by death. On the other hand, the disability feature of group insurance has been generally discontinued in recent years so that the large volume of experience which has been accumulated is likewise unsuitable for recent contracts. It will take some time before the experience on group contracts without the disability benefit will be adequate for rate calculations. In general, the actual claim experience of contracts with the disability feature, including both death and disability claims, has been below the mortality rate of the AM(5) Table at attained ages up to 40 and over 60 but above that table between these ages.

The Combined Group Mortality Experience shows mortality ratios (by numbers of lives) for about 125 industrial groups combined into five major occupational classifications of which two (clerical and "other non-hazardous") are standard and the others substandard.

---

[1] Sec. 101-A, Par. 3.

[2] The disability clause in group insurance provided for payment of the face amount of insurance in instalments—not the additional waiver and income benefits as in ordinary policies.

In the clerical classification the experience (including disability) has been about 80 per cent of the expected mortality by the AM(5) Table. Individual groups may, however, show quite different experience because of chance fluctuations or differences in age distribution or because they are composed of different classes of employees or because of differences in working conditions. Mortality in the "other non-hazardous" class has been close to the expected. Except in the clerical classification there has therefore been little margin for dividends from mortality on standard business.

Although there is no medical examination, mortality experience shows some evidence of selection in a tendency of the mortality rate by attained age to increase with duration. This increase is offset as in ordinary insurance by the decreasing expense rate.

In connection with mortality experience account must be taken (as shown later) of loss from excess mortality on conversions. This loss appears in the experience of the ordinary business but is, in effect, the cost of an additional benefit under the group coverage which should be paid for out of the group premiums. Mortality on insurance converted to ordinary plans on termination of employment is in the aggregate about 300 per cent of the mortality on regularly issued business and in the first year about 600 per cent. Cammack has shown[1] that the capitalized cost of excess mortality on conversions is about $50 per $1,000 of converted insurance. More recent experience indicates a higher figure. The New York Insurance Department has provided by ruling for a transfer in the Gain and Loss Exhibit on this account of at least $40 per $1,000. Where conversion losses are charged directly against premiums as an expense the necessary charge is variously estimated at from 0.5 per cent to as much as 3 per cent. The latter figure is, however, considered too high by most authorities and the best estimate appears to approximate an average of 1 per cent.

Expense. The loading in the *T* rate is a decreasing percentage of the gross premium as the premium increases. If the gross premium is $10 the loading is 23½ per cent; if $15, it is nearly 18 per cent; if $20, it is 15 per cent. The average gross premium

[1] T.A.S.A., XXXIII, 347.

on total group insurance in force is about $12 per $1,000 and the average loading is over 20 per cent. This is substantially in excess of the aggregate expense rate, thus leaving *in the aggregate* a margin of loading over expenses which is available for contingency fund, conversion loss and dividends.

The actual situation, however, is that while the loading in the *T* rate is more than adequate for large or medium sized groups it is insufficient for small groups. In group dividend or rate reduction formulas a proper assessment of expenses based on the *size* of the group is essential, because of the wide difference in expense rates on different groups and because of the system of distribution. As a basis for such an assessment, expenses of the group department must be analyzed in such a way as to furnish definite unit costs per group, per certificate or life involved, per premium payment, per dollar of premium, and so on, distinguishing new and renewal expense. Such a process is outlined by Bjorn.[1] These unit costs then furnish a means of establishing an expense rate applicable to total premium and depending on the size of the group, i.e., the number of lives, as well as on the average amount of insurance per certificate. A general verification of these factors may be obtained by recomputing the aggregate group expenses on the basis of the unit costs.

Bjorn, D. S. Craig[2] and Williamson[3] give tabulations of expense rates according to size computed in this way. The table given by Williamson, which is based on a study of unit costs per contract, per certificate and per dollar of premium, new and renewal, in four companies, is reproduced below. It shows (as do the other tabulations referred to) the *average* expenses, first and renewal, spread over the first five years. Although it is desirable to charge off initial expense as quickly as possible, such expense may, because of the relatively high persistency rate of group business, properly be equalized over that period or even a longer period.

On the basis of these figures and assuming an average loading of 20 per cent, groups of less than 100 lives do not provide their own expenses unless the average amount of insurance per certificate is at least $2,000, while groups of over 200 lives are self-

[1] T.A.S.A., XXXI, 74.
[2] T.A.S.A., XXXI, 312.
[3] T.I.C.A., X, I, 627.

AVERAGE EXPENSES AS PERCENTAGE OF PREMIUMS—FIRST FIVE YEARS

Number of Lives in Group	Average Insurance per Life				
	$500	$1,000	$1,500	$2,000	$3,000
Under 50	76%	45%	33%	30%	25%
50–59	61	37	30	25	21
60–74	55	35	28	24	20
75–99	50	31	25	22	18
100–149	42	28	23	20	17
150–199	37	25	21	18	16
200 and over	32	22	19	16	14

supporting (so far as expense is concerned) if the average insurance exceeds, in general, $1,000. The percentages do not change materially when the number of lives exceeds 200 as is shown by the chart given by Craig and referred to above. It must be remembered in considering such figures that in the past little margin has arisen in the aggregate from mortality profit, so that provision for contingency fund and conversion losses as well as expenses has had to be made largely from loading.

It is evident that variation of the expense charge by size is essential in any equitable system of dividends so long as the *T* rate is in use and also that in very small groups or in medium sized groups with very small average insurance per certificate the *T* rate is actually insufficient so that excess expenses on such groups must be charged against the excess loading of larger groups.

SYSTEM OF DISTRIBUTION—GENERAL BASIS

In ordinary insurance the unit of selection is the individual life. The plan of operation is based on the insurance of a sufficiently large number of lives to secure an average mortality experience and, generally speaking, the fund is accumulated as a whole on the basis of the average rates of mortality, interest and expense. Dividends are computed at a uniform rate on all policies within the same classification. In particular, no distinction is made in the *rate* of dividend because of different amounts of insurance although, in theory at least, such a distinction could and should be made. The elements of the dividends allotted to individual policies thus depend on average rates of

mortality, interest and expense based on the experience of the whole company or of classes comprising large numbers of policies or "units."

The same general principles could be applied to group insurance. The unit of selection is the group and the successful operation of the plan depends on the insurance of a large number of groups. It is not essential that all groups should show a favorable mortality experience. The underlying insurance principle is one of averaging whereby the additional cost from the relatively unfavorable experience of some groups is offset by favorable experience on others. It would not, however, be suitable or practicable to distribute surplus or to compute rate reductions on a uniform basis for all groups in the same general classification. In some cases dividends were, however, for a short time, allotted on the basis of a uniform percentage of the premiums paid irrespective of size or claim experience, and, on a purely mutual basis, there is much to be said for uniform dividends particularly in the case of small or medium sized groups. The practical difficulty is in regard to larger groups with favorable experience. It is evident that from a practical point of view a group is essentially different from a single individual policy (which is apparently as good as any other individual policy of the same class while it remains in force) since good or bad experience develops in regard to each group policy during its currency.

There is thus no necessity (because of lack of knowledge) of permitting unprofitable groups to share equally in the divisible surplus with profitable groups on a broad "insurance" basis. From a practical standpoint, too, it is not considered equitable to allot to a large group which has had a loss ratio of, say, 75 per cent, a mortality dividend based on a general loss ratio of 90 per cent. Nor is it considered practicable where, as we have seen, the effective ratio of expenses to premiums varies so widely according to size of group, to assess expenses at a uniform rate to groups of all sizes even if that were theoretically defensible.

This is partly due to the fact that group insurance has been and usually is sold with the understanding that the ultimate cost will be based to a large extent on the individual experience of the group.

These considerations have led to a recognition of the general principle that the method of surplus distribution in group

insurance must take into account not only the experience of the whole business or of the occupational or industrial classification but also the experience of the individual group itself and that a greater degree of recognition of the individual experience is necessary in the case of large groups than small ones; also that such recognition must extend both to the mortality experience and to the expense rate. It is, of course, obvious that, whatever system may be adopted of giving recognition to the experience of individual groups, the premium income as a whole must provide for all claims and expenses (including a proper share of the general overhead expense of the company) as well as for a suitable contingency fund and that losses on bad groups must therefore be paid for in part out of profits on good ones.

Group dividend and rate reduction formulas have therefore developed along the line of *experience rating*. The process of experience rating aims at determining individual risk rates depending in whole or in part on the risk's own experience.[1] The theory is based on the idea that, among a number of individual risks included in a single classification, there is a certain degree of variation apart from the purely chance variations which would, in any case, exist if all the risks were entirely homogeneous and typical. The principle is applicable to group risks wherever variations in the experience of the individual groups exist such as may be considered characteristic of the individual risks rather than of the class.

The theory of experience rating is discussed by Whitney[2] and in other papers in the Proceedings of the Casualty Actuarial Society; also in a paper by Keffer,[3] to all of which the student may refer. The practical application of the theory to group insurance is in the method of determining the weight or, as it is called, the "credibility" to be assigned to the actual mortality experience of individual groups. As we have seen it is not practical to charge each group with mortality on the basis of the entire business or class experience and it would be equally impractical to rate each group on the basis of its own mortality experience. In the dividend or rate reduction formula, therefore, the actual mortality experience must be modified or weighted by a *credibility factor* which will

[1] Dorweiler, P.C.A.S., XXI, 2.
[2] P.C.A.S., IV, 274.
[3] T.A.S.A., XXX, 130.

depend primarily on the total exposure or *size* of the group. In addition to this modification there must also be a further adjustment of the surplus on the experience basis in the form of a *reduction factor* to provide for accumulation of a contingency fund, for losses in some groups and for conversion losses.

