

ACTUARIAL STUDIES

No.1

SOURCES AND CHARACTERISTICS
OF THE
PRINCIPAL
MORTALITY TABLES
SECOND EDITION

JAMES S. ELSTON

THE ACTUARIAL SOCIETY OF AMERICA

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MORTALITY
TABLES



Original Edition

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GENERAL INTRODUCTION

IN view of the fact that, with the exception of a very few modern text books, the literature of Actuarial Science is contained in scattered original papers, The Actuarial Society of America issued a series of small volumes upon important actuarial subjects. Each volume was intended to bring together, as far as space permitted, the more important points of information on the subject discussed. The objects in issuing the series were twofold: (1) to assist students of Actuarial Science, and (2) to furnish a means of ready reference for Actuaries. The various subjects were allocated to Fellows of the Society by the Committee in Charge; and associated with the principal contributor, who was primarily responsible for the matter included and the views expressed, were one or more "Associate Contributors." These were appointed for the purpose of aiding and criticizing the work before publication. The title chosen to represent the character of this series was "Actuarial Studies."

The thanks of the Society and of the Committee in Charge are due to all the contributors who have freely given of their time and labor, with the sole purpose of helping others—especially students.

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CHAPTER I.

INTRODUCTION

THE mortality table is the scientific basis of all life insurance contracts. By means of it the probabilities of living and of dying can be ascertained, on the assumption that what has taken place in the past will be approximately reproduced in the future. A mortality table consists of a schedule showing for each age the number of persons who die and the number who survive out of a known number under observation. Usually the table starts with an arbitrary number, such as 100,000 at the youngest available age, this arbitrary number being called the *radix*, and gives for each year of age the deaths and the survivors from this original number. By adopting a radix the varying numbers which may have been under observation from year to year are reduced to a common basis suitable for comparison and convenient for computations of annuity values, premiums, and other functions.

The older mortality tables were formed from population statistics, but for about eighty years the tables which have been most valuable for insurance purposes have been derived from the records of life insurance companies. The information available to companies is usually accurate as well as complete in the numbers under observation at each age and the numbers dying, so that the rates of mortality represented by the ratio $\frac{d_x}{l_x}$ can be correctly stated for insured lives. The accuracy of this ratio is the fundamental requirement of a good mortality table; and it is of more importance that the deaths be stated at the proper ages and times than that the record of the number living should be so strictly correct, since any error in the numerator of the fraction has much more effect than a similar error in the denominator—at the younger

ages the effect is more than one hundred times greater.

The technical methods of constructing mortality tables from the records of insured lives are described in detail in Actuarial Study No. 2 of this series entitled "Construction of Mortality Tables from the Records of Insured Lives" by Ray D. Murphy and Percy C. H. Papps.

In forming a mortality table from population statistics, it is necessary, in order to get trustworthy figures, to have a census of the people showing the numbers living at each age, as well as the mortuary registers giving the numbers who have died at each age. The mere death rate, without distinguishing the age distribution of the people, is of practically no scientific value; the ages at death of those dying and also the ages of all who are included in the observations are essential factors. The principles and methods of constructing mortality tables from the data of the general population are discussed in detail in Actuarial Study No. 3 of this series entitled "Population Statistics and their Compilation" by Hugh H. Wolfenden.

The following account of the Sources and Characteristics of the Principal Mortality Tables is not intended to give more than an outline of the general features of the experiences discussed. More complete details of the technique of the construction and graduation of some of the tables are given in Actuarial Studies Nos. 2 and 3 mentioned above, and also in "Graduation of Mortality and Other Tables" by Robert Henderson which is No. 4 of this series.

The earliest mortality tables of which we read were those in use by the Romans for determining the values of life estates. Under the Falcidian law a man could not bequeath more than three-fourths of his property away from the direct heir; and it was a common practice to bequeath annuities or life interests, the values of which had to be determined by computation. Tables of a_x or e_x were given by the Praetorian Praefect Ulpian and the Jurisconsult Æmilius Macer about the third century (see T. A. S. A., Vol. XXVII, p. 471-2). These tables, although they cannot be viewed as anything more than a rough approximation, nevertheless are of con-

siderable historical interest, especially as Ulpian's table continued to be used officially in Northern Italy until the end of the 18th century.

No records have been disclosed, from the days of the Romans until the close of the seventeenth century, of any reliable tables of mortality. In 1693 Dr. Halley (of comet fame), the British Astronomer Royal, published the first tables of any importance which are now known.* He formed them from the registers of deaths in the city of Breslau in Silesia, taken for a period of five years. They were published in the Transactions of the Royal Society (Great Britain). Before the middle of the eighteenth century a table of mortality was formed from observations made in the city of London, and from this table the original premiums adopted by the old Equitable Society were computed. The mortality rates were very high.

* See J. I. A., Vol. I, pp. 43-46 and Actuarial Study No. 3, p. 85.

CHAPTER II.

POPULATION MORTALITY TABLES GREAT BRITAIN

CHESTER TABLES

IN the early development of scientific mortality tables we owe more to Dr. Richard Price than to any other individual; yet he is popularly remembered as the author of the Northampton Table which is held out as an example of faulty construction. Dr. Price himself was well aware of the difficulties with which he had to contend and he strongly urged the adoption of better systems of registration so that accurate mortality tables could be formed. Amongst other tables formed by Dr. Price were the CHESTER TABLES,* which gave the rates of mortality separately for males and females. For the time at which they were prepared they gave a fair conception of human life. The tables were formed from the birth and burial registers of the small town of Chester, in England. The accuracy of the assumptions was tested by an enumeration of the people which included a record of the numbers living at each age. Unfortunately the tables never came into general use, and the Northampton Tables became much better known.

NORTHAMPTON TABLE

The first tables used to any great extent for life insurance purposes were the Northampton Tables, detailed information regarding which is here given as the mode of construction calls attention to one or two errors which should be avoided. Moreover they are still prescribed under certain old statutes as the basis for determining the values of life estates for taxation purposes.

* See the later editions of "Price on Annuities."

There were two tables formed by Dr. Price, the first published in 1771 in his "Observations on Reversionary Payments," and the second, a more complete table, in 1783, the latter being known as *The Northampton Table*. The following extract from Dr. Price's remarks will partially explain the Tables:

"In the parish of All Saints, containing the greatest part of the town of Northampton, an account has been kept since 1735 of the ages at which all have died, also an account of the number of males and females that have been christened.

Christened 4220	Buried 4689
"Of these died under 2 yrs. of age	1529
aged 2 to 5	362
5 to 10	201
10 to 20	189
20 to 30	373 (351)
30 to 40	329 (351)
40 to 50	365
50 to 60	384
60 to 70	378
70 to 80	358
80 to 90	199
90 to 100	22
	<u>4689</u>

"In the fourth edition of this treatise, the following corrections were made:

"1st. The table printed in the first three editions having been formed from the Northampton Bills (of Mortality) for 36 years (i. e., the first Northampton Table 1735-1770) this table was rendered a little more correct in consequence of being formed from the same Bills for 46 years (1735-1780).

"2nd. The Bills give the number dying between 20 and 30 greater than between 30 and 40, but this being a circumstance which does not exist in any other register of mortality, and undoubtedly owing to some accidental and local causes, the decrements were made equal between 20 and 40, preserving the total of deaths the same that the Bills have them.

"3rd. The Bills giving only the total of deaths under two years of age and between 2 and 5, the proportion of deaths for every particular year between 2 and 5, and for every quarter of a year after birth till one year of age, were made the same nearly that the Chester register makes them."

Dr. Price formed the table by taking account only of the deaths and without using any enumeration of the population. If a census had been used a different result would have been reached and the tables would have been much more accurate.* In view of the difference between the number of christenings and of burials (469 or 10% of the burials) he assumed that not only 10% but 13% of the deaths were those of persons who immigrated to Northampton at the age

* See Actuarial Study No. 3, p. 85.

of twenty; his reason for adopting the higher percentage is not stated. The excess of deaths over births was probably caused almost entirely by deaths among Dissenters, whose children were not entered in the Parish Register of Christenings. Dr. Farr pointed out that, if effect had been given to the increasing nature of the population, the table would have shown lower and more accurate mortality rates and would have been much more valuable. The mortality amongst male children appears heavier than amongst females, a feature which has been confirmed in other tables.

CONSTRUCTION OF THE NORTHAMPTON TABLE.*

Age	Deaths	Numbers Living at First Age in Group (if Population had been Stationary)	Numbers in Column (3) Raised to Radix of 10000	Column (4) Less 1300 up to Age 20 (i. e. 13% of Radix)	Column (5) Raised in the Proportion of $\frac{11}{13}$ up to Age 20
(1)	(2)	(3)	(4)	(5)	(6)
0- 2	1529	4689	10000	8700	11649
2- 5	362	3160	6739	5439	7283
5-10	201	2798	5967	4667	6249
10-20	189	2597	5538	4238	5675
20-30	373 (351)	2408	5135	3835	5135
30-40	329 (351)	2057	4387		4387
40-50	365	1706	3638		3638
50-60	384	1341	2860		2860
60-70	378	957	2041		2041
70-80	358	579	1235		1235
80-90	199	221	471		471
90-100	22	22	47		47
	4689				

* See J. I. A., Vol. XVIII, p. 107 et seq.

The figures given by Dr. Price differ very slightly from those given in the last column and are probably the result of an arbitrary adjustment.

The mortality shown by the Northampton Table was excessively high. This led to large profits being made by the early insurance companies, in which a much lighter rate of mortality was experienced than the table led them to expect. On the other hand annuity companies and the government through their annuities lost heavily. Dr. Price was himself sensible of the very high mortality rates shown by the Northampton Table, and in 1782 he recommended the Equitable

Society to adopt the lower rates given in the Chester Experience, but an excess of caution on the part of the Society gave the Northampton Table a standing which it never deserved.

The mortality shown by this table has been found to correspond roughly to the mortality of certain colored races; and, as very complete tables of monetary values are available on the Northampton basis, the tables may in rare instances still be used with advantage, when investigation shows that some class of substandard mortality approximates to that of the old table.

As far back as 1823 the faults of these tables were well known, as the following quotation shows:

"Dr. Price did as much as the nature of his materials would allow. For in those days no census or enumeration of the population had been made; and without a comparison of a census (in which the ages are carefully distinguished) with the Bills of Mortality, an accurate Table of Observations cannot possibly be obtained." FARREN on *Life Assurance*, 1823.

THE CARLISLE TABLE OF MORTALITY*

(Published 1815)

This table was formed by Joshua Milne, Actuary to the Sun Life Office, London, from materials contained in a tract published by Dr. John Heysham at Carlisle in 1797. These materials consisted of a careful census in January, 1780, and a less accurate enumeration in December, 1787, of the two parishes in Carlisle, St. Mary and St. Cuthbert, together with the Register of the deaths in those parishes during the nine years, 1779 to 1787.

The original statistics were divided into male and female; and the deaths showed also the condition of each sex as regards marriage; but the graduated tables dealt only with the total of both sexes, details of which are given in the table quoted on the next page.

It has been pointed out that in the second enumeration the ages do not appear to have been taken, but that the incre-

* Milne on Annuities and Assurances, London, 1815, Article 704, p. 404 et seq. See also King, J. I. A., XXIV, 186.

ment 1000 seems to have been proportionately added to the first enumeration, i. e., in the ratio $1000 \div 7677$. This incre-

Between the Ages	Population		Deaths in the 9 years 1779-1787	Between the Ages	Population		Deaths in the 9 years 1779-1787
	in Jan., 1780	in Dec., 1787			in Jan., 1780	in Dec., 1787	
0-1			390		3327	3761	979
1-2			173	20-30	1328	1501	96
2-3			128	30-40	877	991	89
3-4			70	40-50	858	970	118
4-5			51	50-60	588	665	103
				60-70	438	494	173
0-5	1029	1164	812	70-80	191	216	152
5-10	908	1026	89	80-90	58	66	98
10-15	715	808	34	90-100	10	11	28
15-20	675	763	44	100-105	2	2	4
	3327	3761	979		7677	8677	1840

ment looks like an approximation to the true population and Dr. Heysham said of it, after correcting some errors, "I am persuaded the enumeration is *now pretty exact*."

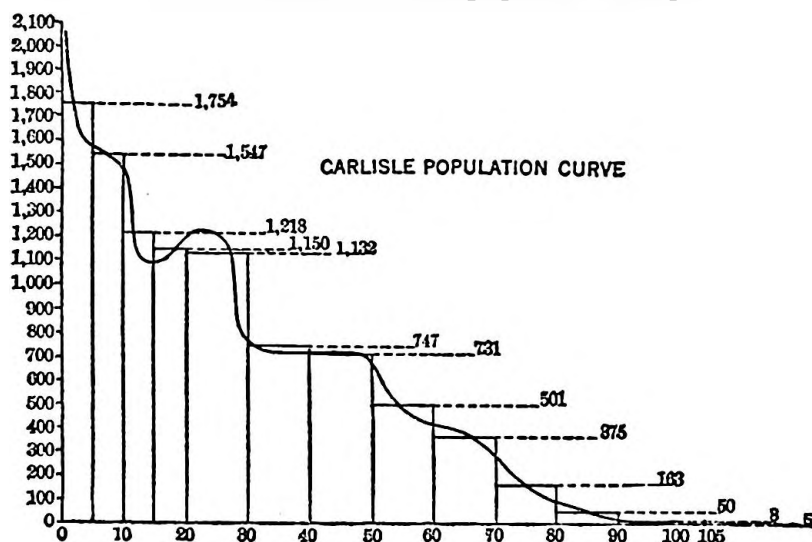
Adding the totals ($7677 + 8677$) together, the result was multiplied by four in order to place the population and the deaths on a common basis for comparison, the latter being reduced by one-ninth in order to effect this. The result was an approximation to the sum of the total population exposed during eight years, and this was compared with an approximation to the sum of the deaths during those eight years.