The general type of dividend formula (which, as we shall see later, has many variations) may be expressed, for purposes of illustration, in the form

$$a[S' + z(C - D)] \tag{1}$$

where S' = the surplus of the group computed on the assumption of mortality experience in accordance with that of the whole industrial classification to which the group belongs or (where there is insufficient exposure by industries) of the entire group business, i.e., premiums less assessed expenses and "expected" claims by class.

C = "expected" claims by class experience.

D = actual claims.

z = "credibility factor," approaching (but not necessarily reaching) unity as the size of the group increases.

a = a reduction factor to adjust total dividends to the aggregate amount of divisible surplus.

If $z = 1$ (i.e., maximum credibility) formula (1) becomes aS, where S is the *actual* surplus, i.e., premiums less expenses and actual claims, the dividend being then based entirely on the actual experience. If $z = 0$ (zero credibility) the expression becomes aS' and the actual mortality is assigned no weight. Zero credibility may be frequent in practice where exposure is small; full credibility is rarely given. The credibility factor is an important element in the group dividend formula and is considered in more detail in the following section.

Credibility Factor

The credibility factor z, the purpose of which is to give due weight to the actual mortality experience, is properly a function of the number of years of life exposed to risk both in the individual group and in the entire experience. The determination of a scale of values of z according to exposure involves theoretical considerations which are discussed in the papers referred to above. Theo-

retical formulas for computing z are, in general, not suitable for practical application and in practice various approximate methods have been used.

Keffer states that the form of z preferred for practical use is

$$z = \frac{P_n}{P_n + k} \tag{2}$$

where P_n is the total premium for the risk and k is a constant to be determined by judgment and inspection. He also develops a formula for the mean ratio of the true mortality rate to the average rate which, when expressed in terms of amounts, is

$$r = \frac{(m+1)A + D}{(m+1)A + C} \tag{3}$$

where A is the average amount of insurance on each life, C and D are the expected and actual claims as before and m is a constant. The relation between formula (3) and formula (2) may be shown by substituting C, i.e., the expected claims (which are approximately the total *net* premiums) for P_n and $(m+1)A$ for k in formula (2), which then becomes

$$z = \frac{C}{C + (m+1)A}$$

The formula for the adjustment of death claims to the experience basis is then

$$C + \frac{C}{C + (m+1)A}(D - C) = rC$$

where r has the value shown in formula (3).

Another method of computing the values of z is based on the probable deviation of the ratio of actual to expected claims.[1] In one company this method has been used, the value of z being the probability that the actual claims will be within a certain percentage of those expected.

Glenn[2] gives a simple formula which he states is commonly used in certain forms of casualty insurance and which may be applicable to group insurance where the class mortality is given full credibility. This formula, which is based on the fact that the accuracy

---

[1] See Medical Imp. Study, 1929, p. 12.
[2] T.A.S.A., XXX, 599.

of an average increases as the square root of the number of observations, is

$$z = c\sqrt{n} \tag{4}$$

where c is a constant the value of which is determined by inserting in the formula the value of n entitled to full credibility, for which $z = 1$. For example, if one million life years' exposure is required for 100 per cent credibility (i.e., $z = 1$) the value of c is 0.001. On this basis the values of z for other exposures would be as shown in the table given below. In order to illustrate the values which may be assigned to z two other specimen scales are added. All these figures are, of course, purely illustrative and are not given as an indication of the actual degree of credibility which should be adopted.

ILLUSTRATIVE VALUES OF z

Exposure in Life Years, n	$0.001\sqrt{n}$ (on mean amount)	Scale A	Scale B
500 and under 1,000	0.027	0.075	0.000
1,000 and under 2,500	0.042	0.125	0.000
2,500 and under 5,000	0.061	0.175	0.050
5,000 and under 10,000	0.087	0.250	0.100
10,000 and under 25,000	0.132	0.400	0.150
25,000 and under 50,000	0.194	0.600	0.200
50,000 and under 100,000	0.274	0.850	0.300
100,000 and under 200,000	0.387	1.000	0.400
etc.			
850,000 and under 1,000,000	0.962	1.000	0.950

In practice a considerably lower basis of exposure required for maximum credibility than any of the scales shown seems to be not uncommon and in some cases the exposure for maximum credibility may be as low as 5,000 life years or even lower. The maximum credibility allowed is, however, not necessarily 100 per cent for any exposure. In rate reduction calculations the trend seems to be to require a substantial exposure before allowing *any* credibility to past experience. A point to bear in mind is that when a relatively low maximum credibility factor is used, a higher charge must be made against the groups with good experience to provide for the "negative asset shares" of the groups with poor mortality. The selection of a suitable scale of credibility factors is thus a practical as well as a theoretical problem and one upon which, in practice, there is considerable difference of opinion.

It will be noted that the application of a credibility factor to the mortality experience in the manner indicated by formula (1) is equivalent to applying a weighting factor to the actual claims and the complementary weighting factor to the expected claims, i.e.,

$$\text{Adjusted claims} = zD + (1 - z)C.$$

An important point in the practical application of the z factor is that in computing dividends and rate reductions all elements entering into the calculation are computed on the basis of *total* exposure from issue, i.e., total premiums, claims and expenses, the current dividend being obtained by deducting from the total surplus so obtained all dividends previously allotted. The weight assigned to actual experience in each group thus increases with duration. One result of this is that where a liberal scale of credibility is adopted the credibility from *total* exposure in smaller groups may result in too great fluctuations for later individual years, in which case some further adjustment may be required.

Reduction Factor

The reduction factor, denoted by a in formula (1), is required to provide for necessary charges against the business as a whole as distinguished from the specific charges for claims and expenses applicable to individual groups and also to control the aggregate amount of surplus distributed. In practice the reduction factor may be graded by size of group, increasing (i.e., rate of reduction diminishing) as the size of the group increases, with the same general object, as in the credibility factor, of making dividends correspond more nearly to the actual accumulated surplus in the case of large groups.

The principal things to be considered in determining the reduction factor in addition to conversion losses, already discussed, are (1) losses on groups where the guaranteed rate has been insufficient; and (2) provision for a contingency fund.

Losses. A dividend formula of the general type described may produce negative dividends due to unfavorable experience. These are, of course, uncollectible and the positive dividends must be reduced to a corresponding extent. In the case of rate reductions it is necessary to take account of future unfavorable experience on cases where an insufficient rate cannot be increased because of a rate guarantee.[1]

[1] In some companies premium rates are guaranteed for the first five years. In two large companies the rate guarantee is limited to one year.

Contingency Fund. Provision for a contingency reserve may be made in the case of participating policies as here suggested by means of a reduction factor applied either to the calculated surplus or to the total premium. In calculating rate reductions provision is generally made by a direct charge against the premium as illustrated later. Another method provides for an increase in the renewal expense charge for all policies, with a greater charge in the case of groups showing a loss.[1]

It is generally considered that a contingency fund of at least 50 per cent of one year's premium should be aimed at. Williamson indicates that, in the absence of any other special provision for epidemics or other catastrophes, the figure should be nearer 100 per cent. The influenza epidemic of 1918 caused an *increase* of 60 per cent in group mortality and, on this basis, assuming that such an epidemic may occur about every thirty years, an annual charge of 2 per cent of net premiums would be necessary for epidemics alone. Besides the epidemic and catastrophe hazard the contingency fund must provide for other mortality fluctuations including such as might arise from the changing conditions of industry. Probably an annual charge of from 3 to 5 per cent of premiums would not be excessive. General opinion at present, however, is that an annual charge of 2 per cent of each year's premium with a total objective of 50 per cent of one year's premium is a reasonable provision for contingencies.

Examples of Dividend and Rate Reduction Formulas

Dividend Formulas. In practice, dividend formulas, although in general based on the principles indicated above, vary widely in detail. Most companies have made many changes and modifications in their dividend methods. The following formula[2] (subsequently modified, as described below) was one of the earliest to be used for dividend calculations.

$$\text{Current dividend} = a[S' + z(S - S')] - \text{total previous dividends} \qquad (5)$$

where S' = total earned premiums on individual group since issue, multiplied by the ratio of the total divisible

[1] See T.A.S.A., XXX, 604.

[2] See P.C.A.S., VIII, 308. The notation has been changed to correspond with that adopted above.

surplus to the total group premiums (calendar year basis);[1]

S = actual "surplus" of individual group, i.e., total earned premiums less total actual claims and assessed expenses;

and a and z have the meanings already indicated.

In describing this formula Bassford points out that in computing S' (surplus on basis of experience of whole business or class) the use of the experience of the industrial classification rather than that of the whole group business would be more correct, because more homogeneous, but may be impracticable because of insufficient experience by industries.

A modification of formula (5) is arrived at as follows: Denoting accumulated actual and expected claims (i.e., total since issue) by D and C respectively and total premiums and expenses by P and E, we have:

$$S = P - E - D$$

and, if we assume that $S' = P - E - C$ (as in definition on page 112) formula (5) may be written, (omitting deduction of previous dividends)

$$\begin{aligned} & a[S + (D - C) - z(D - C)] \\ &= a[S + (1 - z)(D - C)] \qquad (6) \\ &= a(P - E - x) \qquad (7) \end{aligned}$$

where $x = C + z(D - C)$. This is the formula described by Benjamin.[2]

In this form it is not necessary to compute the "expected surplus" (i.e., S') as well as the actual surplus on each group as in formula (5).

In another method used in practice the distinguishing feature is the accumulation of a surplus account on a definite basis for each group policy, the "reduction factor" being expressed as a "contribution to surplus account." Acquisition expenses are, in the first place, charged against the surplus account so that the initial "surplus" is negative. The surplus account is

[1] It will be noted that this definition of "expected surplus" is not quite the same as that on p. 112.

[2] T.A.S.A., XXX, 601.

charged with "adjusted claims," expenses and dividends and is credited with premiums and interest. A credit or charge is made as necessary to distribute to the surplus account of each continuing group its share of the aggregate net profit or loss on terminating groups. The contributions to surplus account deducted in the dividend formula consist of (1) the amount required to amortize the initial expense charge and (2) an amount to build up a contingent surplus. The latter amount depends on (*a*) the size of the policy; (*b*) the absolute amount of surplus existing at the beginning of the policy year, a reduction being made in the amount of the contribution in the case of a positive surplus and an addition in the case of a negative surplus; and (*c*) the difference between the claim charge in the dividend formula and the "adjusted claims" charged in the surplus account. The aggregate of the "adjusted claims" for all groups equals the aggregate actual claims for each year, the adjustments being made so as to reduce fluctuations of claims in individual groups from a weighted average.

The basic assumption underlying this process is that the premium rate under each policy is adequate to cover, over a long period of years, both expenses and claims under that policy with substantially the same ratio of claims to premiums regardless of size or class. The dividend formula used is of the same general *type* as above described, i.e., the dividend is the excess of the premium over (1) assessed expenses based on size of policy; (2) the "adjusted claims" described above; and (3) a reduction factor—expressed in this case as "contribution to surplus." It will be noted that the second item, "adjusted claims," appears also in the "contribution to surplus," these two amounts cancelling in the dividend formula, leaving as the net claim charge the reported claims modified by a credibility factor based upon the accumulated past experience of the group. Provision for excess mortality under conversions is included with reported claims.

As an example of a different procedure the following description is given of a method which is based on the principle that groups of the same size (number of lives) should contribute the same proportion of premiums toward creating a contingency fund of a definite percentage of the premium. By this method the dividend payable is the sum of (1) a basic dividend depending on the

actual surplus of the group and (2) a percentage dividend depending on the size of the group, each weighted as described below.

The basic dividend, expressed as a percentage of the total surplus, is the amount which can be returned after deducting from the average rate of surplus earnings for the whole duration of the group, i.e., S/P, the contribution necessary to create the desired contingency fund according to the size of the group.

The percentage dividend is a low minimum percentage of the annual premium (say 3 to 5 per cent), the percentage increasing with duration, plus an additional 1 per cent for each $100,000 (say) of total insurance in the group, up to a stated maximum.

The weighting factor used for the basic dividend is 1 per cent for each m total life years of exposure and, for the percentage dividend, the complementary factor. The formula for the annual dividend on this basis may be expressed as

$$zdS + (1 - z)\left(p + \overline{n - 1} + \frac{A}{100{,}000}\right)0.01P_n$$

where z = $0.01/m$ times total life years exposure ($z \not> 1$).
d = rate of annual basic dividend applicable to total actual surplus and depending on average surplus earnings (S/P) and size of group.
S = total actual surplus earnings of the group.
p = minimum percentage in "percentage dividend."
n = duration.
A = total insurance in force in the group.
P_n = total annual premium of the group.

The foregoing formula as actually applied in practice is subject to various modifications which are of a practical character and not essential to the general basis. Additional factors may be introduced into any of the formulas described above such as (1) to allow for different age distribution in otherwise similar groups or (2) to allow for differences in mortality experience due to different formulas of insurance. In one case the latter adjustment is made by introducing a multiplier of the form

$$\frac{1}{2}\left(1 + \frac{\text{average insurance per life}}{\text{maximum insurance per life}}\right).$$

Frequently no dividends are paid under very small groups (because of the heavier expense rate) and sometimes modifications

of the dividend according to formula are made where the dividend would exceed the current year's surplus earnings. No general rules can be given as to such modifications which are partly arbitrary and usually on a quite complicated basis.

Rate Reductions. The elements entering into the calculation of a rate reduction are, of course, the same as in the calculation of a dividend. As already indicated, however, there should be less emphasis on past experience and more on probable future experience. Consequently, in rate reduction formulas there should be a requirement of a substantial total exposure before the actual experience is given any credibility.

In the formula given by Bjorn[1] the reduction is based on the total premiums and earnings since issue and the rate of reduction obtained is therefore the average rate which could have been granted for the whole period. The formula does not differ in principle from the dividend formulas already described. The adjusted surplus used (after five years) in determining the ratio of reduction may be expressed, in the notation used above, as

$$(P - E - r_1D - r_2C - aP)$$

where C is the expected mortality based on the experience of the industry taken as $s \times$ total net premiums, s being the ratio of the actual mortality of the industrial classification to the expected by the premium basis [AM(5) Table]. No information is given as to the basis of the weighting factors r_1 and r_2 (corresponding to the z factor) but the degree of credibility indicated (namely 100 per cent for an exposure of 1,000 life years) is extreme.

A different method, more characteristic of rate reduction as distinguished from dividend calculation, may be indicated in a very general way as shown on page 121.

In this example it is seen that the major part of the rate credit comes from class experience rather than from case experience.

Group Annuities

The general principles of surplus distribution applied to group life insurance may also be applied to group annuities, whether issued on the usual single premium deferred annuity basis or on a straight accumulation plan. This would imply the calculation

[1] See T.A.S.A., XXXI, 79.

	Percentage of Premium Required	
	For Expense	For Mortality
(1) Average premium of group (*T* rate) provides, on premium basis (say) ("Expense" includes 5% for contingency reserve and conversion loss allowance)	20	80
(2) If the total group mortality is below that expected by the AM(5) Table preliminary allocation of the premium between mortality and expense may be modified to (say)	25	75
(3) On the basis of expense analysis, the actual percentage of premium required for expenses (according to size of group) is (say) 17% plus 5% for contingency reserve, etc. [as in (1)] a total of 22%, permitting an adjustment of	−3	
(4) Mortality of the particular industrial classification being *lower* than total group mortality allows adjustment of (say)		−5
(5) The "Expected" requirements of the group (as a percentage of the premium payable) are therefore (Some other adjustments are omitted for simplicity)	22	70
(6) The actual rates of expense (including contingency reserve and conversion loss allowances) and of mortality (total experience) have been	22	60
(7) *Rate adjustment* (per cent of premium) is calculated as the sum of		
(*a*) Difference between total requirements per line (5) and 100 per cent	8% of premium	
(*b*) Difference between expected (class) mortality and actual mortality, multiplied by credibility factor (according to size), i.e., (70 − 60) × (say) 0.10	1% of premium	
	9% of premium	

of the actual surplus on each group annuity contract by accumulating all receipts and payments, interest being, of course, in this case, a necessary part of the calculation and expenses being carefully assessed. Such an accumulation may most conveniently be made on a calendar year basis because of the necessity

of including reserves and interest. After making provision for a contingency fund the balance of surplus would (if annuities are on the deferred annuity basis) be multiplied by a credibility factor based on size and duration, in order to obtain the dividend payable.

A contingency fund is required (1) to provide for possible reduction in interest earnings below those assumed; (2) to provide for mortality fluctuations in smaller groups[1]; (3) to provide for extreme fluctuations in cases where a few employees are covered for large amounts of annuity.

In practice the question of dividends on group annuities has, so far, been largely an academic one. The bases upon which group annuities have been issued in the past have, for the most part, not been such as to yield surplus for dividends in view of improved mortality, increased expense and reduced interest earnings. This applies particularly to smaller groups in which the expense ratios are relatively high and also to larger groups on the accumulative plan where expenses are assessed against interest earnings.

---

[1] See T.A.S.A., XXXIV, 240.

CHAPTER VIII

INDUSTRIAL INSURANCE

In view of the very large number of policies, the small amounts involved, the class of persons insured and the debit system of accounting it is essential that the system of distributing surplus on industrial insurance be simple, i.e., easily applied and easily understood, and economical of administration. Equity, in a broad sense, is a primary consideration, but in industrial insurance, even more than in ordinary, the degree of theoretical equity obtainable is limited by expense and other practical considerations.

Simplicity and economy are secured (1) by the adoption of a *uniform* method of allotting surplus, all dividend *options* being eliminated; (2) by the use of a simple formula which recognizes duration and, to some extent and under certain methods, also the plan of insurance, but not generally the age; (3) in some cases, by allotting dividends as of a fixed date to all policies within the classification affected in force on that date without reference to the policy anniversary.

Equity might be secured in a broad way by basing dividend scales, so far as possible, on financial analyses or asset share computations by year of issue and, perhaps, plan. Although individual dividends are small the aggregate amounts are very great and the general plan of distribution should therefore be capable of some theoretical justification.

A somewhat more conservative dividend practice has grown up in respect to industrial insurance than that followed by most companies on ordinary insurance. Industrial dividends are not usually paid until the surplus accumulations have been sufficient to wipe out the initial expense, i.e., not earlier than the end of the third year and in some cases not until the end of the fourth or fifth year.