The following table shows the results thus obtained:*

Between the Ages	Population	Deaths	Between the Ages	Population	Deaths
0-1		347.0		28352	870.5
1-2		153.7	20-30	11316	85.3
2-3		113.9	30-40	7472	79.1
3-4		62.2	40-50	7312	104.9
4-5		45.3	50-60	5012	91.6
			60-70	3728	153.8
0-5	8772	722.1	70-80	1628	135.1
5-10	7736	79.1	80-90	496	87.1
10-15	6092	30.2	90-100	84	24.9
15-20	5752	39.1	100-	16	3.6
	28352	870.5		65416	1635.9

* Sutton, J. I. A., Vol. XXIV, pp. 110-122.

The results given in the above table were graduated by a graphic method of adjustment, two graphs being prepared, one for the population and one for the deaths. The following is a geometrical illustration of the population graph:



The base line represents the age attained while the upright lines (called ordinates) represent the numbers alive at the various ages. The areas of the parallelograms represent the total population in the several groups; thus, the base of the first parallelogram being 5 years, and its area 8772, its altitude is $8772 \div 5 = 1754$. Similarly, the altitudes of the next 3 parallelograms are $7736 \div 5$, $6092 \div 5$, and $5752 \div 5$ respectively, but the base of the fifth being 10, its altitude is $11316 \div 10$, and so on for the other altitudes.

A curved line was then drawn through the parallelograms proceeding smoothly from the one to the other, adding a portion to each, and cutting off an equal part. The areas were then subdivided, each subdivision representing a period of one year. These years were bisected, and the length of the ordinate then erected gave the graduated population at the middle of the year, i. e., L_x , which Mr. Milne wrote $\bar{L}_x$. The deaths were treated in exactly the same way, and $\bar{D}_x$, i. e., the mean number that died between the

ages x and $x+1$, was obtained. From these mean values Mr. Milne formed the values of d_x . The radix was taken as 10,000 and the figures after age 100 were adjusted arbitrarily.

Females represented about 55% of the population, which had the effect of showing light mortality at the older ages, one of the characteristics of female mortality being that it is much lighter than male after the age of 50. Amongst insured lives only about 10% are females, so that the table was never a good one for life insurance premiums or valuations, and it is not now used for such purposes. The original statistics and remarks published by Mr. Milne in 1815 contain a great deal of valuable information, including comparative mortality—(1) from different diseases, (2) during different seasons of the year, and (3) of the two sexes. The original publication is well worthy of careful perusal even at this date.

The results of the graduation left several irregularities, especially at the older ages. For example, at age 91 the expectation of life is 3.26 years, while at age 95, when it should be less, it is 3.83. This was caused by the system of graduating separately the population and the deaths, and the difficulty of reading the values from the graduated curve (see Actuarial Study No. 3, p. 98). The irregularities were completely removed in a graduation which was successfully accomplished by Messrs. King & Hardy* in 1880 by applying Makeham's formula. This formula gives a perfectly smooth curve when the results are presented graphically and the general characteristics of the original Carlisle Table were maintained from childhood till old age. It is only when approaching age 90 that the difference between the two tables becomes considerable. Facilities were thus given for the calculation of the more intricate benefits.

The table has been very extensively used, and even yet is of considerable value in calculating the values of Reversions, etc. As the published tables based on Carlisle Mortality are unusually extensive, it is often adopted as a matter of convenience in connection with special calculations.

* J. I. A., Vol. XXII, p. 221.

ENGLISH LIFE TABLE No. 1*

The compulsory registration of births, marriages, and deaths was introduced in England in 1836, and this requirement of law, which became operative from July, 1837, gave a proper basis for the formation of a national table of mortality.

Dr. William Farr was appointed Compiler of Abstracts in the General Register Office in 1839 and for the next forty years he was engaged in developing a national system of vital statistics. He had no official connection with the census enumeration of 1841, the observations being then made on a plan recommended by Professor de Morgan and Mr. Griffith Davies. But Dr. Farr used the census enumeration in conjunction with the Registers of deaths to compile the first English Life Table, which was published in the Registrar General's fifth annual report, 1843. This table was based upon the Census Returns for England and Wales made on June 6, 1841, and the deaths taken from the Registers of that year. As the deaths related to the calendar year, it was necessary to make a correction in the population for the period from June 6 to July 1—the middle of the year. An increase at the annual rate of 1.334% had taken place in the ten years from 1831. It was assumed that this rate of increase continued during the 25 days, or .07 of a year, until July 1st. The population as adjusted to July 1st was 15,927,867 and the deaths 343,847. In the census enumeration some of the ages were not specified and in such cases they were assumed to be in the same proportion as those whose ages were given. The ages at death were not given in 521 cases and these were distributed in a similar manner.

The rates of mortality in a single year cannot be viewed as a proper basis for a national table, because even with a large population of nearly 16,000,000 the variations caused by climatic conditions, epidemics, etc., are great. It appears, however, from subsequent results that the year 1841 did prove to be a fair average one.

Owing to the tendency for persons at any age between 30

* Registrar General's Returns as quoted. Walford's *Cyclopædia*. Farr's "Vital Statistics, 1885." Actuarial Study No. 3, p. 99.

and 40 to state their age as 30, Dr. Farr said that in quinquennial groups the death rate for the period from age 30 to age 35 would be too small and for the period from 35 to 40 too large. He further said: "It is my opinion that the ages of the people have been returned with sufficient accuracy for all practical uses in *decennial periods*."

To overcome such difficulties and obtain rates of mortality for each age, Dr. Farr dealt with the statistics as if they were two series of geometric progressions—the first from age 15 to age 55 and the second from age 55 to age 95. He obtained the ratios for these geometric series by comparing the increases in the mortality in the groups—

First	15-20	25-30	35-40	etc.
Second	20-25	30-35	40-45	etc.

Since the errors in these two series would be of a similar nature there was a great probability that the ratio would be satisfactory. From such quinquennial grouping a central death rate was obtained and treated as the death rate applicable to the central age of the group. The table was based upon 100,000 births of whom 51,274 were males and 48,726 females—being the proportion in which the births of the two years 1840 and 1841 were distributed. The table therefore represents a generation of 100,000 individuals born at the same instant and shows the relative numbers of males and females at each age, as well as their probabilities of living and expectations of life.

The period of infancy was differently treated and Dr. Farr explained the method as follows:

"The births of boys in the two years 1840-41 amounted to 520,157; which was at the rate of 260,078 a year; whence it may be assumed that 260,078 were born in the year, of which 1 January, 1841, was the middle—or that the mean date of their birth was January 1, 1841. We can then reason upon the assumption that 260,078 boys—the mean ann. number of boys born in 1840 and in 1841—were born 1 January, 1841. But all the boys who *died* in 1841, under 1 year of age, must have been boys born in 1840 and in 1841. The deaths occurred in the year 1841—in one year—and they must therefore be compared with the births in one year, viz, with the births in the year of which January 1, 1841, was the middle. We have then this result, that 41,444 of 260,078 boys born died in the first year after birth. . . .

"All the births are not registered: the deaths in the first year must have occurred out of more than the number of births returned; and the mortality in the first year must have been less than that given in the table, which is, however, lower than

the mortality deduced immediately from the children stated at the enumeration to be living at the first year of age, and the deaths registered at the same age. As it is, the mortality in the first year stands lower than in any other authentic table."

As large masses of figures were involved in each age-group a graduation of the group results was neither necessary nor desirable. To get results at individual ages, "the mortality at every age was interpolated by the log which expresses the ratio of increase in the mortality at every year of life, and the chance of living each year was de-

duced from $\frac{1 - \frac{1}{2}m}{1 + \frac{1}{2}m}$."

ENGLISH LIFE TABLE No. 2

This table was published in the 12th Report of the Registrar-General, 1853, the census of 1841 again furnishing the basis for the exposures, but the deaths were taken for a period of seven years from January 1, 1838, to December 31, 1844, inclusive, the total deaths in this period being 2,436,648. During that time there were no epidemics of any consequence. Dr. Farr calls attention to the rapid variation in the rates of mortality in infancy, "when the rate of mortality varied so rapidly that every year and even month is marked by a change." At the earlier ages the mortality rates were directly taken from the return but after age five, quinquennial, and after age fifteen decennial periods were used. The table is of little importance in comparison with the later compilation with which the results are in general agreement. The final tables were based upon a radix of 10,000,000 persons—5,126,235 males and 4,873,765 females.

ENGLISH LIFE TABLE No. 3*

This was published in 1864 as a distinct work by Dr. Farr. Several improvements were introduced in construction and the Table is worthy of description in greater detail than either No. 1 or No. 2.

* "English Life Table," Longmans, Green & Co., London, 1864.

Walford's Insurance Cyclopædia.

Farr's "Vital Statistics," 1885.

Important. J. I. A., Vol. XLII, p. 228 et seq. (King.) Actuarial Study No. 3, p. 99.

Two censuses were used—those of 1841 and 1851, comprising 15,929,492 persons as adjusted to the middle of 1841 and 17,982,849 in 1851, while the deaths extended over the 17 years from 1838 to 1854—that is, approximately $3\frac{1}{2}$ years before the first census and a similar period after the second census. During that time there were 6,470,720 deaths.

The population was carried back to the beginning of 1838 by using the ratio of increase in the population from 1841 to 1851, assuming that such increase was in geometrical progression. Thus, having obtained the ratio from the formula

$$\log r = \frac{\log P_{51} - \log P_{41}}{10},$$

where P_{51} represents the population in 1851, the population at the beginning of 1838 was estimated by the formula

$$\log P_{38} = \log P_{41} - 3\frac{1}{2} \log r.$$

The following table shows the population of England and Wales estimated in the middle of 1841 and 1851 in Dr. Farr's groupings.

Ages	Persons		Males		Females	
	1841	1851	1841	1851	1841	1851
All Ages	15,929,492	17,982,849	7,784,883	8,808,662	8,144,609	9,174,187
0-	2,107,008	2,355,345	1,048,270	1,180,430	1,058,738	1,174,915
5-	1,865,856	2,098,808	953,235	1,053,510	912,621	1,045,298
10-	1,772,913	1,919,255	880,567	967,007	892,346	952,248
15-	3,145,541	3,418,488	1,511,602	1,671,634	1,633,939	1,746,854
25-	2,450,322	2,740,919	1,174,473	1,323,621	1,275,849	1,417,298
35-	1,778,737	2,089,629	875,874	1,017,018	902,863	1,072,611
45-	1,270,178	1,516,324	617,113	734,314	653,065	782,010
55-	832,692	1,010,973	399,490	482,788	433,202	528,185
65-	483,593	579,187	224,310	268,052	259,283	311,135
75-	190,443	220,618	86,736	97,008	103,707	123,610
85-	30,541	31,754	12,635	12,745	17,906	19,009
95	1,668	1,549	578	535	1,090	1,014
and up-wards						

By arranging the groups in this way a correction is applied to the tendency in census returns to state the ages at the nearest decennial, as 30, 40, etc. The deaths were arranged to correspond and thus average annual rates of mortality were obtained by dividing the deaths during the 17 years

by $8\frac{1}{2}$ times the sum of the populations in 1841 and 1851. This was apparently a departure from the principle of geometric increase; but it was found by actual trial that the use of the geometric mean (1838-1854) would have produced results almost identical with those of the method adopted.

Having thus obtained an average death rate for each age-group Dr. Farr calculated the values of p_x at each age as explained in Actuarial Study No. 3, par. 97 (ii).

As in Table No. 1 the statistics were scheduled in three parts:

1. Persons—Male and Female.
2. Males.
3. Females.

The radix of the first part was 1,000,000 children born alive, of the second part 511,745, and of the third part, 488,255, being the proportions in which boys and girls were born during the seventeen years under observation. The male and female tables were constructed independently; that of the "persons" was obtained by combining the other two.

Out of 1,000,000 persons born, it was shown that 149,493 died in the first year; 53,680 in the second; with decreasing numbers of deaths until the age of 13, when 3,382 died between that age and 14; then this number increased steadily for each age until it reached 15,469 who died between 73 and 74. Thereafter, although the rates of mortality increased rapidly the annual deaths diminished, 92 dying at the age of 100 and the last one at the age of 108—"so 109 years is the limit of age of this table."

Notwithstanding that census tables must be computed from data which are admittedly imperfect, being based on voluntary unverified statements, this table is looked upon as a great advance on older methods, and many of the ideas used for the first time have since been copied and adopted by other statisticians.