Until the mutualization of two of the large American industrial companies about 1915 most industrial policies were non-participating but in some cases dividends, or "bonuses," were

paid on these policies. In other cases the policies provided for deferred participation. Of the stock companies issuing industrial insurance the smaller companies in the United States do not, as a rule, pay dividends but some of the larger United States companies, and companies in Canada and also in Great Britain, either provide for participation in their policies or allow dividends under a voluntary "profit-sharing" plan.

Since 1915 the law in New York[1] has required annual distribution of surplus on industrial as well as ordinary participating policies with the provision that "in the case of industrial policies the share of surplus so apportioned shall be payable annually in such manner as may be determined by the company with the approval of the Superintendent of Insurance." The law of Massachusetts[2] specifies that "on policies of industrial insurance on which premiums are payable weekly the annual surplus distribution shall begin not later than the end of the fifth policy year and be applied to the payment of any premiums or, at the option of the holder of the policy, be made in cash. . . . " In Massachusetts, therefore, domestic companies cannot allot surplus on industrial policies (weekly premium) in the form of additional insurance as may be done elsewhere. It may be noted that the definition of industrial insurance in the New York law includes policies with premiums payable *monthly* where the amount of insurance is less than $1,000 and the policy is issued specifically as an industrial policy.

In Canada annual distribution of surplus is not required either for ordinary or industrial business and in the case of at least one large company dividends on industrial business are declared quinquennially.

Methods of Distribution

In each company there is, as indicated above, one main uniform system applied to all policies which do not specifically provide for some other system. In addition to the regular dividend under the uniform plan there are, sometimes, additional allotments made for special reasons or under special conditions.

There are two main plans for the regular annual dividend system, namely, (1) premium credits and (2) paid-up additions.

[1] N. Y. Ins. Law, Sec. 83.
[2] Chapter 175, Sec. 140.

Of the three largest United States companies writing industrial business, one uses the premium credit system, another the paid-up additions system, while the third company uses the premium credit system for weekly premium business and the paid-up additions system for monthly premium business. Additional, i.e., supplementary, allotments are chiefly in the form of (1) mortuary and maturity dividends, which are additional dividends payable at the time of settlement of the policy, and (2) "equalization" dividends, which represent adjustments of the amounts of insurance on policies issued on an older and higher premium schedule to the larger amounts based on current, or more recent, rates. Other forms of surplus distribution are referred to below.

Premium Credits. On this plan the dividend consists of a credit of a specified number of weeks' premiums, the number depending only on the year of issue (increasing with duration). The same number of weeks' credit applies to all plans and to all ages at issue in the same year. This plan is not applied to policies with monthly premiums because multiples of the (larger) monthly premiums do not furnish a sufficiently flexible basis for surplus distribution.[1] For these the dividend addition plan has been considered more suitable.

One company formerly allotted dividends as a uniform number of weeks' premium credits irrespective of duration but in all cases such credits are now graded on an increasing scale according to duration. As a practical simplification the scale of credits is, in one case, made uniform for five year duration intervals.

The scale of premium credits might be determined by a form of contribution formula under which the surplus is calculated by years of issue and, perhaps, plan, on the basis of average age. Such calculations could take into account profits from mortality, loading and interest. Mortality profit might be expressed as a percentage of the cost of insurance on the basis of the Standard Industrial Table according to the company's experience and expenses as a uniform percentage of the premium. Specimen dividends may thus be calculated for different durations and, perhaps, plans. The variation of scale by plan arrived at by any reasonably practical basis of computation will probably not be very great and as a practical matter the same scale is necessarily

[1] See Report of N. Y. Ins. Dept., Part II, 1932, p. 198.

applied to all plans. This is further justified by the fact that the number of different plans is limited so that, on the whole, substantial justice is done, which is all that can be attempted.

While some variation with age would naturally appear if calculations were made by age as well as by plan and duration, it would be impossible in practice to vary the scale by age. Any such attempt would lead to greatly increased expense and to numerous errors in the crediting of dividends by agents in the field. The number of weeks' credit usually ranges from four or more weeks at the minimum duration entitled to dividend to a maximum of twenty-six weeks for all policies over a certain number of years in force.

The advantages usually claimed for the premium-credit system are:

(1) It is popular with policyholders and agents.
(2) It gives an immediate and apparent benefit which can be appreciated by the policyholder.
(3) It results in a progressively decreasing cost if the dividend scale so provides and if the same dividend scale is continued.
(4) It very frequently prevents lapse and aids in securing reinstatement of lapsed policies; this is of particular importance during times of depression.
(5) It relieves the agents of the work of collecting the premiums credited.
(6) The insured, in appropriate cases, may use the credits for the purpose of taking out additional weekly premium insurance, thus obtaining more immediate protection than he would have under a dividend addition plan.

It will be noted that the advantages of the premium credit plan are largely, if not entirely, *practical.* The plan has certain disadvantages, some of which are as follows:

(1) The system is not very flexible. For example, it is impracticable to differentiate according to plan.
(2) The system is relatively expensive to operate requiring much additional bookkeeping and clerical help. The company must see that the policyholder is not overlooked and gets the credit to which he is entitled. This requires the maintenance of a force of field inspectors.

(3) The amounts of dividends in most cases are small.
(4) A decrease in the dividend scale is more evident under this plan and more likely to be noticed and felt by the policyholders. However, by declaring the same number of weeks without any increase for duration, a moderate reduction can be accomplished without causing any serious dissatisfaction.

Paid-up Additions. Under this system dividends are declared as a percentage addition to the face amount of insurance. The percentage increases with duration, and different scales of percentages may be applicable to broad groups of plans. For example, in one case, one scale of percentages applies to Whole Life, Twenty Payment Life and long-term Endowments and a second scale (about 1 per cent lower at each duration) to other endowment policies. The percentages can be varied by plan without difficulty.

Dividend additions (like premium credits) take effect generally on a stated date, but sometimes on policy anniversaries. A dividend certificate is issued to each policyholder. This certificate does not show the actual addition made to the individual policy, but does illustrate the percentage addition for the current year and the total percentage addition standing to the credit of each policy according to plan and year of issue.

The equity of a scale of dividend additions may be checked by computing true gross premiums on the experience bases of interest, mortality and expense assuming a continuation of the proposed scale of additions. If the proposed scale is reasonably equitable the results should approximately reproduce the premiums charged. Such calculations may indicate a sufficiently close agreement by ages to justify a uniform scale for all ages on the same plan (i.e., with due allowance for the practical requirements of any dividend plan for industrial insurance) but show more variation between plans. A rough division by plans, as indicated above, is therefore made.

The arguments advanced in favor of paid-up additions may be summarized as follows:

(1) A greater degree of flexibility and, perhaps, of equity is obtainable. This is the most important advantage of the plan. The scale of percentages can be graded with any

desired degree of accuracy and, if desired, may be varied according to plan of insurance in order to give effect to different surplus earning rates.

(2) The method is simpler and less expensive in operation, the only expense being the valuation of the dividend additions plus the trifling cost of the printed certificates.

(3) Since dividend additions are usually computed on a conservative basis they are themselves a source of substantial surplus earnings, particularly when interest rates are high. Dividend additions are not on a participating basis but surplus earned thereon provides a fund for further dividends.

(4) The dividend as additional insurance at a low rate is a more substantial benefit in dollars and cents than a few weeks' premium credit, and produces an increase in cash and other non-forfeiture values.

(5) A reduction in scale is not noticed or felt by the policyholder to the same extent as under the premium credit system.

(6) The dividend provides an additional sum to the beneficiary when most needed, as well as increased cash value to the insured in case of emergency.

Disadvantages of paid-up additions are:

(1) The benefit is not so apparent as a premium credit. The policyholder may receive the impression that he is not getting any dividends.

(2) The system may tend to encourage surrender because of the increase in cash values by additions.

(3) The insured does not receive any cash return until surrender or maturity.

As between the two main dividend systems, the balance of advantage, from the company's point of view at least, would seem to lie with the paid-up addition plan because of the greater flexibility attainable and the greater economy and simplicity of application. The practical advantages of the premium credit system from the standpoint of the policyholder and the agent, and, to some extent also, the company, are, however, very considerable.

Mortuary and Maturity Dividends. These are additional dividends payable only at death or upon maturity (either as an endowment or, in some cases, upon occurrence of disability) and represent either a supplementary allotment of profit or a return of a portion of the contingency reserve. The dividend is usually a uniform percentage of the face amount of insurance, the percentage depending only on the year of issue. In at least one case these additional dividends are paid on claims at all durations at which regular dividends are payable and not only on policies of a certain minimum duration as is sometimes the case in ordinary insurance where such dividends are definitely a share of the contingency reserve. They are also sometimes paid on claims under (non-participating) reduced paid-up or extended term insurance although in one company such payments are apparently limited to claims occurring within one year of lapse.

In Great Britain the system of mortuary and maturity dividends has been adopted by one or two companies as the main plan of distribution. On this basis (in the absence of any other supplementary form of participation) the only policies which receive dividends would be those which become claims or mature and in the opinion of some this is the most satisfactory system of distribution for industrial policies. It has the great advantage that very much larger amounts are paid on claims since only policies which remain in force receive dividends. The cost of administration, also, would be exceedingly low and possibly a trifle lower even than under the paid-up additions system.

Similar to the regular mortuary dividends are the one year term additions which have been, or are being, paid in certain cases. Such additions are, of course, very much greater than regular paid-up additions.

Equalization Dividends. These dividends are also supplementary to the regular system and are applicable to policies issued in past years with lower benefits than those under current policies. Equalization dividends may be, and usually are, paid as a special form of maturity or mortuary dividend, i.e., only upon settlement, but are sometimes also allotted as a dividend addition prior to maturity. They usually consist of an increase of the face amount of insurance to the amount provided by the same premium on the basis of the current schedule of premiums.

Where a reduction of premium schedule takes place it may be advisable to grant an increase of insurance or "equalization dividend" at least to all recent issues in order to prevent wholesale lapse followed by replacement on the new basis. This has been done frequently in England according to Spratling.[1]

An equalization dividend gives an adjustment which implies that conditions throughout the duration of the policy have been as good as they have been in recent years whereas presumably the experience has improved gradually. Moreover, the relative surplus for different ages at issue is probably quite different. These objections are, however, unimportant and from a practical standpoint such an adjustment is simple and readily understood and appreciated by policyholders.

Where equalization dividends have been allotted to continuing policies, regular or other dividends allotted thereafter would be based on the increased amount of insurance so that in effect the equalization dividends are on a participating basis. They are, in fact, a definite adjustment of the amount of insurance rather than a paid-up dividend addition.

Limitation of Premiums. Another form of supplementary allotment of surplus consists in making the premiums under life policies cease at a stated age. This may be done only in the case of policies which reach the stated age, or surplus may be applied to limit future premiums on all policies subject to a minimum total duration. For some time practically all whole life industrial policies issued in America have been written with provision for cessation of premiums at 70 or 75 and in some cases all policies previously issued and providing for life premiums have now been made automatically paid up at the limiting age so that this method of distribution is no longer of importance.

Maturity as Endowment. An extension of the idea of limiting premiums is a provision in the dividend schedule that all policies or all of a certain minimum duration reaching (say) age 80 during the current calendar year shall be paid in full as endowments.

Special Benefits. The inclusion in industrial policies (or the addition to existing policies as a special "concession") of a provision for additional benefits in event of total and permanent

---

[1] J.S.S., IV, 234.

disability or in event of death by accident, without any specific premium charge therefor, constitutes, in effect, a form of surplus distribution.

Because of the nature and amount of the insurance such "special benefits" are necessarily on a different basis from those included in ordinary policies. The disability benefit is payable only in event of the type of disablement usually described in ordinary policies as a "specified" disability, i.e., loss of sight of both eyes, etc. The benefit may be waiver of premium or immediate payment of one-half or the whole amount of insurance and, in addition, endorsement of the policy as fully paid up for the full amount or for the balance of the sum insured. Disability income benefits would, of course, be impractical in industrial policies. The "double indemnity" benefit, when granted, is very similar to that in ordinary policies except that where death occurs while the insured is engaged in certain occupations including unusual hazards, the additional amount payable may be reduced by one-half.

The free nursing or other health service furnished to their industrial policyholders by certain companies may also be regarded as a form of surplus distribution.

SUPPLEMENTARY NOTE ON THE PRACTICAL DETERMINATION OF THE DIVIDEND FORMULA

By Edward W. Marshall

The subject of Distribution of Surplus admits of a considerable difference of opinion at many points, as Mr. Maclean has indicated in the preceding pages. Thus various approaches to the subject are possible and the following discussion is presented in order that certain points of view may be brought out in more detail.

As a preliminary, let us assume that the *total* amount of surplus available for distribution has been determined according to the principles laid down in Chapters I and II. It then remains to develop a dividend formula which will result in a total distribution approximately equal to that contemplated. Just how shall the factors in the formula be determined?

Some Disadvantages of the Usual Approach to the Contribution Dividend Formula

The usual approach to the three-factor dividend formula described in Chapter III possesses certain disadvantages, as follows:

(1) As the mortality and expense factors usually employed do not follow actual policy year experience, there is no assurance that the resulting dividends are in reasonable accordance with those actually earned. A further check by asset shares and a revision of the formula are usually needed if realistic dividends are to be obtained.

(2) There is an unduly wide latitude possible for determining the dividend factors from a given experience, indicating the practical advantage of a more controlled approach.

(3) Equity between persisting and withdrawing policyholders is not a primary prerequisite and is not automatically maintained by the formula.

As these disadvantages are both important and fundamental they will be discussed in some detail.

Obscure Mortality and Expense Factors

As a rule, under the three-factor contribution formula in widespread use, the mortality, expense and surrender gain or loss factors are not separately applied on a select basis, but to a degree are merged and determined according to an ultimate or semi-aggregate basis largely developed by custom or convenience as follows:

Mortality Factor. The mortality factor usually is supposed to reflect the excess of the tabular over actual ultimate mortality by attained age and is frequently expressed in the dividend formula as $kq_{x+n-1}(1-{}_nV_x)$ where k varies by attained age. The remainder of the mortality gains, namely, the excess of ultimate mortality over select mortality, is used to offset initial expense and thus distributed indirectly in accordance with the method used to assess expenses. Whether the plan even approximately returns these select mortality gains to the policies contributing them seems obscure.

Furthermore, the boundary line between select and ultimate mortality is often not clear-cut. As is well known, under modern experience the mortality rate for a given attained age frequently continues to increase by duration even after fifteen policy years. The choice of an ultimate mortality rate thus involves a considerable latitude for discretion. Also the proper determination of the mortality gains due to medical selection requires some discretion so that the resulting amount of credit against expenses is subject to variation according to the individual viewpoint. Thus the determination of the mortality factor is by no means as simple as might be expected.

Expense Factor. The amount of expenses to be allocated is supposed to be found by subtracting from the appropriate total expenses the gains from surrenders and the aforesaid gains from excess of ultimate over select mortality. The allocation of the resulting net expenses through any of the orthodox formulas of expense charge now used under the contribution formula seems rather empirical.

For example, the actuary has to decide whether to assess the net expenses in proportion to gross premium or net premium,

loading or constant, or percentage of premium for given plan plus another percentage of ordinary life premium, etc. The size of the percentages or constants and whether or not they vary by duration also have to be decided. If it is desired that expense charges should reflect to some extent the average size of the policy for various plans, ages and years of issue, how should this be accomplished? Obviously there can be a wide variation in the resulting dividend according to the formula for expense charge employed, and unfortunately there is no clear-cut scientific approach to enable the student to decide which formula is most equitable.

As a result of these wide areas for personal judgment, the student is likely to be somewhat at a loss to know just what would be an equitable dividend from mortality and loading. The landmarks are very empirical. In place of homogeneous factors based on select mortality, expense and surrender gain or loss, there are substituted heterogeneous factors which (unless controlled through asset share or analogous studies) may easily produce dividends by plan, ages at issue and duration greatly different from those which are actually earned.

In Appendix B there are shown the 1936 dividend formulas of twenty-four representative companies. It will be seen that there is a very wide variation in both the mortality and loading factors. One of the tendencies in recent years has been for the loading factor to become more complicated, indicating that the dividend problem is not so simple as might be supposed from a casual inspection of the expense charge formulas often suggested in descriptions of the three-factor contribution formula.

Equity between Persisting and Withdrawing Policyholders

The recent heavy voluntary withdrawals which reached a maximum in the years 1932–1933 impressed many company executives with the belief that it is both unsound and unwise to pay unearned dividends or surrender values to withdrawing policyholders. As a result many companies have reduced their surrender values and either eliminated the dividend at the end of the first policy year or made it conditional upon the payment of the second year's premium.

In order to maintain equity between persisting and withdrawing policyholders, the cash values and dividends must be

determined in such a way that the withdrawing policyholder receives no more than the asset share of his policy on a realistic basis. Otherwise he is being favored and the persisting policyholder discriminated against.

In the usual approach to the three-factor contribution formula, the principle used is practically equivalent to deducting the present value of all select mortality gains from initial expenses and assessing the balance (less surrender gains) as expense on an aggregate basis. This principle is not unreasonable if surrender values are consistently low. In many cases, however, the net effect now is that on surrendered policies select mortality gains beyond the date of surrender, which thus will never be realized on such policies, have been anticipated to offset initial expenses already incurred—a rather questionable procedure which penalizes the persisting policyholders.

Inasmuch as the three-factor contribution formula as usually employed does not reflect the incidence of mortality and expense according to duration, this lack of equity between persisting and withdrawing policyholders cannot be detected from the formula itself, but only through asset share or analogous studies.

At times the relation of dividends and cash surrender values on new and old policies in some companies has been such as to encourage the replacement of old insurance by new insurance, an unsatisfactory and unsound situation. Obviously, if the acquisition expenses of the new and old policies were realistically assessed by duration, and all other elements of gain or loss were similarly taken into account, the above condition would largely disappear.

A Controlled Approach to the Dividend Formula

For these and other reasons, some companies have found it desirable to approach the dividend formula from the standpoint of the asset share or analogous contribution rather than merely to use the asset share as a final check on a predetermined three-factor dividend formula of the usual type. The advantage of such an approach is that the resulting dividend formula thus can be "tailor made" from the outset, and any adjustments introduced can be fully visualized and kept under greater control at all stages. Thus there is a greater feeling of assurance and satisfaction in knowing that the resulting formula is not one of many

possible guesses within a rather wide area, but is based on the underlying facts by age at issue, plan and duration. Essentially, of course, the fundamental principle of the contribution plan is retained, i.e., surplus is distributed to policies from their contributions thereto.