Census tables represent the mortality of all classes mixed. This means a preponderance of the laboring and industrial classes; but the table cannot be said to represent even these classes, because of the percentage of well-to-do people. For financial calculations applicable to individuals the tables do not meet present-day requirements.

Mortality in Infancy.—From birth until the age of 1 the deaths were given for each month of age. They were extracted from the registers of births and deaths directly, the census figures being of no value for this particular purpose. For the 17 years under observation the deaths under age 1 were given for (a) the first 3 months, (b) the second 3 months, and (c) the next 6 months; but for 8 years of the period Dr. Farr abstracted the deaths for each one of the first three months, and then distributed in the same proportions the deaths during the 17 years occurring in the first three months. The other figures for the first twelve months were obtained by interpolation. These deaths in conjunction with the total number born admitted of a good approximation to the probabilities of living and dying.

The English Life Table No. 3 shows slightly higher mortality rates than either No. 1 or 2, as indicated by the expectations of life at age 30 (male). The cause of this slight difference lies probably in the processes employed, rather than in reduced vitality.

English Life No. 1=33.13 years.

No. 2=33.21 "

No. 3=32.76 "

The volume published in 1864 not only explained the construction of the experience but gave also about 600 pages of tables with monetary values and commutation columns from 3% to 10%, also complete joint-life values at 3% for (1) two males, (2) two females, and (3) male and female. The tables have been freely used by Industrial Insurance Companies.

HEALTHY ENGLISH LIFE TABLE No. 1*

After the completion of the English Life Table No. 3 Dr. Farr set himself to the construction of another table based upon the experience of 63 of the healthiest English districts inhabited by nearly a million people, dividing the experience into males and females. He gave a brief description in his "Vital Statistics" as follows:

* J. I. A., Vol. IX, pp. 124, 188, etc. Farr's "Vital Statistics."
Transactions of the Royal Society, 1859, pp. 838-41. Actuarial Study No. 3,
p. 99.

"The Healthy District Life Table was constructed in 1859 from the Census enumeration of 1851 and from mortality observations extending over the five years 1849 to 1854 in 63 districts of England and Wales which showed during the ten years 1841-50 a mean annual death-rate not exceeding 17 per 1,000 persons living. It has been found by experience that this Healthy District Life Table expresses very accurately the actual duration of life among the clergy and other classes of the community living under favourable circumstances."

In general, the methods of construction were those used in compiling the English Life Table No. 3, with slight modifications.

The probabilities of life for the first four years, namely p_0 , p_1 , p_2 , and p_3 , were also obtained from the births and deaths without reference to the census figures. For example, half the number of births in two calendar years was taken as the number exposed to risk from January 1 of the second year, while the death records gave the numbers dying under age 1, from 1 to 2, from 2 to 3, etc., in the subsequent years, whence it was easy to form l_x and d_x columns, and get the early values of p_x . Three such computations were independently made by using different years, and the values finally adopted were the mean of the three. Values of p_7 , p_{12} and p_{20} were obtained from the census figures, and the intervening figures were computed by mathematical interpolation from p_3 to p_{20} . Dr. Farr pointed out that if there were much emigration from a community this would result in an understatement of the death rates, because the births would be properly recorded, but children would be taken away by their parents and would die elsewhere. This effect would be heightened each year after birth, and at ages 4 and 5 the death rates might be appreciably affected. On the other hand a failure to register births might have the opposite effect.

ENGLISH LIFE TABLES NOS. 4, 5 AND 6*

The fourth English Life Table was based on the mortality for the decennium 1871 to 1880 and founded upon Table No. 3 by making allowance for the different rates of mortality in the new period. Details regarding it are of little interest.

* These Tables are all published in the supplements to the annual reports of the Registrar General of Births, Deaths and Marriages in England and Wales and appear in the 45th, 55th, and the 65th (1907) annual reports respectively.

The fifth and sixth English Life Tables were prepared by Dr. John Tatham, who introduced important improvements in the methods of construction, which are described in Actuarial Study No. 3, par. 98 (iii). Table No. 5 was based on the censuses of 1881 and 1891 and the deaths of the ten years 1881 to 1890; No. 6 was derived by very similar methods from the censuses of 1891 and 1901 and the deaths of 1891-1900. A Life Table for London was also constructed from the census data of 1891 and 1901 and the deaths of 1891-1900. Monetary tables at 3% for the English Life Table No. 6 have been published in J. I. A., Vol. XLIII, p. 351.

Probably the most important innovation in the Mortality Statistics of the 5th and 6th English Life Tables dealt with *Occupational Mortality*.—The varying rates of mortality in different occupations first formed the subject of an inquiry by Dr. Farr, and his ideas were developed and improved upon until a very complete analysis of this nature was given for the triennial period 1900 to 1902, based upon the census figures of 1901. The numbers living in 1901 were obtained from the census, and the deaths in each occupation from the registers. The information, which related only to male lives, was then tabulated for four principal areas:

1. London,
2. Industrial districts,
3. Agricultural districts, and
4. Other parts.

Comparisons of this nature were generally made in other countries from "crude" death rates, which show the number of deaths per 1,000 of population without giving effect to age or sex distribution and are thus often misleading; but these publications of English Mortality give "corrected" figures. A standard table is adopted showing the numbers alive in decennial periods in an average population, and a corrected death rate for the particular occupation or locality is then deduced by reducing the population to the standard basis for comparison. The necessity for such treatment is well illustrated in the case of farmers. The crude death rate amongst farmers, ages 15 and upwards, for 1900-2 was 16.44 per 1,000,

whereas the general death rate amongst all males of 15 and upwards was only 16.23. From this it would appear as if farmers were subject to a higher death rate than males generally; yet at each age group the death rate amongst farmers is much *lower* than the death rate of males generally, and the high crude death rate is caused by the large number of farmers at the older ages, and, relatively, the small number at the younger ages. (See Actuarial Study No. 3, p. 125 et seq.)

The principal comparisons were made during the active working period of life, in four decennial age groups from 25 to 65. Over 100 occupations were dealt with—in some instances industries of like character being grouped under the same heading. The census tables of 1901 also supply information about those who had retired from any particular calling—the word “retired” covering those who had retired in comfort as well as those whose health had broken down and who had thus lost the ability to earn a living. The occupation dealt with was that which had last been followed, and no system has yet been devised for dealing with changes.

In the population from ages 25 to 65 there had been a consistent decline in mortality between 1890–2 and 1900–2, this decline varying from 11 per cent at ages 55 to 65 to 17 per cent at ages 25 to 35. Graphic illustrations are given showing the gradations of the mortality for the age group 25–65 in different occupations, commencing with clergymen, gardeners, game keepers, and farmers, who show the lowest corrected death rates, and ending with hotel servants, costermongers, tin miners, and general laborers, who show the highest death rates.

In some instances the figures must be used with great caution because the classified numbers become so small that accurate deductions from them are not warranted. Moreover, some occupations requiring muscular strength and vigor are forsaken in favor of an occupation of lighter character when a man falls into ill health in the former. These occupations, therefore, exercise a species of selection favorable or adverse by attracting to themselves either the unusually vigorous or the weak. This probably had much to do with the heavy death rates in the miscellaneous occupations

like costermongers and general laborers. During the period 1890-1892 there was prevalent a serious epidemic of influenza, which made the mortality at that time abnormally high. Accordingly the lower rates shown ten years later are probably not caused solely by improvement in longevity. The census date—1st April, 1901—was not exactly in the middle of the three years during which the deaths were computed, but it was thought better to neglect the small error thus introduced rather than to make estimates of the subdivided populations in various occupations to the exact middle of the year 1901, and it was found impracticable to take the deaths otherwise than for the three complete calendar years.

In the statistical tables the deaths in the various occupations are not only given in seven age groups, but are given under "causes of death," and are also published separately for those who are occupied and those who are "occupied and retired." The mortality of industrial districts is above, and of agricultural districts below, the average.

HEALTHY ENGLISH LIFE TABLES NOS. 2 AND 3

The utility of a mortality table deduced from healthy districts only, as recommended and originated by Dr. Farr, has been confirmed and approved by later statisticians. Dr. Farr based his table upon those districts in England and Wales whose *crude* death rates did not exceed 17 per 1,000, one district with larger death rate having been included through error. As the death rates were tabulated to the nearest whole number, this practically meant under 17.5 per 1,000.

A second Healthy English Table was formed after the census of 1891, and at that time a "corrected" death rate was used showing the number of deaths per 1,000 in each registration district if the ages and sex distribution had been in the same proportion as the general population at the preceding census. The second Healthy Districts Table was formed from the statistics of England and Wales over the period from 1881 to 1890, and a third Healthy Districts Table was formed for the period from 1891 to 1900.

Lower rates of mortality were recorded during each of these decennial periods than had been shown in Farr's earlier

table. In forming the *second* table those districts only were used where the *corrected* death rate was *15 per 1,000 or less*. There were 263 of such districts, with an aggregate mean population of 4,603,055, representing during the ten years over 46,000,000 years of life, or more than nine times as many as the older table.

In forming the *third* table the standard of healthiness was again raised and 260 districts were included whose aggregate population was 4,477,485 with *corrected death rates not exceeding 14 per 1,000*. Of the 260 selected districts 222 were common to the second and the third tables. Accordingly 41 of the districts used for the second table were excluded in forming the third table, and 38 new districts were included. When reduced to a common standard, after allowance for age and sex distribution, the corrected death rate of the entire group of healthy districts for the third table was .85 per 1,000 lower than that of the second table. Had the corrected death rate of 15 per 1,000 been used (in 1891–1900) there would have been 352 districts (with an aggregate mean population of 7,326,280 persons) qualifying under such standard for the third table.

The districts which experienced the low death rates were almost exclusively those which are rural in character or contain only small towns with rural surroundings. No part of London is represented in this table, yet of the healthy districts, a large proportion is in the section of England of which London might be taken as the center.

NATIONAL INSURANCE ACT (1911) LIFE TABLES

Before the appearance of the next in the series of English Life Tables, namely Nos. 7 and 8, tables for males and females were constructed on the basis of the estimated population of England and Wales on June 30, 1909, and the deaths registered in 1908–1910, for the purpose of computing reserve values in connection with the National Insurance Act, 1911. They may be found in J. I. A., Vol. XLVII, p. 548, and should be noted not only because of the object for which they were compiled but also on account of the use of a curve-fitting method of construction, which is based on a modification of Makeham's second formula (*loc cit.*, and Actuarial Study

No. 3, p. 114). The rates of mortality of these tables are considerably lower than those of English Life Table No. 6, except at the very oldest ages.

ENGLISH LIFE TABLES NOS. 7 AND 8

These two tables were published at the same time, in 1914, in Part I of the Supplement to the 75th Annual Report of the Registrar-General of Births, Deaths, and Marriages in England and Wales. In an Appendix all of the statistics are published which were used to form the final tables as well as the mathematical formulas employed.

Table No. 7 consists of the experience during an intercensal period of ten years from 1901 to 1910 inclusive. Table No. 8 deals with the deaths in three calendar years 1910, 1911, and 1912, grouped around the census point in 1911, the population being adjusted to 1st July, 1911. Both investigations were made by Mr. George King, and his report on the work contains a lucid description of the tables and of the reasons for the decisions which were reached on points of difficulty.

Table No. 7 follows in its general principles the main lines of investigation which characterized, and were developed from, the earlier English Life Tables; but a radical change in mode of construction was made in the adoption of King's method. The population for each year to age 4 inclusive was enumerated, then in quinquennial groups 5 to 9 last birthday, 10 to 14, etc. The deaths from 1901 to 1910 were given in decennial groups from age 25 onwards, but were subdivided into quinquennial groups in the proportions existing among the deaths of 1910 to 1912. From these values, quinquennial pivotal values of population and deaths were obtained, ages 12 to 97 inclusive, and hence m_x , etc. From these central death rates, quinquennial values of q_x were deduced. Then by osculatory interpolation, using the function $\log(q_x + .1)$, the values from 18 to 97 were obtained.

The rates of mortality at ages 0 to 4 inclusive were determined from the death returns at individual ages and the readjusted average population. A readjustment was necessary because at ages under two last birthday there was a marked deficiency in the numbers enumerated in the census

returns. The theoretical "numbers living" at each age were accordingly calculated from the birth and death returns only, and in constructing Table No. 7 were then reduced in the proportion necessary to make the total "numbers living" under age 5 coincide with the numbers enumerated by the census. This reduction assumes that the total of the census populations under 5 was correct. This method was adopted because a similar assumption had been made in the construction of English Life Table No. 6, and it was desired that the two should be comparable. Mr. King, however, shows clearly that this method is based on an erroneous assumption—for the deficiency in the numbers enumerated at ages 0 and 1 is not made up at ages 2 to 4; and therefore in preparing English Life Table No. 8 he used the "numbers living" computed from the birth and death returns without adjusting them to conform to the census total.*

At the extreme old ages no assumption was made with regard to the termination of the table. The original data were used to age 92, with a pivotal value at age 97. A fourth difference series of values of $\log p_x$ was determined from the values at ages 89, 90, 91, 92 and 97, and projected onwards. While there is no theoretical limit the practical limiting age is about 110 for men, and 114 or 115 for women. It is claimed that the new tables are more accurate than previous English Life Tables at the older ages.