Inasmuch as the controlled approach now employed by some companies varies from the asset share check referred to by Mr. Maclean, a brief summary of one method used follows:

Somewhat along the lines of the report contained in the Life Office Management Association Proceedings for 1932, the expenses of the company apart from special features such as accidental death benefits, disability insurance, annuities, etc., are subdivided through a yearly cost accounting analysis, care being taken to see that all expenses attributable to the first policy year are so allocated. A further study is made of the differential expense between large and small policies.

The following expense factors are obtained: (1) selection and issue cost expressed as cost per new policy plus a cost per $1,000 new insurance found from the above-mentioned differential study; (2) acquisition cost as a percentage of first year commission; (3) maintenance expense on premium-paying policies expressed as a cost per mean premium-paying policy in force plus a cost per $1,000 of premium-paying insurance in force; (4) the corresponding maintenance expense factors on non-premium-paying policies. For each representative age, plan and duration group the cost per policy is converted to cost per $1,000 insurance giving some weight to the average amount of insurance for the group. The resulting expense factors per $1,000 insurance together with appropriate premium tax rates and commission rates for given plans and policy years are used for the asset share study.

A net interest rate, and graduated select mortality and lapse rates varying by plan, reasonably representing the current trend, are employed, with appropriate modification for term insurance or preferred risk policies, the extra mortality after conversion of term policies being assessed as an additional charge on the term policies. The resulting factors (on other than term policies) are employed in the following formula, in which π' is gross premium, r_n and e_n are nth year's expense factors of premium percentage and constant, respectively, i' is the earned interest

rate, q' is the select mortality rate, CV_n is the cash value nth year, and w is the select lapse rate. The formula assumes that a prorata dividend is paid at time of death. The "contribution to surplus" during the nth year is then:

$$\frac{[\pi'(1 - r_n) + CV_{n-1} - e_n](1 + i') - q'_{n-1}\left(1 + \frac{i'}{2} - CV_n\right) - CV_n}{1 - \frac{q'_{n-1}}{2} - w_{n-1}*}$$

This formula is applied for several representative ages, plans of insurance and durations to produce a network of "contributions to surplus." The resulting contributions indicate the maximum dividend payable at the various points if loss is to be avoided on withdrawing policyholders. Owing to the fact that the surrender values may not be ideal and the contributions thus distorted, it is sometimes necessary to graduate the contributions for a number of policy years after issue. For each plan they are then scheduled on a sheet along with the corresponding dividends according to the previous dividend scale of the company and those of other companies of particular interest. These dividends of other companies, however, are first adjusted by increasing or decreasing the actual dividends by the difference between the gross premiums of one's own company and those of the other company, the resulting dividends thus being what would be required on the premiums of one's own company to produce the net cost of the other company. Thus a direct comparison between the dividends of the various companies is made possible.

The actuary then has before him on one sheet a schedule of (1) contributions indicating approximately the maximum dividends theoretically payable at the various points; (2) dividends according to the old dividend scale; and (3) the corresponding dividends of other companies. With this information he is in a position to write down "model dividends" which reflect the trend of the contributions as well as the commercial practice indicated by the dividends of other companies. He also can tell how the new model dividends would compare with the dividends according to the previous scale. This is the point in the whole operation where both theory and practical

* Omit when dividend is not conditional on payment of next premium.

considerations can be given their proper weight in the determination of the new dividend scale.

With these model dividends available the next step is to find a convenient three-factor formula to reproduce them reasonably closely. To accomplish this, for each representative age at issue, plan and duration point an excess interest dividend and an empirical mortality dividend (based on an assumed ultimate mortality factor graded by attained age applicable to all regular plans except term) are obtained and deducted from the corresponding model dividend. The balance is assumed for convenience to be dividend from loading.

By experiment there is obtained an empirical formula for loading less expense charge such that the resulting loading dividends at the various points reproduce reasonably closely the above-mentioned assumed loading dividends. The expense charges thus found are likely to decrease by duration and the formula producing them is usually more complicated than the more simple types of expense charge formula. As previously stated, this situation already exists in the expense charge formulas of a number of companies.

Finally, after the three-factor dividend formula has thus been derived, with suitable variation for term and preferred risk policies, the usual test is made to compare the total prospective outlay under the dividend formula with the total amount available for distribution during the coming dividend year. This comparison is obtained by the application of factors to valuation summaries. If the prospective outlay is much different from that available for distribution, the factors have to be modified correspondingly. However, as the outlay resulting from the continuation of the previous scale is already known, thus affording a landmark when the model dividends are written down, this modification is usually relatively slight. All the dividend factors may be modified proportionately, or a further consideration of the relation of the model dividends to the contributions may suggest that a small change in one or two of the factors may be sufficient.

It will be seen that the above formula assumes that, for each policy year, the contribution thus determined is distributed currently as a dividend, so that the cash value at the end of the year is the only fund remaining in the company's hands. While

this formula is useful in maintaining approximate equity between withdrawing and persisting policyholders, it does not attempt to relate the dividends to reserves. If reserves were substituted for cash values, in most companies there would be no dividend earned for perhaps at least four or five policy years after issue if the heavy deficit in the first year were carried forward until extinguished. As the cash value is often a rough approximation of the asset share, the formula produces contributions which afford a reasonable basis upon which to apportion the amount available for distribution.

The main advantage of the method is that it gives a closer control of the incidence of dividends by age at issue, duration and plan and enables them to be kept in reasonably close touch with reality. It is particularly useful when a complete readjustment of dividend factors is necessary because of a material change in the incidence of surplus gains from various sources.

The greatest difficulty encountered in the approach above described is that the "contributions to surplus" used as a basis for the model dividends may be substantially different at important points from those of nearly all other companies with approximately the same gross premiums. There are perhaps two main reasons for this condition. One is that many participating companies apparently assess expenses giving little if any weight to the variation in average size of policy by age at issue or plan. This means that the company giving more weight to this variation would pay smaller dividends for the younger and older ages at issue, and for the more expensive plans, where the average amount of insurance is lower than the general average. Prevailing commercial practice cannot be entirely disregarded because, as stated in Chapter II, a scale should be in reasonable "harmony with current usage." Thus the best that a company can do which wishes to give weight to the average amount of policy, seems to be to pursue an intermediate course, deviating only partly from the usual commercial basis in the direction of maintaining the various groups of policies on a self-supporting basis.

The second difficulty arises from the fact that, as Mr. Maclean points out, some companies do not assess against the first policy year of contracts issued in a given calendar year all the expenses which, strictly speaking, were incurred in the company's office

and elsewhere as a result of doing new business during the year. Consequently, during the early policy years, the cash surrender values and the dividends of such companies are too large. Withdrawing policyholders may receive substantially more than their policies have earned, which thus makes the dividends to persisting policyholders lower than they otherwise would be over the years. Companies which endeavor to assess the first year's expenses more accurately are likely to find that, other things being equal, their dividends or cash values in the early years are not so great as those of the other companies, although in the long run their total benefits over a period of years would be definitely more favorable to the persisting policyholder.

It is recognized that the exact method of assessing expenses is open to some difference of opinion and that items such as agency expenses may be based on different principles in different companies. However, if it is assumed that each year's issues should be self-supporting over the years, and that any part of expenses incurred on account of withdrawing policyholders should not be borne by persisting policyholders, the variations caused by differences of opinion regarding the assessment of expenses become comparatively limited. As already stated different viewpoints exist regarding this whole subject, but the tendency in recent years has been to give more consideration to the question of incidence of expense and the preservation of equity between persisting and withdrawing policyholders on the basis of asset share studies. Thus some of the foregoing elements of difference in commercial practice may become of diminishing importance as time goes on.

APPENDIX A

DIVIDEND CALCULATIONS AND OFFICE RECORDS

Office procedure and the methods for calculating and recording dividends naturally vary considerably in different companies. The following notes, which are intended chiefly for the student who has no opportunity to observe office practice, give a general outline of the work to be done and the method of performing it.

After the dividend formula has been determined it will be necessary (1) to prepare a complete schedule of dividend rates per $1,000 of insurance (or other unit) for all plans, ages at issue and durations corresponding to the business in force; (2) to calculate the actual amount of the new dividend on each policy; and (3) to make a permanent record of the amount of each dividend and its disposition.

Calculation of Dividend Schedule. The dividend will normally be the sum of independently computed factors. Tables of initial (or other) reserves, cost of insurance, loadings, etc., as required, can be prepared in advance and will be available from year to year. The actual work is usually done on schedules in columnar form in accordance with the particular dividend formula and using the material available from the calculations of previous years' dividends. Sometimes exact calculations are made only of individual dividends at quinquennial ages and durations, the schedules being completed by interpolation, but most companies prefer to make exact calculations covering all cases.

If desired, the work of preparing the complete dividend schedule may be reduced in certain cases either (1) by combining the interest and mortality factors using McClintock's formula[1] or (2) by adopting the process described by Grout.[2]

Where the interest factor is in the form $(i' - i)({}_nV + \pi)$ and the mortality factor in the form $(q - q')(1 - {}_{n+1}V)$, McClintock

[1] See R.A.I.A., XII, 46.
[2] T.A.S.A., XXVII, 59.

showed that the combined interest and mortality factor may be expressed in the form

$$A_{x+n} + B_{x+n} \cdot {}_{n+1}V_x$$

where A and B are constant for any attained age at the beginning of the dividend year and have the following values:

$$A_{x+n} = \frac{1+i'}{1+i} q_{x+n} - q'_{x+n}$$

$$B_{x+n} = \frac{i'-i}{1+i} - A_{x+n}$$

The formula may, in fact, be further simplified and expressed in the form

$$A + B'_{x+n}(1 - {}_{n+1}V_x)$$

where $A = \frac{i'-i}{1+i}$ and is a constant depending only on the interest rate, and

$$B'_{x+n} = \frac{1+i'}{1+i} q_{x+n} - q'_{x+n} - \frac{i'-i}{1+i}.$$

The combined interest and mortality factors may be calculated by either of these formulas and added to the loading factor which is computed separately.

Grout's method provides a continuous process for calculating dividend schedules. The method is applicable where the factors in the formula can be expressed as follows:

Interest Factor: Constant times Initial Reserve.

Mortality Factor: Constant times Cost of Insurance plus constant times Amount at risk.

(i.e., a percentage of the cost of insurance in the form $a/q + b$)

Loading Factor: Constant times Net Premium plus constant times Ordinary Life Net Premium.

Some of the conditions under which the method would not be applicable are stated by Hart in the discussion of Grout's paper (T.A.S.A., XXVII, 413). The principal case is where either the loading factor or cost percentage is varied by plan.

Grout's formula (3a), which is the general formula for the nth dividend, may be written

$$A' + B'P_x - (\pi_x + d)C'_{x+n-1} + \theta_x D'_{x+n-1}$$

where A' and B' are absolute constants, C' and D' are constants for any attained age and θ is the "Karup Function," used in the attained age valuation method, which is a constant for any age at issue and plan. This formula may be applied continuously. For example, if dividends at each duration are to be calculated for a specified age at issue and plan, the successive differences are the values of

$$(\pi_x + d)(C'_{x+n-1} - C'_{x+n}) - \theta(D'_{x+n-1} - D'_{x+n}).$$

While the advantages of having a continuous process and of reducing the time required to complete the schedules are worth considering, these methods have the disadvantage that the identity of the respective factors in individual dividends is lost. It may be considered preferable to calculate each factor of the dividend separately.

Mechanical Calculation of Dividends by Continuous Process. A recent adaptation by Camp[1] of the standard three-factor contribution formula has made it possible to calculate the schedule of dividend rates by a continuous process using in principle the accumulation, by a forborne annuity, of the difference in premiums between respective plans for any given age. The method assumes that a standard loading formula is used for the calculation of the gross premium, and that this formula does not vary between plans for a given age but may vary for each age at issue. Thus the orthodox method of obtaining a gross premium by loading the net premium a certain percentage plus a percentage of the corresponding ordinary life net premium readily fits in with the basic scheme. The method may be used by calculating a dividend for a zero premium policy and mechanically calculating the rest of the dividends for the same age at issue and duration by multiplying the premium for the next higher plan by a factor which may depend upon both age and duration, and building the next dividend upon this dividend by again taking the difference between the last preceding premium and the next higher plan and repeating the process. It may also be used by calculating the basic ordinary life dividend and building the rest of the dividend scale for the same age at issue and duration by taking the difference between the ordinary life net

[1] T.A.S.A., XXXVII, 64

premium and the net premium for the particular plan, thus getting each dividend directly without building upon the dividend for the last preceding plan.

The method is easily adapted to the use of either the Burroughs Printer-Calculating machine, or tabulating equipment with punched cards. In either case, after the factors and difference in premiums have been prepared the product of the factor and the difference in premiums is calculated, and by means of the cross-footing device this product is added to the basic dividend previously calculated, thus producing the dividend for the desired plan. Further extensions of the plan include the calculation of dividend accumulations for a period of years, and, again by use of the cross-footing device, the calculation of net costs mechanically for agents' use.

Calculation of Dividends on Individual Policies. When the schedule of dividend rates is completed the dividends on individual policies will be calculated by multiplying the amount of insurance by the appropriate dividend rate. If existing prior dividend additions are participating the new dividend on additions will be separately computed and added to the dividend on the face amount, the total being multiplied, if necessary, by the proper "additions purchased" factor (reciprocal of net single premium) to obtain the new dividend addition.

The mode of performing these operations in bulk varies according to the different systems of dividend records used. Where the calculations are made and entered directly on the permanent dividend record cards these may first be sorted to correspond to the rate schedules. If the dividend record cards are to be kept in numerical or anniversary order by agencies (for convenience in preparing notices) the dividend rate schedules must be tabulated in convenient form by duration so that dividends for different plans and ages can be taken off.

Many companies have now adopted the punched card system for the calculation of dividends on individual policies in bulk. This system is briefly described by West[1] and by others in the discussion of West's paper and is probably the most expeditious and efficient method.

The general procedure where the most modern punched card system is used for calculating individual dividends is as follows:

[1] T.A.S.A., XXXV, 275.

A Hollerith card is prepared from existing dividend records for every participating policy. The file is kept up to date in relation to terminations, restorations, changes, etc., and is normally arranged in due date (i.e., anniversary date) order.

Each year a set of dividend rate cards is prepared from the detailed rate schedule.

Sufficiently in advance of the anniversary dates, the dividend cards are removed (by months) from the files and the following operations performed:

(1) Dividend and rate cards are sorted according to plan, age and year of issue.
(2) In this order the cards are placed in the automatic multiplying punch for multiplication of amount of insurance by cash dividend factor. The result is automatically recorded on the card.
(3) Where dividend additions are participating, the cards are again placed in the multiplying punch and the amount of existing dividend additions is multiplied by the additions dividend factor, in the same manner as for the dividend on the face amount, the result being added to product of operation (2) and automatically recorded on the card in one operation as total cash dividend. The new dividend addition purchased is obtained by sorting the dividend cards in attained age order (or attained age and term to run in case of endowments) with the appropriate "rate" card for the additions purchased factor and punching the new addition by the multiplying punch.

For preparation of dividend notices the cards are sorted in numerical order by agencies (at which time the rate cards are automatically separated from dividend cards) and placed in the "Posting Tabulator" which writes the notices automatically, also adding the number of notices and the total amounts of dividends.[1]

The cards are then filed until required for the following year's dividend operations, at which time they are reproduced before being used for calculations or other general purposes.

---

[1] See T.A.S.A., XXXVII, 392.

Permanent Dividend Record. A permanent dividend record for each policy is necessary. This will usually be a written card. The form of record to be used depends on the circumstances of each company, including, for example, such things as the practice in regard to permitting change of dividend option at each anniversary or the provisions of the contract as to dividend additions. The record should show, for each year, particulars of dividend allotted, used or deposited at interest and the balance of dividend additions in force. For valuation of dividend additions the record should include (office) year of birth and, for endowments, year of maturity, as well as the usual description of the policy, namely, age, date of issue, kind and amount.

The permanent record cards will most conveniently be filed in numerical order or, if dividend notices are prepared from them, in numerical order by agency.

APPENDIX B

1936 DIVIDEND FORMULAS OF 24 COMPANIES

(See table on page 150)

I = initial reserve
M = mean reserve
L = loading
k = constant
π' and π = gross and net premium for plan
P' and P = gross and net premium Ordinary Life
C = cost of insurance
AR = amount at risk
x = variable (not age)

Explanation of Symbols for Gross Interest Rate. The first letter in the column shows the gross interest rate used in the basic dividend scale, the second letter the maximum rate for settlement options, and the third letter the rate for dividends left at interest. The meaning of the letters is:

a = 5.0%	d = 4.5%	g = 4.25%	j = 3.8%	n = 3.5%
b = 4.75	e = 4.4	h = 4.0	l = 3.75	o = 3.0
c = 4.6	f = 4.3	i = 3.9	m = 3.6	

Symbols are explained on page 149

Flat Cut,* %	Interest		Mortality	Loading†
	Gross Rate	Excess On		
	lhh	*I*	$.003AR$	$.25L$
47.5	*all*	*M*	xAR $x = .0035$ up	$L - .14\pi' - x$ $x = 1.$ to $.50$
	ihh	*I*	$.0015AR$	$L - x\pi'$ x var. by age, plan & dur.
	jnn	*I*	xC $x = .405$ down	$(L - x \cdot \overline{\pi' + P})(1 + i')$ x decr. by dur.
	hll	*I*	$xC + 1.40$ $x = .15$ to $.075$	$.175L$
	lhh	*I*	$.003AR$	$.25L$
	ohh	*I*	xAR $x = .005$ down	$L - .08\pi' - .03P'$ (adj.)
	fln	*I*	$.0025AR$	$.03\pi$
40–20	*ahh*	*I*	xAR x adj. for each age by Hyp. Table	$L - k\pi' - 1.25$ $k = .08$ approx. but var. by plan
27.5	*chl*	*I*	xC $x = .401$ down	$1.046(L - .115\pi')$
	mmm	*I*	xC $x = .423$ down	$L - .14P' - x\pi'$ x graded by dur.
	ijj	*I*	$.004AR$	$L - .1125\pi - .0175P - x$ $x = 1.$ to 0
10	*dhl*	*I*	$[x]C$	xP $x = .045$ to $.09$ accord. to dur.
	dff	*I*	xC $x = .50$ to $.10$	$(L - .08\pi' - .05P')(1 + i')$
	ohh	*I*	xC $x = .45$ to 0	$L - x\pi' - 1.$ x var. for kind & dur.
22	*bhh*	*I*	xC $x = .57$ to $.20$	$L - .105P' - .005\pi'$
	hhh	*I*	xC $x = .27$ to $.005$	$.75(L - .1\pi') - x$ $x = 1.$ to $.50$
22	*eln*	*I*	xC $x = .682$ to $.004$	$L - .09\pi' - .07P - .002AR$
	lhn	*I*	xC $x = .50$ to $.10$	$L - .08\pi' - .04P' - .0025AR$
	hnn	*I*	$x$per$M$ x var. with attained age	$k\pi$ k var. by plan
50‡	*?ll*	*M*	See Loading	$\pi' -$ Co. Exp. π' at val. int. rate.
	hln	*Spl.*	See Loading	$\pi' -$ Co. Exp. π' at val. int. rate
	nnn	*M*	See Loading	$.3L$ (on AM Select, val. int.) $-k\pi$ k var. by plan
	hno	*M*	xAR x adj. by Spl. Co. Exp. Table	$\left(L - x\frac{\pi' + P'}{2}\right)1.03$ $x = .125$ to $.075$