Important changes in methods of construction were made in forming Table No. 8. These may be briefly summarized as follows:

1. The reduced period gives a more definite measure of the mortality;
2. The census of 1911 gave numbers for individual ages instead of quinquennial or decennial age periods. It was possible therefore to group these in a more scientific way and the groupings adopted were ages 4 to 8, inclusive, 9 to 13, etc., instead of 5 to 9, 10 to 14, etc., as formerly.
3. Marital condition was given in tabulating the deaths of

* See also Actuarial Study No. 3, pars. 30–32 and 92.

women during that period. Wives show lighter mortality rates than widows, and spinsters lighter than wives, except between ages 45 and 55, though in the latter case the differences are not very marked.

4. The rates of mortality for the first six years of life were calculated from the birth and death returns without reference to the census figures.
5. Healthy Districts Tables were discontinued in favor of Tables based upon the mortality in
 - a.* London;
 - b.* County boroughs;
 - c.* Urban Districts; and
 - d.* Rural Districts.

These four groups represent approximately diminishing degrees of urbanization.

The census population in 1911 was approximately 17,400,000 males and 18,600,000 females. Under the general title of English Life Table No. 8, there are published the following:

1. Life table for the entire area of England and Wales—males and females respectively, with the usual mortality functions.
2. Life tables for females only according to marital condition—(*a*) Spinsters, (*b*) Wives, and (*c*) Widows.
3. Sectional life tables—males and females respectively—(*a*) Administrative County of London; (*b*) Aggregate of County Boroughs; (*c*) Aggregate of Urban Districts; (*d*) Aggregate of Rural Districts.

Throughout the greater part of life London shows lighter mortality than the County Boroughs; both of these show heavier mortality than the Urban Districts, while the Rural Districts show the lightest mortality of all. Female mortality in London up to age 25 is better than any of the others.

The effect of the emigration of young adults, and of immigration caused by the return of older British subjects from overseas; the effect of old age pensions on the statements of age; deliberate misstatements by women, etc., are discussed

with other minor criticisms in a review of the Tables in J. I. A., Vol. XLIX, pp. 96-107. The suggestion is made to attempt elimination of such errors—a dangerous proceeding unless based on carefully ascertained facts.*

The notation was changed from the old system used in English Life Tables to the regular actuarial notation, Text Book form. If Mr. King had done nothing in actuarial science except to standardize our notation, all future generations of actuaries would be in his debt.

An interesting part of the report is that the mortality shown by the various tables was reduced to one common basis for comparison. The rates were lower at all ages and for both sexes by the English Life Tables No. 7 than by No. 6, and lower at practically all ages in No. 8 than in No. 7.

EXPECTED DEATHS PER ANNUM IN A POPULATION DISTRIBUTED ACCORDING TO THE CENSUS RETURNS OF 1911—AGES 0 TO 89, INCLUSIVE.

	Ages Last Birthday	No. 6	No. 7	No. 8
Males.....	0- 4	106,677	92,197	79,357
	5-59	122,830	101,995	91,628
	60-89	94,369	87,114	84,216
	0-89	323,876	281,306	255,201
Females.....	0- 4	89,394	76,879	65,980
	5-59	112,806	93,667	81,497
	60-89	108,344	94,472	91,091
	0-89	310,544	265,018	238,568

Monetary tables based on Table No. 8 are given in J. I. A., Vol. LIII, p. 313.

The occupational mortality of men in the three years 1910, 1911, and 1912 was dealt with in Part IV of the Supplement to the 75th Annual Report of the Registrar-General, along lines somewhat similar to those followed in connection with previous censuses (See J. I. A., Vol. LV, p. 266).

ENGLISH LIFE TABLE No. 8-A†

A very interesting method of studying mortality by means of "analyzed" mortality tables was suggested by Mr. Eric B.

* See further discussion by Mr. King and others, J. I. A., Vol. XLIX, p. 297 et seq.; J. I. A., Vol. XLVIII, p. 207, on "Graduation of Ages."

†J. F. A., Vol. X, p. 45.

Nathan. This consisted in constructing separate mortality tables for different causes of death. Mr. Nathan based his studies on the deaths in England and Wales in 1911 and 1912 and the population estimated as of January 1, 1912. He studied twenty groups of causes of death, tuberculosis and cancer being further divided into two and five subgroups. Some of the other causes studied were, epidemic diseases, excluding influenza; influenza; rheumatism; diabetes; nervous diseases; heart and arterial diseases; bronchitis; pneumonia and other respiratory diseases; nephritis and Bright's disease; diseases of the stomach; old age and senile decay; infantile diseases; accidents, violence, and suicides; and minor and non-specified diseases. Graduation of each of the 20 tables was by osculatory interpolation in such a way that the total of deaths at each age from individual causes equaled the total deaths at that age. Male and female mortality was studied separately.

Mr. Nathan called the combined table the English Life Table No. 8-A because it corresponded closely with the English Life Table No. 8. The deaths of 1910, used in No. 8, were excluded here because the causes of death of that year were not suitably separated by cause. This resulted in a total death rate slightly greater than in English Life Table No. 8.

Such an analysis by "analyzed" mortality tables was proposed as offering interesting possibilities in investigating improvement of mortality, in studying select mortality and the relation of select mortality to ultimate, in modifying known mortality to fit other needs such as use in connection with pension funds, in analyzing the mortality among sub-standard lives, and even in graduating mortality tables.

DR. E. C. SNOW'S ABRIDGED LIFE TABLES

These tables were published in 1920, after the war, as Part II of the Supplement to the 75th Annual Report of the Registrar-General of England and Wales. They comprise 28 abridged male and female life tables based on the mortalities of 1911-12 in the North, Northern Boroughs, other Northern Urban Districts, and Northern Rural Districts, and also in the separate counties and in County Boroughs with popula-

tions over 100,000. The method of construction was new, and is described in par. 124 (v) of Actuarial Study No. 3 where a criticism of the method is also discussed.

ENGLISH LIFE TABLES No. 9

These tables, prepared by the Government Actuary, Sir Alfred Watson, were published in Part I of the Registrar-General's Decennial Supplement, 1921. The data for ages 6 and up were the census returns of 1921 and the deaths for the three years 1920–22; for ages under 6 they were the births for 1914–22 and the deaths for 1915–21. No tables were constructed for the decennium 1911–21, largely on account of the disturbances due to the war, so that the long series of tables of this type, culminating in No. 7, has been interrupted. The general principles adopted were those used by Mr. King in the construction of English Life Table No. 8, to which No. 9 is therefore closely related. The following variations in method were, however, introduced: (1) the births for each quarter instead of for each year were used in computing the infantile rates of mortality, on account of the fluctuations in the numbers of births, not only from year to year, but also from quarter to quarter; (2) it was found that the best age grouping to counteract the influence of minor misstatements was 2–6, 7–11, etc., instead of the 4–8, 9–13, etc., grouping used by Mr. King; (3) after proper tests a Gompertz graduation was used at ages 85 and up because it had been observed that the ratio $\frac{\log_{10} p_{89}}{\log_{10} p_{84}}$ was approximately equal to the ratio $\frac{\log_{10} p_{94}}{\log_{10} p_{89}}$; and (4) after various experiments it was decided to use the four values of q_x for ages 5, 9, 14, and 15 already found and to obtain from these the intervening values by means of a third difference formula.

An experimental graduation based on quinary groups of q_x , instead of the deaths and populations separately, was made. As the results agreed very closely with those of Mr. King's method, the latter was retained, partly to make the tables as nearly comparable as possible with English Life Table No. 8 and partly to encourage its study and use as a model by Officials of Local Authorities who would be dis-

couraged by more abstruse mathematical processes. Another reason for the adoption of King's method of osculatory interpolation between quinquennial pivotal values was that such a method, producing curves which adhere closely to the original data, was more than usually necessary, owing to the extensive and well defined depression in the natural sequence of numbers representing the male population at ages between 20 and 45 owing to the large number of deaths resulting from the Great War.

As the 1921 census was taken only eleven days prior to the central point of the three-year period, no adjustment was made for this fact. It might be noted that Sir Alfred Watson used the interpolation between quinquennial pivotal values, although he had available the population and the deaths at individual ages and might have obtained the crude values of q_x and graduated these. No allowance was made for the possible effect of migration.

Complete life tables are given in the Report for males and females separately for the whole of England and Wales and for Greater London; and monetary tables at 3% and 3½% are to be found in J. I. A., Vol. LIX, p. 164. Comparisons are also shown between English Life Tables Nos. 7, 8, and 9, by means of decennial values of q_x , l_x , e_x and $10p_x$. The values of $10p_x$, for which the author expresses a preference as a basis of comparison, are as follows:

PROBABILITY OF SURVIVING 10 YEARS, $10p_x$

Age	MALES			FEMALES		
	English Life Table No. 7	English Life Table No. 8	English Life Table No. 9	English Life Table No. 7	English Life Table No. 8	English Life Table No. 9
0	.78083	.81241	.85693	.80756	.83598	.87909
10	.97477	.97664	.97730	.97524	.97707	.97758
20	.95570	.96067	.96180	.96220	.96660	.96603
30	.93027	.94031	.94718	.94100	.94977	.95618
40	.88525	.89760	.91640	.90792	.91856	.93531
50	.79401	.81001	.84107	.83650	.85161	.87819
60	.62859	.64154	.67217	.69512	.71066	.74232
70	.35479	.36474	.38038	.41291	.43385	.46063
80	.10526	.11160	.11373	.13883	.15283	.15461

The improvement in mortality in England and Wales may be indicated partly by the following tables of expectations,

but it should be borne in mind that the figures are affected by the inaccuracies in the various tables previously discussed.

Table—Males	Complete Expectations of Life				
	Age 0	Age 20	Age 35	Age 50	Age 65
English Life No. 3 (1838–1854)	39.91	39.48	29.40	19.54	10.82
“ “ “ 4 (1871–1880)	41.35	39.40	28.64	18.93	10.55
“ “ “ 5 (1881–1890)	43.66	40.27	28.91	18.82	10.31
“ “ “ 6 (1891–1900)	44.13	41.02	29.24	18.90	10.34
“ “ “ 7 (1901–1910)	48.53	43.01	30.79	19.76	10.80
“ “ “ 8 (1910–1912)	51.50	44.21	31.71	20.29	10.99
“ “ “ 9 (1920–1922)	55.62	45.78	33.25	21.36	11.36
Healthy Districts No. 1 (1849–1853) . .	48.56	43.40	32.90	22.03	12.00
“ “ “ 2 (1881–1890) . .	51.48	44.41	32.70	21.53	11.60
“ “ “ 3 (1891–1900) . .	52.87	45.37	33.32	21.74	11.61

Table—Females	Complete Expectations of Life				
	Age 0	Age 20	Age 35	Age 50	Age 65
English Life No. 3 (1838–1854)	41.85	40.29	30.59	20.75	11.51
“ “ “ 4 (1871–1880)	44.62	41.66	30.90	20.68	11.42
“ “ “ 5 (1881–1890)	47.18	42.42	31.16	20.56	11.26
“ “ “ 6 (1891–1900)	47.77	43.44	31.52	20.64	11.27
“ “ “ 7 (1901–1910)	52.38	45.77	33.31	21.81	11.99
“ “ “ 8 (1910–1912)	55.35	47.10	34.37	22.51	12.36
“ “ “ 9 (1920–1922)	59.58	48.73	36.05	23.69	12.90
Healthy Districts No. 1 (1849–1853) . .	49.45	43.50	33.46	22.87	12.58
“ “ “ 2 (1881–1890) . .	54.04	45.62	34.16	22.75	12.36
“ “ “ 3 (1891–1900) . .	55.71	46.93	34.79	22.92	12.36

The foregoing tables illustrated progressive improvement in the vitality of the population. The decrease in the rates of mortality by the No. 9 tables as compared with the No. 8 tables is very pronounced at the early ages; it is but slight at the ages of adolescence and from ages 20 to 30 the rates do not differ greatly. For ages 30 to 80 the decrease is considerable, but at ages over 80 there is little improvement. A peculiar feature is that the curves for both males and females for the No. 9 tables rise a little above those for the No. 8 tables at the early adult ages, and particularly is this noticeable in the case of females for ages 18 to 27. The increase in deaths from tuberculosis probably accounts for part of this set-back. Furthermore, these groups of lives were subjected to wartime food rationing at ages when their food requirements were especially important. In the case of males some

lives may have been affected by the rigors of war service. Many thousands of healthy lives had been lost by death on active service.

The improvement shown in the period of about 15 years which separates the midway points of Tables 7 and 9 respectively is striking at all ages, but particularly in the first year of life, where the probability of a child dying had decreased about 40%.