* Basic insurance dividends by formula to be reduced by percentage shown.

† Values for x below, when denoting a separate item in the expense charge, are given in dollars per thousand.

‡ Resulting dividends held back one policy year.

APPENDIX C

BIBLIOGRAPHY

(The list given below is not complete but comprises most of the principal papers, etc., to which reference might be made. Papers which are required reading for the Fellowship Examination are listed in the Year Book of the Society.)

T.A.S.A.—Transactions of the Actuarial Society of America.
R.A.I.A.—Record of the American Institute of Actuaries.
J.I.A.—Journal of the Institute of Actuaries.
J.S.S.—Journal of the Institute of Actuaries Students' Society.
P.C.A.S.—Proceedings of the Casualty Actuarial Society.
T.I.C.A.—Transactions of the International Congress of Actuaries.

1. *Historical and General.*

Fackler, D. P.	An Account of the Various Dividend Systems Adopted by American Life Insurance Companies up to 1889, etc.	T.A.S.A., I (Pt. 2), 3
Ryan, G. H.	Methods of Valuation and Distribution of Profits in the United Kingdom	J.I.A., XXXVIII, 69 and T.I.C.A., III, 252
Henderson, R.	Equity in Surplus Distribution	T.A.S.A., XXIV, 291
Rietz J. C.	A Review of Some of the Earlier Methods of Surplus Distribution	R.A.I.A., XI, 106
Lochhead, R. K.	Valuation and Surplus	Inst. of Act. Stud. Soc. Consolidation of Reading Series
Elderton, W. P.	Fair Distribution of Bonus	T.I.C.A., IX, I, 158
Hoskins, J. E.	Some Fundamental Characteristics of Mutual Life Insurance	T.A.S.A., XXXI, 253

2. *The Contribution Plan.*

Homans, S.	On the Equitable Distribution of Surplus	J.I.A., XI, 121
Sprague, T. B.	Editorials	J.I.A., XIV, 322 and XV, 31
Wells, D. H.	Application of the Contribution Plan to the Distribution of Surplus	T.A.S.A., II, 361
Weeks, R. W.	A Practical Rule for Calculating Annual Dividends	T.A.S.A., IX, 310
Dawson, M. M.	A Distribution Formula	T.A.S.A., IX, 344

Moir, H.	Valuation and Distribution	T.A.S.A., X, 179
Papps, P. C. H.	A Note on the Original Contribution Plan	T.A.S.A., XI, 49
Nichols, W. S.	Mortality Factor in the Allocation of Insurance Surplus	T.A.S.A., XI, 497
Moir, H.	Mortality Surplus and Its Distribution	T.A.S.A., XIV, 17
Kattell, S. C.	A Study in Surplus Distribution Using the American Men Table as a Guide	T.A.S.A., XXII, 418
Coburn, A.	Distribution	R.A.I.A., XII, 35
Hunter, A.	Methods of Distributing Annual Dividends	T.A.S.A., XXIV, 14
Maclean, J. B.	Notes on the Practical Application of the Contribution Method of Distributing Surplus	J.I.A., LXII, 243
Jackson, H. H.	The Wisdom of Mutual Life Insurance	T.A.S.A., XXXIII, 116
Hollenberg, M. R.	Recent Developments in the Distribution of Surplus to Policyholders	T.A.S.A., XXXIV, 277
Elderton, W. P.	Bonus Distribution—A Study in Differences	J.S.S., IV, 71

3. *Contribution Plan in Great Britain.*

Sprague, T. B.	Editorial Note	J.I.A., XIV, 396
Browne, T. G. C.	Correspondence—Method of Division of Profits	J.I.A., XXXII, 194
Lidstone, G. J.	On the Distribution of the Divisible Surplus of a Life Assurance Company	J.I.A., XXXII, 73
Ryan, G. H.	See above	
Hustwitt, W. E.	Bonus and Surrender Value Systems in the United Kingdom	T.I.C.A., IX, I, 166

4. *Distribution in Relation to Valuation Basis.*

Rothery, H. J.	On a System of Bonus Distribution considered in Relation to the Office Premiums and Modes of Valuation	J.I.A., XXX, 131
Maltby, C. H.	Results of an Investigation into the Effect of Different Valuation Bases on Surplus	J.I.A., LIII, 167

5. *Analysis and Assessment of Expenses.*

Starr, W. E.	The Apportionment of Expenses and Its Relation to the Distribution of Surplus	T.A.S.A., III, 171
Papps, P. C. H.	A Few Suggestions as to the Assessment of Expenses	T.A.S.A., IX, 232

Craig, J. D.	Allocation of Expenses	P.C.A.S., X, 9
Elston, J. S.	Expense Investigations	T.A.S.A., XXIII, 83
Turnbull, J.	Separation of Accounts	T.A.S.A., XXVIII, 216
Rietchel, H. J.	Analysis and Apportionment of the Expenses of Management of a Life Office	J.I.A., XLIV, 415
Maltby, C. H.	The Analysis of Life Office Expenses	J.I.A., XLIX, 111

6. *Asset Share Accumulations.*

Weeks, R. W.	An Equitable Method of Keeping the Accounts of Deferred Dividend	T.A.S.A., IX, 93
Bowerman, W. G.	Contribution of Dividends	T.I.C.A., IX, I, 78

7. *Office Methods.*

Grout, H. A.	A Process of Calculating Annual Dividends by Use of the Karup Method of Attained Age Valuation	T.A.S.A., XXVII, 59
West, C. E.	Some New Uses for Modern Punched Card Equipment	T.A.S.A., XXXV, 265
Camp, K.	A New Method for Calculating Dividends According to the Contribution Formula	T.A.S.A., XXXVII, 64
Pettegrew, A. A.	The Production of Dividend Notices by Modern Machine Methods	T.A.S.A., XXXVII, 394

8. *Annuities.*

Fackler, D. P.	Annuities with Participation	T.A.S.A., XV, 40
Ferguson, C. C.	Participating Annuities	T.A.S.A., XV, 354
Fackler, D. P.	Annuities with Participation Based upon Select and Ultimate McClintock Tables	T.A.S.A., XVI, 279
Camp, K.	Participating Immediate Annuities	T.A.S.A., XXXVI, 214

9. *Supplementary Contracts.*

Fassel, E. G.	Excess Interest under Annuity-Certain	R.A.I.A., XIV, 21
———	Discussion: "Dividends to Beneficiaries under Instalment Policies"	R.A.I.A., X, 274

10. *Group Insurance.*

Craig, J. D.	Discussion	P.C.A.S., VIII, 307
Keffer, R.	An Experience Rating Formula	T.A.S.A., XXX, 130
Bjorn, W.	Distribution of Surplus under Group Life Contracts	T.A.S.A., XXXI, 72

The following papers deal with the theory and practice of experience rating in casualty insurance.

Whitney, A. W.	The Theory of Experience Rating	P.C.A.S., IV, 274
Leslie, W.	Distribution of Surplus by Casualty Companies writing Participating Insurance	P.C.A.S., VIII, 54
Michelbacher, G. F.	The Practice of Experience Rating	P.C.A.S., IV, 293

11. *Industrial Insurance.*

Carpenter, R. V.	Industrial Insurance in the United States and Canada	T.I.C.A., VIII, I, 62
Kineke, F. D.	Some Aspects of Modern Industrial Insurance	R.A.I.A., XXI, 23
Britt, K. J.	Industrial Assurance Profit-sharing in Great Britain	T.I.C.A., IX, I, 141
Briscoe, H. J.	Participating and Non-participating Industrial Assurance	T.I.C.A., IX, I, 396
Spratling, F. H.	Industrial Assurance Profit-sharing	J.S.S., IV, 234

12. *Methods Used in Foreign Countries.*

	Various Papers (Great Britain, France, Holland, Switzerland and Scandinavian countries)	T.I.C.A., III and IX
Manes, A.	In *Versicherungslexicon*	(Germany)
Coutts, C. R. V.	On the Distribution of Life Office Profits (Bonus Reserve Valuations)	J.I.A., LVII, 159