As in the English Life Table No. 8, rates of mortality were also computed and graduated by the same methods used in the main tables (at each age from 16 to 84) for three separate female groups, namely, Spinsters, Married Women, and Widows. Life tables, however, were not prepared—the reasons for this decision being stated in the following words: “Whilst there appears to be full justification for deriving the rates of mortality, q_x , for each of these classes, the legitimacy of a life table showing the number of survivors at each age in an l_x column must be regarded as doubtful. The rate of mortality which has prevailed among spinsters of middle age is no doubt ascertainable with exactitude, but cannot be regarded as the rate to which the survivors of those who are just entering womanhood will be subject—even assuming no improvement in mortality to be experienced in the intervening period—in 20 or 30 years’ time. In the interval a large proportion of the present youthful spinsters will have entered the married section of the population, and on the reasonable assumption that marriage is a selective force, the effects of which will be seen in an enhanced rate of mortality among those remaining in the spinster class, the utmost that can be said for a table of the rates of mortality among spinsters is that it shows the course of the death rate among a body which is continuously being depleted by two forces, death and marriage, the first of which is withdrawing the worse, and the second the better, lives. In regard to such a body a life table in the ordinary form appears to have no definite meaning. Similar considerations apply in respect of separate life tables for married women and widows.” Spinsters show higher mortality than wives between ages 37 and 64, and after age 78. The rates for widows are throughout considerably heavier than for spinsters or wives. For each

section the rates are lower than for the corresponding experiences of Table No. 8, except for spinsters below about age 30.

The Sectional Life Tables constitute an outstanding feature of the report. The four groups, representing varying degrees of urbanization, which were adopted by Mr. King in Table No. 8 instead of a Healthy Districts Table, were considered in the light of the known dependence of mortality upon geographical distribution, density of population, and occupation. Separate mortality investigations were accordingly carried out for both males and females in (1) County Boroughs, (2) other Urban Districts, and (3) Rural Districts in each of six Geographical Divisions, designated Northern Counties, Central Counties, Southern Counties, Eastern Counties, South Wales, and North and West Wales. The results were presented in the form of comparisons at ages over 5 of actual deaths with those expected by Table No. 9, the individual ages being used below 14 and above 80 and quinquennial ages elsewhere. Complete life tables were constructed for Greater London including the area within a radius of about fifteen miles and embracing about 7,500,000 lives. The mortality experienced approximated that of the whole country, that of the males being slightly higher and that of the females slightly lower than the general average. In the case of both sexes the rates of mortality between ages 15 and 30 are found to be relatively light, attributable presumably to the fact that the population of cities is being continuously recruited by the accession of healthy lives from other districts. It is suggested that the demand for domestic help may account for the fact that this feature is more strongly marked amongst women than amongst men. Life tables were given for the groups of heaviest and lightest mortality, namely, Northumberland and Durham (County Boroughs) and Eastern Counties (Rural Districts), and for County Boroughs, Urban Districts, and Rural Districts.

A very full discussion of all the tables appears in J. I. A., Vol. LIX, p. 125, and a review of the basic data in T. A. S. A., Vol. XXIX, p. 327.

Values of a_x at $3\frac{1}{2}\%$ for males and for females for the Northumberland and Durham County Boroughs and for

Eastern Counties Rural Districts are given in J. I. A., Vol. LIX, p. 186. Commutation columns and values of a_x and A_x at $2\frac{1}{2}\%$, 3% , 4% and 5% for males only for Eastern Counties Rural Districts are printed in J. I. A., Vol. LIX, p. 323.

The report on Occupational Mortality in connection with the 1921 census was issued as Part II of the Registrar-General's Decennial Supplement. Several important changes were made in the method of treatment of the data. (See T. A. S. A., Vol. XXIX, p. 331.)

OTHER BRITISH TABLES

Various British mortality tables have not even been mentioned. Thomas Simpson's London Life Table was based on bills of mortality for the years 1728-37, with an adjustment to allow for adults settling in London.

There have also been numerous tables of a purely local character, such as the Surrey and Liverpool Tables of Dr. Farr, the Manchester Tables of Dr. Tatham, and Brighton Tables of Dr. Newsholm, and the Haydock Tables of Dr. Hayward.

SCOTLAND

Dr. Robertson's table was published in the Nineteenth Annual Report of the Registrar-General for Scotland. This was compiled from the Census and Death Returns for the year 1871.

Dr. Adam's table was published in the Journal of the Royal Statistical Society, Vol. LXVII, p. 448. This was based on the mean population and deaths in Scotland for the decennium, 1891-1901.

Dr. Dunlop's tables for the decennial periods 1861-70, 1871-80, 1881-90, and 1891-1900 were published in the Supplement to the Forty-eighth Annual Report of the Registrar-General for Scotland. They were constructed by Dr. Hayward's "short" method. (See Actuarial Study No. 3, p. 118.)

In T. F. A., Vol. VII, p. 357, Dr. Dunlop and Mr. R. M. Hunter gave male and female tables for Scotland for 1911 with many details as to the methods of construction and

comparisons that could be deduced from them. These tables were based on the population of Scotland as ascertained by the census of 1911 and on the deaths registered in the three years 1910, 1911 and 1912.

Dr. Dunlop, the Registrar-General for Scotland, published in T. F. A., Vol. X, p. 1 the "Scottish Life Tables, 1921," based on the 1921 census of Scotland and the deaths of 1920-22. He employed in general the methods used by Mr. King for English Life Table No. 8, and gave life tables not only for males and females separately, but also for single males, married males, single females, and married females.

The new Scottish tables, compared with the corresponding 1911 tables, show substantial reductions in the rates of mortality up to about age 70, but thereafter the 1921 rates are generally heavier than the 1911 rates. If comparison of the Scottish 1921 rates is made with those of English Life Table No. 9 rates, it is found that the English rates are lower throughout. Infantile mortality is much heavier in Scotland, and female mortality at the early adult ages is very heavy.

GREAT BRITAIN LIFE TABLES, 1920-22

These tables, given by Sir Alfred Watson in J. I. A., Vol. LIX, p. 318, show for males and females separately the mortality experience of the population of Great Britain (England, Scotland and Wales) during the period 1920-22. The methods employed in constructing the tables were identical with those adopted for the English Life Tables No. 9 except that at infantile ages, mortality rates were derived from the births and deaths recorded in calendar years because the numbers of births in Scotland were given for calendar years only.

IRISH FREE STATE

SAORSTAT LIFE TABLE NO. 1

These tables* were constructed by Mr. R. C. Geary and included in the *Census of Population, 1926*, Volume V, compiled by the Department of Industry and Commerce of the Irish Free State. The data for males and females separately were

* See J. I. A., Vol. LXI, p. 96.

based on the census of 1926 and the number of deaths in the three years 1925-1927. The methods of construction were those, with slight modifications, used by Mr. George King in the construction of English Life Tables Nos. 7 and 8 and adopted by Sir Alfred Watson in the construction of English Life Table No. 9. Comparisons show that, if the tables reflect actual facts, Irish mortality was much lighter than the English at infantile and at advanced ages, that for males at other ages it is sometimes lighter and sometimes heavier, that for females at all other ages it is heavier, and that up to age 50 the mortality of females is heavier than that of males. There is more than the usual doubt, however, about the accuracy, especially of the ages, of the census and registration data.

CHAPTER III.

POPULATION MORTALITY TABLES AMERICA

WIGGLESWORTH'S MORTALITY TABLE

THE earliest American mortality table was compiled by Professor Wigglesworth of Harvard and published in 1789 in *Memoirs of the American Academy of Arts and Sciences*, Vol. II, Part I, p. 132. It was formed from sixty-two bills of mortality of towns in Massachusetts and New Hampshire and was based on death records alone without allowance for migration or for the age distribution of the populations of the towns. As a result it overstates the mortality. It is one of seven tables from whose average the early premiums of the New England Mutual were derived. It was used for many years by the Supreme Court of Massachusetts in calculating the present values of estates and interests of heirs.

UNITED STATES LIFE TABLES, 1910*

UNITED STATES LIFE TABLES, 1890, 1901, 1910 AND 1901-1910†

These tables are the first of any scientific value prepared by the U. S. Government from census returns. When the census of 1910 was taken the Bureau of the Census called into consultation a committee of the Actuarial Society of America, and this committee gave general advice with reference to the taking of the census, the tabulation of the data, and the preparation of life tables. Although not all the committee's recommendations were followed, these tables, which were prepared under the supervision of Prof. James W.

* United States Life Tables, 1910. Department of Commerce, Washington, 1916.

† United States Life Tables, 1890, 1901, 1910, and 1901-1910. Department of Commerce, Washington, 1921.

Glover, mark a notable epoch in the history of mortality investigations in the United States.

The first volume, published in 1916, contained only the U. S. Life Tables, 1910, which were based on the census of 1910 and the deaths of 1909-11 in the original registration states comprising: Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New York, New Jersey, Indiana, Michigan, and the District of Columbia. Accordingly the tables do not exhibit the mortality of the entire area of the United States, but deal chiefly with the northeastern section. This area contained a population estimated to be 24,131,759 as of July 1, 1910, or roughly one-quarter of the population of the whole country. In much of the remaining area the registration system was either faulty or practically non-existent. In this volume twenty-five tables were published—for the entire area for both sexes combined, for males and females separately in the entire area and in the several groups White, Negro, Native White, Foreign-born White, Cities and Rural, and also for the individual states of Indiana, Massachusetts, Michigan, New Jersey, and New York.

The second volume mentioned above contained a reproduction of these twenty-five tables, together with forty-nine others. In this group further tables are given, for the original registration states, which are based upon the estimated population of 1901 and the deaths of 1900-1902. Tables for White males, White females, Negro males and Negro females were based on the populations of 1901 and 1910 and the deaths of the intervening period. For certain selected registration states and cities the 1910 tables are similarly supplemented by others constructed from the 1901 data, and for Massachusetts alone the 1890 census has also been employed. The various characteristics of these tables are well brought out in a number of illustrative questions and answers which deal very completely with the comparative rates of mortality of males and females, urban and rural populations, Whites and Negroes, etc. To these are added comparisons with the life tables of twelve other countries, and also with the various tables based upon insured lives. Fifty pages of graphs are also shown; and commutation columns at 3, $3\frac{1}{2}$, 4, 5 and 6

per cent, and P_x and A_x at 3, $3\frac{1}{2}$ and 4 per cent are derived from the 1910 life tables of white males and females in the original registration states.

The tables were constructed between ages 5 and 85 by Sprague's fifth difference osculatory interpolation formula, and at the advanced ages a modification of Wittstein's formula was used—Spencer's 21-term formula also being employed to smooth the values at the end of the table (see Actuarial Study No. 3, pars. 53-4, 61 (iii), 99(iv), 100, 115 and 117). The process employed at the infantile ages (for the first year of which the rates of mortality are given for each month of age) is described in detail in Actuarial Study No. 3, par. 93.

It must be remembered that only the data of the "original registration states" enumerated above were included, and that those were, with the exceptions of Indiana and Michigan (and District of Columbia), in the northeastern portion of the country where the manufacturing populations are most dense. The tables therefore do not represent the general mortality of the entire country. For the portion which was covered, however, the mortality among males is generally higher than among females, the rural mortality is very much lower than the urban, and negro mortality is very pronouncedly higher than that affecting Caucasians. The following table gives some indication of the relative rates:

UNITED STATES LIFE TABLES—1910.
Death Rates per 1,000 of Population—i. e.: 1,000 q_x .

Age	Native White Male	Foreign-Born White Male	Negro Male	Native White Female
10	2.37	2.47	5.02	2.06
30	7.14	5.80	14.96	6.13
40	10.02	10.53	21.03	7.76
60	27.21	36.81	50.79	22.06
80	132.43	141.76	131.27*	121.23
<i>Corresponding Values of e_x.</i>				
10	51.93	50.30	40.65	54.43
30	35.61	33.71	27.33	37.98
40	28.33	26.03	21.57	30.33
60	14.58	13.06	11.67	15.78
80	5.15	4.98	5.53*	5.47

* Ages probably exaggerated.

The tables dealing with different states indicate that mortality in Indiana and Michigan is notably lower than in the older communities of Massachusetts, New Jersey and New York. This may partly be accounted for by the larger proportion of rural population in the former and by the larger proportion of foreign born in the industrial centers. Other sections of the United States are understood to show still lower death rates, while the South is not represented at all in the compilation herein described.

A comparison between the life tables for 1901 and 1910 indicates there had been a general improvement in mortality for all classes up to about ages 40 for men and 50 for women, except for Negroes, but that above those ages there had been no improvement, and in some cases the 1910 rates were even greater than those of 1901.

The tables are valuable as being the first of the kind prepared in the United States, but, in studying the differences in the various rates of mortality, those factors likely to produce differences should not be forgotten. For instance, in comparing the expectations of life of people living in the cities with those living in the country, it must be remembered that the cities contain a larger proportion of foreign born, and that hospitals and sanitariums are mostly situated in towns and cities. The tables having been prepared from an estimated population and from reported deaths, the errors likely to have occurred in registration and tabulation, even as late as 1910, may be considerable. Where all the deaths are not recorded, as is generally the case in registration statistics, the error tends to decrease the death rate, and this factor may explain part of the difference between rural and urban communities, since town and city registration is more complete than rural. The statutory obligation to register any death before a burial permit will be granted can be enforced better in city districts.

UNITED STATES ABRIDGED LIFE TABLES, 1919-1920*

These tables were prepared by the Bureau of the Census, upon the approval of the Census Advisory Committee, to

* United States Abridged Life Tables, 1919-1920. Department of Commerce, Washington, 1922.

study the mortality of different sections of the country and to compare it with the somewhat similar previous tables. The construction was under the immediate supervision of Miss Elbertie Foudray, but the methods were approved by Prof. J. W. Glover, acknowledgments also being made to Mr. Robert Henderson. The term "abridged" was included in the title because of the abridged nature of construction—life table values, such as l_x , d_x , q_x , $\bar{e}_x$, being given only at ages 0, 1, 2, 7, 12, and every subsequent fifth age to 92.

These tables, published in 1922, were based on the 1920 enumerated population and on the births and deaths of 1919 and 1920 in 27 states, the District of Columbia, and the Territory of Hawaii—representing about 74 per cent of the population of the United States and this Territory. 92 tables were made up, 46 for males and 46 for females. The tables for each sex were further classified as follows:

- | | |
|----------------------|--|
| 40 for white lives | —23 for individual states. |
| | 1 for aggregate of 27 states. |
| | 1 for aggregate of original registration states. |
| | 14 for individual cities. |
| | 1 for aggregate of 14 cities. |
| 4 for colored lives | — 1 for states with less than 4% Negroes. |
| | 1 for states with more than 5% Negroes. |
| | 1 for 12 large cities. |
| | 1 for original registration states. |
| 2 for Hawaiian lives | — 1 for Japanese in Hawaii. |
| | 1 for all races combined. |

The states included in addition to the original registration states were California, Illinois, Kansas, Kentucky, Maryland, Minnesota, Missouri, North Carolina, Ohio, Oregon, Pennsylvania, South Carolina, Tennessee, Utah, Virginia, Washington and Wisconsin. The above separation was adopted for colored lives because the colored population varied from .6 per cent in Wisconsin to 51.4 per cent in South Carolina. No tables for aggregate White nor aggregate Negro populations were given. The largest exposure in any one table involved thirty-seven million White males.

Graduated values were found by Mr. King's abridged method for the central age of each of the groups, 10–14,

15-19, . . . 90-94, by the formula $u_{x+7} = .2w_{x+5} - .008\Delta^2w_x$, where $w_{x+n} = \sum_{t=n}^{t=n+4} u_{x+t}$, and for the group 5-9, by the formula $u_{x+2} = .2w_x - .008\Delta^2w_x$. In extending the table to old age, the logarithms of the last seven probabilities of living were differenced four times and a constant fourth difference used to extend these probabilities of living for five-year periods up to age 112. For the infantile ages, 0, 1, 2, the deficient population, P_0^{+1} , was first corrected on the assumption of a fixed percentage deficit of 9% for Whites and 25% for Negroes. These percentages were the results of inquiries made in Washington, D. C., concerning birth statistics and enumerated population; and it was assumed that they were applicable for other sections of the country. For further description of the graduation see Actuarial Study No. 3, par. 122(iii). The process which was devised for the infantile ages is explained in detail in the same reference, par. 93 (iii).

The life values obtained are set out for all the 92 life tables in four male and four female tables—comparisons being indicated by giving to each value its "rank" in the table of 46 male or female values amongst which it appears. Comparisons are also shown with the White and Negro life tables of the original registration area in 1909-11. The general conclusion is stated that "the 1919-20 rates of mortality show . . . marked improvement among white males for each age given, except 17, 27 and 92, and among white females for each age given except 17, 22, 27 and 32. The values for Negroes show similar departures at the same ages, and also some at ages 52 to 62, 72 and 82. This lack of improvement in the young adult years was due to the influenza epidemic of 1919 and 1920." The Japanese in Hawaii are also compared with those in Japan as shown in the 1908-1913 life tables of that country; and "it appears that infant mortality is much lower among the Japanese in Hawaii than in the Japanese Empire . . . but after age 27 and up to old age, mortality rates among Japanese appear to be more favorable in the Empire than in Hawaii"—although it is to be noted that "the number of Japanese over 50 years of age in Hawaii is too small to afford reliable life table values."

SPECIAL NEW YORK STATE TABLES*

A mortality table for males in New York State, outside of New York City, based on the 1920 census and the deaths of 1920 was given in *Monthly Vital Statistics Review*, New York State Department of Health, Vol. V, p. 2. From this it appeared that expectations of life are appreciably greater for all ages in up-state New York than in New York City. Similar tables (from age 20 upwards) for foreign born white males in New York State, for native born white males, for foreign born white females and for native born white females were given later.* These tables were based on the 1920 census and on the deaths in 1919, 1920 and 1921.

* *Monthly Vital Statistics Review*, New York State Department of Health, Vol. VII, p. 70 and p. 90; Vol. VIII, p. 2 and p. 22.

CHAPTER IV.

TABLES FORMED *from* LIFE INSURANCE EXPERIENCE GREAT BRITAIN

DAVIES' EQUITABLE TABLE

THE Equitable Society was formed in 1762, and is the oldest Life Office at present having a separate existence. Some years after its formation this Society adopted the Northampton Table as the basis of premiums, and Dr. Price recommended that the actual deaths experienced should be compared with those expected by the Northampton Table; this was accordingly done annually.

In 1800, Mr. Morgan, then Actuary of the Equitable, stated that the deaths among the members for the preceding thirty years had been as follows:

Ages 10-20 One-half of the number expected by the Northampton Table.		
Ages 20-30 One-half	do.	do.
Ages 30-40 Three-fifths	do.	do.
Ages 40-50 Three-fifths	do.	do.
Ages 50-60 Five-sevenths	do.	do.
Ages 60-80 Four-fifths	do.	do.

This table shows that the actual deaths were very much less than the expected, and Mr. Griffith Davies adjusted the Northampton Table to follow the experience of the Society, his results being published in 1825. He obtained the numbers living for each tenth age, and interpolated by finite differences for the intermediate ages, but the methods he used are now unimportant. His table was a modification of the Northampton Table to obtain figures more nearly in conformity with actual conditions.

MORGAN'S EQUITABLE TABLE*

This table, published in 1834, was formed by Mr. Arthur Morgan, Actuary of the Equitable Life Office, from the experience of the Society from its commencement in September, 1762, to January 1, 1829. It is the first table deduced on correct principles from the records of a life insurance company.

There were 21,398 lives, of whom 6,930 were living on January 1, 1829; 9,324 had surrendered, forfeited, or discontinued their policies; and 5,144 had died. Altogether there were 266,872 years of life under observation and the average period of exposure was $12\frac{1}{4}$ years.† The effect of the selection of lives was discussed and the rapid increase in mortality after date of entry was referred to.

The following shows the way in which the facts were tabulated.

Age at Entry	Age Attained 30				Age Attained 31	
	Attained the Above Age	Living on Jan. 1, 1829, at Above Age	Discontinued their Assces, at Above Age	Died	Attained the Above Age	Living, etc.
.	.					
24	.					
25	421	10	29	2	380	etc.
26	.					
27	.					etc.
.	.					

Thus opposite each age at entry under the Column headed "Age Attained" were put:

1. The number living who attained that age.
2. Those who were existing at that age on January 1, 1829, and who thus passed out of observation.
3. The number who ceased to be members at that age, and who thus also passed out of observation.
4. The number who died between that age and the next.

Under *Age Attained 30*, opposite age 25, we find 421 in the column showing the number living, which is the number out

* J. I. A., Vol. XXIX, pp. 113-17, by James Chatham.

† J. I. A., Vol. II, p. 202.

of those entering at age 25 who continued members of the Society until they attained age 30; of these, 10 were existing on January 1, 1829, while 29 ceased to be members between the ages 30 and 31, and 2 died. Deducting $10 + 29 + 2$, or 41, from 421 gives 380, the number who entered upon their thirty-second year of age, which was placed under "Age Attained 31" opposite 25, etc. Members were assumed to attain the age stated at entry (i. e., age *next* birthday) on the first of January following entry. This was therefore the first "Calendar Year" investigation.

The table is now of historic interest only, and it is therefore unnecessary to describe the processes in full.

ACTUARIES' TABLE; COMBINED EXPERIENCE TABLE; OR
SEVENTEEN OFFICES' EXPERIENCE, 1843

As it was felt that the rate of mortality among insured lives must be different from that of the general population on account of the former having to pass a medical examination, it was resolved, at a meeting of actuaries and others connected with life insurance held in London in 1838, to ask various companies to contribute their experience. A committee was formed and issued schedules in the following form to the contributing offices:

For Use of Office	Current Age at Entry	Year of		If by Death, D.	Sex, if Female, F.	Distinction into Town, (T) Country, (C) Irish, (I)	Cause of Death	Special Risks and Remarks
		Entry	Exit					

The forms were thus confined to a record of the *policies* issued; but the Equitable and Amicable Societies apparently furnished their observations on lives only, so the general results were mixed.*

It was believed that the rate of mortality would not be sensibly affected by the fact that a man might often effect several policies at various ages, and might at death be

* J. I. A., Vol. II, p. 203.

treated as several different persons. It has been said that this has the effect of diminishing the rate of mortality at the early ages, and increasing it at the old ages; but the extent of this tendency must be slight.

The returns embraced 83,905 policies, of which 44,877 were existing at the close of the observations (December 31, 1837), 25,247 had been discontinued, and 13,781 had been terminated by death.*

The committee stated that the tables represent a lower rate of mortality than could be expected during a longer period of time than that over which the observations extended, for the average duration of all the policies was less than $8\frac{1}{2}$ years, notwithstanding that the two oldest companies then existing (the Amicable and the Equitable) were included. The average duration of policies embraced in nearly one-half of the experience was under $5\frac{1}{2}$ years. More than one-half of the policies effected were existing at the close of the observations, and nearly one-third had been discontinued.

The tables were constructed by a "calendar year" process, and were adjusted according to a formula given by Mr. Woolhouse. The mortality amongst Irish lives was found to be heavy as compared with those resident in Great Britain. The mortality amongst assured females from ages 20 to 50 was considerably greater than amongst males of the same ages. From 50 to 70, it was less, while above 70 it was found to be sometimes greater and sometimes less, but at the advanced ages the statistics were quite inadequate to enable one to form a reliable opinion.

The table was adopted as the standard for valuation purposes in Massachusetts about 65 years ago, and continued in very general use till 1901, as it was considered a "safe" table. Industrial policies were also valued on this basis, so it became necessary to extend the mortality rates down to age 0, and the National Convention of Insurance Commissioners in 1883 adopted the following rates for ages 0 to 9, recommended by D. P. Fackler, W. S. Smith, D. W. Whitney and A. F. Harvey:

* J. I. A., Vol. X, p. 197.

Age	l_x	d_x	q_x
0	143,400	22,184	.1547
1	121,216	7,697	.0635
2	113,519	4,030	.0355
3	109,489	2,617	.0239
4	106,872	1,892	.0177
5	104,980	1,428	.0136
6	103,552	1,123	.0108
7	102,429	922	.0090
8	101,507	792	.0078
9	100,715	715	.0071

CRAIG'S MODIFIED ACTUARIES' TABLE FOR IMPAIRED LIVES

This was an empirical table constructed by James M. Craig for use in connection with impaired lives. It was obtained by doubling the rate of mortality shown in the Actuaries' or Combined Experience Table up to age 60 and thereafter using a gradually decreasing scale up to age 84 after which the rate coincided with that in the Combined Experience Table. This table is given with commutation columns at 4% interest in T. A. S. A., Vol. I, No. 4, p. 52 and in Dawson's "Practical Lessons in Actuarial Science."

INSTITUTE OF ACTUARIES TABLES—PUBLISHED 1869 (H^m AND OTHERS)

In 1862, the Council of the Institute of Actuaries, aided by a joint committee of the Faculty of Actuaries and the Managers' Association (The Associated Scottish Life Offices), undertook to collect and tabulate the material requisite for investigating the rate of mortality among insured lives, and 20 companies agreed to furnish their experience. The particulars of the lives were furnished on 180,000 cards, which gave the following particulars:

Policy No.	Healthy or Diseased	Age at Entry	Cause of Death
Life	Year of Entry	Age at Exit	Remarks.
British, Irish or Foreign	Year of Exit	Mode of Exit	

There were 10 English and 10 Scottish companies contributing their experience, and on the Scottish card, the

following particulars in addition to the above were given:

Sum Assured.
Class.
Exact dates, and
English, Scotch, instead of British.

The office age at entry (i. e., next birthday age) was inserted in the card, which afforded the means of approximating to the actual age by the assumption that the insured attained that age at the end of the year of entry.

The cards were first of all sorted into four divisions, viz.: (1) H^m , (Healthy Males); (2) H^f , (Healthy Females); (3) D^{mf} , (Diseased Male and Female); and (4) those exposed to extra risk from climate, occupation, etc.

The cards in each of these divisions were then arranged according to the name of the life, with the view of bringing together all those relating to the same person and eliminating duplicates. The next step was to sort the cards in each division into three groups according as the insured had passed out of observation—(1) by death, (2) by discontinuance of their insurance, and (3) by surviving the period at which the observations closed, briefly termed “died,” “discontinued” and “existing.” All those of the same age at entry were next brought together, and finally arranged according to age at exit. The number of cards was reduced by elimination of duplicates to 160,426. 26,721 “died,” 45,376 “discontinued,” and 88,329 were “existing” at the close of the observations. The average duration of exposure was more than 9 years. Females formed about 11% of the total. The H^m Table started at age 10, with a radix of 100,000.

The Tables were formed by the calendar year method, it being assumed that the entrants in any year came under observation on the average at the middle of the year and would attain the age at entry (i. e., age next birthday) at the close of the calendar year. This short period from the date of entry to the end of the year of entry (assumed to be six months) was called the year “0”; and in keeping the mortality separate for each age at entry, as already explained, the mortality during this period was assumed to apply to the previous age. Thus, those who entered at age 30 next birthday were assumed to be on an average exposed to risk for

six months in the year "0"; and the mortality for this six months was set down as applying to age 29. Later investigation showed that the average period of risk in the calendar year of entry was less than six months, also that the average true age at issue was less than six months younger than the age next birthday as stated.

The method of keeping the various ages at entry separate from each other was adopted in order that future investigators might make use of the facts in forming "select" tables, afterwards described. The committee made no immediate use of this arrangement, but combined in the H^m Table all those of the same attained age whether they had been insured for only a year or two, or for many years. The effect of selection ceased to be of any practical importance after the expiry of 5 years, and accordingly the $H^{m(5)}$ Table was formed, the experience of the first five calendar years after entry (approximately $4\frac{1}{2}$ years) being neglected.

The method of construction used for the H^m Table (known technically as an "aggregate" table, as opposed to a "select" table) i. e., entering new or select lives at each age with the general body of policy-holders at that age, some of whom must of necessity be unhealthy, has the effect of making the apparent mortality at the older ages higher than would be shown by a table taking account only of the new entrants at such higher ages, because the unhealthy lives are brought forward and included. The method has the opposite effect at the younger ages, because a body of lives, within a few years after selection, becomes "mixed" and contains a proportion of unhealthy lives; but the proportion of such unhealthy lives is reduced by the introduction of more healthy lives year by year, so that the lighter mortality amongst these newly selected entrants reduces the rate in respect of those who entered at the younger age. Premium rates computed from such a table are lower at young ages and higher at old ages than a select experience shows to be correct.

At ages under 45, the H' Table shows much heavier mortality than the H^m , but after that age lighter—thus confirming previous investigations. The tables were graduated by Woolhouse's formula.

These tables came to be very generally employed in Great Britain, and were also adopted as a standard for all life insurance purposes in Canada. They were never officially recognized and have been very seldom used in the United States.

SELECT LIFE TABLES

Insurance companies do not accept all the applicants who offer themselves, but reject a small proportion of the lives proposed for insurance, taking the great majority at the usual premium rates, charging others an increased premium, and declining entirely a few whose prospects of longevity appear to be much below the average. The necessity for this distinction arises from the fact that if the benefits of insurance were free to all comers, there would be an undue proportion of under-average lives, more than the normal proportion in the population, since an unhealthy man needs insurance more than a healthy one. Moreover the healthy would with reason object to being classified with the diseased and to paying the same premium rates. Selection is effected by means of medical and other reports on the health, family history, personal history and habits of the individual whose life is proposed for insurance, with due regard also to occupation and habitat.

For the first two or three years after entry the effect of this selection is quite apparent, the death rate amongst recent entrants being very low as compared with the rate amongst those who have been insured for several years, or as compared with the general population; but, as years pass, the rate of mortality approaches nearer to that of non-select lives, although it is the general opinion that the effects of the first selection never entirely disappear. Mr. Sprague held* that the mortality among select lives gradually increased until it attained a maximum, after which it diminished; this he attributed to two counterbalancing causes:

1. The selection as exercised by the Office when insurance is effected, and
2. The selection exercised by the lives insured in having the right to withdraw, and the consequent with-

* J. I. A., Vol. XV, p. 328.

drawal of a considerable proportion of the healthy lives during the early years of insurance.

More recently* it has been urged that withdrawals do not have the effect of reducing the proportion of healthy lives; indeed the direct contrary is sometimes accepted on the ground that withdrawal from a company in good standing is more frequently a result of financial embarrassment or irregular habits. The opinion that discontinuances have not an adverse influence on the mortality seems to be gaining ground—even Mr. Sprague stating that he had “considerable doubt” as to whether the lives which withdraw are on the average better than those which remain. Until the point is settled by convincing statistics, which are difficult to procure, it remains a matter of opinion with plausible arguments on both sides.

To investigate the effect of selection properly, it is desirable to trace a large number of entrants from each age at entry until they all pass out of observation. This has been done on several occasions, the first important investigation being made by Mr. J. A. Higham prior to the year 1850. In that year he read a paper before the Institute of Actuaries on “The Value of Selection among Assured Lives, etc.” This consisted of an analysis of the experience of the Equitable Society. In 1851 he submitted another paper on “The Value of Selection as Exercised by the Policy Holders Against the Company.”† His deductions were based upon the Actuaries’ or Seventeen Offices’ Experience, but his figures were not much used for practical purposes. Mr. Higham said “the probability of surviving the year immediately after selection is a quantity which we have not at present the means of measuring”; and the means of measuring this quantity accurately was not obtained until about the year 1900.

After the observations from which the H^m Experience Table was compiled were published, Messrs. King and Sprague devoted themselves to a careful analysis of selection and the monetary tables published by Sprague in 1879 were widely used for the succeeding twenty years.

* J. I. A., Vol. XXIX, p. 81; also XXXII, p. 117, etc.

† J. I. A., Vol. I, p. 179.

KING'S ANALYZED TABLES—1876*

The first important discussion of the rate of mortality by duration of insurance, after publication of the H^m mortality experience, was opened by Mr. King in a paper "On the Mortality amongst Assured Lives," read before the Institute in 1876. This paper marked an epoch in the study of mortality statistics. Extracts from Mr. King's own description of his new mortality tables will give an indication of his procedure:

"I have prepared and graduated ten mortality tables for ages at entry 20, 25, etc., up to 65. They were formed from the tables of 'exposed to risk' and 'died,' . . . by tracing separately . . . those who entered at the ages 20, 25, etc., respectively. It was of importance to include as many of the observed facts as possible; I have, therefore, after the first ten years of assurance, combined with those entering at age x those also entering for ages $x-2$, $x-1$, $x+1$ and $x+2$, thus making x the central age of a quinary group. . . . It would have been more satisfactory had larger numbers enabled us to dispense with this grouping; but, after all, it can make very little difference in the mortality among persons of the same age, when they have been assured for some considerable time, whether they entered at a period two years more or less remote. Two slight errors are introduced which tend to neutralize one another.

"In the early years of assurance, however, the case is different. During the first ten years, the medical examination at entry must exercise a decided but rapidly decreasing influence, while the selection against the company caused by discontinuances is at its strongest. It is, consequently, of importance to avoid mixing those who have been assured, say 5 years, with those who have been assured 6 and 7 years, or only 3 and 4. The numbers, moreover, are sufficient to render it unnecessary; and for the first ten years of each table, I have brought into account the exposed to risk and the deaths for that age at entry only which appears in the heading. The table for age 65 is the only one where the numbers were so small as to produce serious fluctuations; but for the sake of uniformity, I have adhered to the same plan throughout."

At the advanced periods of life, commencing after age 60, Mr. King combined the observations for several ages at entry:

Entry ages 20 and 25 being combined at age 60.
" " 20 to 45 " " " " 80.
And all entry ages " " " " 89.

The tables were adjusted by a very simple method of graduation, viz.:

$$d_x = \frac{d'_{x-2} + d'_{x-1} + d'_x + d'_{x+1} + d'_{x+2}}{5}$$

where d' represents ungraduated and d the graduated num-

* J. I. A., Vol. XIX, p. 385.

ber of deaths. When the result was still too irregular the operation was repeated.

Mr. King assumed that the rate of mortality for the fraction of the year of entry, "year 0," would continue for a full year, adding "perhaps by so doing the mortality is slightly underrated." He used the mean of 3 ages at entry, thus obtaining a simple and rough graduation of the first year's mortality.

Mr. King also analyzed the rate of discontinuance which "decreases not only with the duration of the policy, but also with the age at which the policy was effected." The rates of discontinuance at age 35 and at all ages combined were as follows:

Rate % of Discontinuance			Rate % of Discontinuance			Rate % of Discontinuance		
Year	Age 35	All Ages	Year	Age 35	All Ages	Year	Age 35	All Ages
0	2.5	2.7	5	2.7	2.8	10-14	1.4	1.4
1	6.2	7.0	6	2.2	2.4	15-19	1.0	.9
2	4.2	5.0	7	3.2	3.6	20-24	.8	.7
3	3.8	4.1	8	1.9	1.8	25-29	.4	.5
4	2.9	3.3	9	1.5	1.6	30 and over	.4	.4

SPRAGUE'S SELECT MORTALITY TABLES*

In 1879 Mr. T. B. Sprague published the results of an investigation he made from the H^m statistics to determine the effect of selection, and to show the uses to which Select Tables could be put. To obtain the mortality rates he first reduced the figures for each age at entry to a common radix of 100,000; and, in order to lessen the irregularities arising from small numbers, he grouped the entrants at five ages to get the rate of mortality for the central age of the group.

The same assumptions were made as to ages at entry as were made in compiling the Institute Experience, namely, that the entrants at any age next birthday attained that age at the end of the year, also that the initial period, "year 0," covered half a year. To deduce the mortality for age 30, the five year group contained:

* J. I. A., Vol. XXI, p. 229 et seq.

100,000 of exact age $29\frac{1}{2}$ }
 100,000 of exact age $30\frac{1}{2}$ } which gave 200,000 of mean age 30.
 100,000 of exact age $28\frac{1}{2}$ }
 100,000 of exact age $31\frac{1}{2}$ } which gave 200,000 of mean age 30.
 50,000 of exact age $27\frac{1}{2}$ }
 50,000 of exact age $32\frac{1}{2}$ } which gave 100,000 of mean age 30.
 500,000 of mean age 30.

The reason for taking only 50,000 at the two last named ages was of course that the balance of 50,000 in each case entered into the adjacent groups; i. e., the other 50,000 at age $27\frac{1}{2}$ were included in the group for age 25 at entry, and the 50,000 at age $32\frac{1}{2}$ in the group for age 35. By reducing the figures to a common radix, the facts at each age had the same weight assigned to them in the final table.

The following table indicates the method and gives the figures used for age 30:

Time Elapsed	Assumed Age at Entry						Average Age at Entry 30		
	$27\frac{1}{2}$	$28\frac{1}{2}$	$29\frac{1}{2}$	$30\frac{1}{2}$	$31\frac{1}{2}$	$32\frac{1}{2}$	Survivors	Total Deaths	Age Attained
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Years									
0	50,000	100,000	100,000	100,000	100,000	50,000	500,000	0	30
$\frac{1}{2}$	49,962	99,788	99,931	99,754	99,676	49,864	498,975	1,025	$30\frac{1}{2}$
$1\frac{1}{2}$	49,550	99,346	99,394	99,316	99,115	49,609	496,330	3,670	$31\frac{1}{2}$
$2\frac{1}{2}$	49,167	98,655	98,644	98,594	98,347	49,256	492,663	7,337	$32\frac{1}{2}$
$3\frac{1}{2}$	48,672	97,940	97,490	97,839	97,573	48,818	488,332	11,668	$33\frac{1}{2}$
$4\frac{1}{2}$	48,345	96,823	96,166	96,926	96,841	48,315	483,416	16,584	$34\frac{1}{2}$
$5\frac{1}{2}$	47,986	95,858	95,142	96,024	95,960	47,860	478,830	21,170	$35\frac{1}{2}$

This table gives the numbers living at the end of $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, etc., years; and in order to obtain the numbers at the end of full years, Mr. Sprague divided the deaths in column 9 of the above table into half-years by a method of differences. The deaths in the first and second half-years were then added together and divided by the corresponding numbers exposed to risk to give the rate of mortality for the first full year, and so on for subsequent years. (See also Actuarial Study No. 2, p. 66.)

The Tables were computed on the further assumption that the $H^{m(6)}$ Table represented the ultimate rate of mortality. Accordingly, having deduced the select rates for the first five years, and having graduated them by the graphic method as explained in Actuarial Study No. 4, p. 10, these graduated

rates were joined to the $H^{m(5)}$ rates. When the former exceeded the latter in the years one, two, three or four, the $H^{m(5)}$ rates were adopted from that year onwards; after the first five years the $H^{m(5)}$ Table was adopted in all cases. This saved labor in calculations, because all the more useful values had been published for the $H^{m(5)}$ Table, and those for the early years only had to be added.

From the rates of mortality for quinquennial ages, found as above shown, the rates for intermediate ages were obtained by osculatory interpolation which Sprague devised as an original method and presented for the first time in connection with these tables.

These tables quickly came into general use, especially for the calculation of premiums; and, as compared with ordinary life net premiums under the H^m Table, those by the Select Tables are higher up to age 43, but after that age they are lower. King's Analyzed Tables gave similar results. As the method of computing the rate of mortality for the first year introduced a function involving the second year's mortality, the rate deduced by Mr. Sprague was probably too high, while Mr. King's method left the same factor avowedly too low.

In addition to the painstaking work of Mr. Sprague in this investigation there were four original ideas introduced by him at that time which deserve special emphasis:

1. The facility obtained by using a *Common Radix*;
2. The happy *combination* of his figures with the $H^{m(5)}$ Table;
3. The use of the new method of *Osculatory Interpolation*; and
4. The development of an excellent *Notation* for Select Tables.

Moreover, Mr. Sprague, in an elaborate and very valuable paper,* showed the many uses to which select tables could be put whereby information could be obtained on points regarding which nothing could be learnt from the old aggregate tables. Those uses opened a new field of vision to many actuaries, and enabled them to solve problems at which previously some had scarcely even ventured to guess. The

* J. I. A., Vol. XXII; p. 406.

result was the immediate acceptance by nearly all actuaries of the Select Table principle.

BRITISH OFFICES' LIFE TABLES, 1893*

A growing opinion that the mortality of insured lives had developed on lines which made the H^m Table obsolete, with a strong desire for more truly representative tables, especially Select Tables, led to the formation of a Joint Committee of the Institute and the Faculty of Actuaries to compile new statistics. In 1894 cards were issued to the contributing offices, and sixty sent in returns. Males and females were dealt with separately by means of colored cards. Rated-up lives and those subject to extra hazard were excluded, the experience being confined to healthy lives resident in Britain when the policies were issued. If extra premiums were *afterwards* incurred for foreign residence or occupation, the lives were continued under observation without modification on this account. The experience extended from the policy anniversary in 1863, or subsequent date of entry, to the anniversary in 1893, or previous exit. One of the objections to the H^m Table was thus partially avoided, namely, that the data for that table extended back to the earliest history of the oldest companies when sanitary and economic surroundings were entirely different.

The rates of mortality are shown separately for different classes of policies on male lives, as follows:

	Symbols Used	No. of Years of Experience	No. of Lives	Approximate Mortality Ratio†
Whole Life PARTICIPATING.....	OM	7,056,863	551,838	100
Whole Life NON-PARTICIPATING.	ONM	602,591	56,807	110
Endowments.....	OM	897,673	132,043	75
Limited Payments.....	OLM	410,251	36,839	81
Increasing scale premiums.....	OIM	207,709	23,280	109
Temporary.....	OTM	36,489	11,603	
Contingent.....	OCM	15,586	3,482	
Joint Lives.....	OJM	90,171	9,195	

* British Offices' Life Tables—several volumes published by C. & E. Layton, London. See especially "An Account of the Principles and Methods, etc.," 1903.

† This "Approximate mortality ratio" is obtained by comparing the actual deaths between the principal insurance ages, 30–50, with the expected deaths by the $OM^{(10)}$ Table. The select period (first five years) was excluded in making the comparison. In the last three classes the non-select experience is scarcely sufficient to make a satisfactory comparison.

Select Tables are expressed $O^{[NM]}$, $O^{[M]}$, etc.; while for female lives the letter 'P' is substituted in the above symbols instead of 'M'.

Select mortality tables were accurately formed for the first time and show the effect of the company's and the policyholder's selection. The effect of selection is shown to be of little importance after 5 or 6 years, yet unadjusted tables are given which exclude:

1. The first 5 years;
2. The first 6 years;
- etc., etc., up to the first 10 years.

By excluding the first ten years we probably get the *ultimate* rate of mortality. Select values during the first ten years have been graduated to join this ultimate table. Those tables which exclude a portion of the data have been called "truncated"—a word which does not seem expressive for the particular purpose in view.

Duplicate lives were eliminated from the aggregate tables; but all except simultaneous policies were retained in the select statistics, since two policies taken at different dates on the same life do not come into the same section of a select mortality table. Accordingly the select tables purport to deal with a larger number of lives and a greater number of years of exposure. In the aggregate and ultimate tables if two policies appeared on the same life, but with an uninsured interval between the discontinuance of the one and the issue of the other, the first was treated as withdrawn and the second as a new issue.

As may be seen from the tables at the end of the book the rate of mortality, q_x , by the American Experience Table is higher than the O^M at ages under 43, then considerably lower from that age to age 70, after which it is again higher. Net premiums at 3% interest for Whole Life Insurance follow generally the same course; but at the principal entry ages, 25 to 40, there is never so much difference as \$1.00 per thousand. At age 25 the American Experience rate is 87 cents *more* than the O^M and at age 40 it is 49 cents *less*. The expectations of life by the American Experience and the

$O^{(5)}$ show a remarkable agreement while annuity values, premium rates, etc., also differ but slightly.

Female Experience is given for: (1) Ordinary Life participating, (2) Ordinary Life non-participating, and (3) Minor Classes. As compared with male mortality the same characteristics appear as had been noted in earlier investigations; there is also an indication of general superiority of female over male vitality; but medical selection does not appear to be quite so effective. A full analysis of female in comparison with male mortality was made by Mr. C. W. Kenchington* and a further investigation was afterwards made by Mr. A. J. C. Fyfe.†

Amongst the "Old Assurances" in force in 1863 there were comparatively few recent policies. The method whereby such Old Assurances were included, while extending greatly the experience at the older ages, had the effect of introducing a disproportionate number of non-select and aged lives into the aggregate O^* experience, as well as into the select experience. This would make the mortality curve of the aggregate table unduly steep after middle life; it probably accounts for the high reserves by this table. Although the H^m net premiums are lower than Sprague's Select up to age 43, the O^* are lower than the $O^{[M]}$ Select only up to age 29—a result possibly arising from the same cause.

Net premiums by the O^* Table are less than those by the older H^m Table, but policy values are on the average greater, a condition caused by the different slope of the mortality curve. The mortality in the various classes confirms the belief that the larger the rate of premium paid, the lower the mortality—an effective illustration of the selection exercised by policy-holders.

A careful study of the new statistics as compared with earlier compilations appears to show:‡

1. A marked improvement in vitality;
2. The influence of medical selection is more persistent, and especially conspicuous amongst younger men;
3. Self-selection by annuitants had not improved to the

* J. I. A., Vol. XLIV, p. 105.

† T. F. A., Vol. VII, p. 21.

‡ T. F. A., Vol. IV, p. 82.

same extent as medical selection of applicants for insurance. (For annuity experience, see p. 112.)

One of the new features of this experience is the publication for the first time of complete tables of withdrawals. These are given for ages at issue grouped around quinquennial points and for each year of duration from 0 to 9 inclusive, also for "10 and upwards."

The rates of discontinuance per cent according to the O^m experience* were as follows:

Year	Central Age at Entry				Year
	25	35	45	55	
1	3.05	2.42	2.25	1.98	1
2	8.59	6.56	5.70	5.17	2
3	5.81	4.44	3.88	3.46	3
4	4.21	3.40	2.92	2.64	4
5	3.27	2.66	2.44	1.95	5
6	2.78	2.34	2.09	1.77	6
7	2.30	2.08	1.70	1.41	7
8	2.17	1.72	1.49	1.11	8
9	1.91	1.67	1.33	1.02	9
10	1.68	1.52	1.21	1.11	10

The rates of discontinuance per cent on *non-participating* whole life policies differed materially from the above as is shown by the following rates for central age at issue, 35, that is for the group of ages 32-37.

RATES OF DISCONTINUANCE PER CENT, O^{nm} EXPERIENCE
Central Age 35.

Year	Rate
1.....	4.36
2.....	10.13
3.....	7.36
4.....	6.57
5.....	4.90
6.....	3.88
7.....	3.20
8.....	2.72
9.....	2.49
10.....	2.31

The technical methods used in the construction of the mortality tables are discussed in detail in Actuarial Study No. 2. The graduation of the entire experience was placed in

* See J. I. A., Vol. XXXVII, p. 463.

charge of Mr. George F. Hardy; and the $O^{(6)}$ Table was graduated by Makeham's Law, constants being determined by a new method somewhat similar to the method of moments. In the adjustment of the O^* Table the use of Makeham's formula alone was not practicable, but the graduated table was built up on the basis of the graduated $O^{(6)}$ by a double frequency curve connecting the two tables. As it was necessary to abandon or modify Makeham's formula the only object in view was to obtain a perfectly smooth curve for the O^* Table and to represent as nearly as possible the ungraduated facts. But Makeham's Law was applied to the graduation of the $O^{(6)}$ experience* and was found to give good results from age 20 onwards.

In the $O^{(M)}$ Table an ultimate table, excluding the first ten years of the participating experience, was graduated by Makeham's Law and the select values were graduated to join this ultimate table by an extension of the same law, introducing a function dependent on the duration of the insurance.

* J. I. A., XXXVIII, p. 501.

CHAPTER V.

TABLES FORMED . *from* LIFE INSURANCE EXPERIENCE AMERICA

THE AMERICAN EXPERIENCE TABLE, 1868*

THE American Experience Table of Mortality was formed by Sheppard Homans and was first published under its present name in a schedule attached to an Act passed by the legislature of the State of New York on May 6, 1868. The author never gave full particulars of the data employed. He used mortality statistics deduced from the experience of the Mutual Life Insurance Company of New York, but the figures were inadequate at the older ages and accordingly he arbitrarily adjusted the table. The table starts at age 10 with a radix of 100,000 and ends with three deaths between ages 95 and 96, the latter being therefore the limiting age of the table. It was much used for many years because it furnished a safe basis of measurement of American mortality amongst insured lives after the first effects of selection had disappeared. It is still the principal table in use for valuation purposes since it has been prescribed by State Laws for that purpose, but dividend scales of participating companies and non-participating premiums have been much influenced by the American Men Mortality Table since the publication of that investigation.

At the first dinner of the Actuarial Society of America, April 25, 1889,† Mr. Homans said:

* Walford's "Insurance Cyclopædia." T. A. S. A., Vol. X, pp. 509-514.

† T. A. S. A., Vol. I, pp. 33-34.

"The result was that after I had collated the experience of the Mutual Life I drew a curve representing the approximate rates of mortality at different ages; and then found by a simple method of adjustment the rates of mortality now called 'The American Experience Table'—a name, however, that was not given by me. The table has for its basis the experience of the Mutual Life; but it is not an accurate representation of that individual company. In other words, it is not intended to be, and never was claimed to be, an accurate interpretation of the experience of the Mutual Life."

He further stated that the American Table was prepared simply as a study, and that he had made the limiting age 96 because in the records in different countries he could find no instance of any insured individual attaining the age of 100 years.

The Mutual Life experience, also prepared by Mr. Shepard Homans, was published in 1858 and dealt with a period of fifteen years from the commencement of the company in 1843. Mr. D. P. Fackler has pointed out that premiums based upon the new tables were adopted in 1861 so that the modifications contained in the American Experience Table as compared with the mortality rates of the earlier table of 1858 must have been made about the year 1860. They must therefore have depended on the additional experience of the Mutual Life for the years 1858 and 1859 only, or in other respects have been adjusted by Mr. Homans arbitrarily from a study and comparison of other tables. The new table "was intended to represent the death rate among insured lives residing in salubrious districts *after the effects of medical selection were eliminated.*"

The graduation of the Mutual Life Table of 1858 was by a graphic process, but no particulars are given of the method used by Mr. Homans in graduating the American Experience Table. Recent investigations* would seem to indicate that the following may be considered as established facts:

1. The American Experience Table is based on a table of the values of the reciprocal of q_x to three decimal places for each age from 10 to 95 inclusive. The method by which

* T. A. S. A., XII, p. 253; XIV, pp. 27-37 and 354-363.

this table was derived from the original data is unknown.

2. From the values of the reciprocal of q_x the values of the logarithms of the rates of mortality were taken out to seven decimal places.

3. Starting with a radix of 100,000 at age 10, the value of $\log l_{10}$ was taken out to seven decimal places. The logarithm of q_{10} was added to it to form $\log d_{10}$. The natural number of d_{10} was extracted to the nearest unit and subtracted from l_{10} to form l_{11} , etc., the process being repeated for each age.

*Gill's Mortality Table**

The American Experience Table was probably influenced somewhat by the table constructed in 1853 by Professor Charles Gill which was made the basis of the premiums of the Mutual Life for the following fifteen years and which was used also somewhat by other companies. This was an empirical table constructed by plotting several English tables. It followed the Carlisle Table closely below age 60 and the Actuaries' or Combined Experience Table at higher ages.

Makehamized American Experience Table

In 1902 Dr. Arthur Hunter made a very successful graduation of the American Experience Table by the application of Makeham's Law of Mortality, and this Makehamized Table is of great value in simplifying the calculation of benefits depending upon two or more lives.

American Experience Table—Extensions to Age 0

In 1900 J. M. Craig prepared an extension† to age zero, which is in general use and is embodied in the Insurance Laws of Wisconsin. In preparing the extended rates shown in the following table, the rates for ages zero to 5 inclusive were taken without change from the extension of the Combined Experience Table adopted by the National Convention of Insurance Commissioners and rates for ages 6 to 9 were obtained by interpolation.

* T. A. S. A., Vol. XII, p. 253; Vol. XIII, p. 80.

† Given with commutation columns, annuity values and other functions in American Experience Derived Values, $3\frac{1}{2}\%$ interest—Ages 0 to 19, I. P. Mantz.

CRAIG'S EXTENSION TO AGE 0

Age	l_x	d_x	q_x
0	143,819	22,249	.1547
1	121,570	7,719	.0635
2	113,851	4,042	.0355
3	109,809	2,625	.0239
4	107,184	1,897	.0177
5	105,287	1,432	.0136
6	103,855	1,181	.0114
7	102,674	1,001	.0097
8	101,673	877	.0086
9	100,796	796	.0079

A different extension has been used by companies domiciled in Maryland for all their reserve valuations for business issued since January 1, 1903. Farr's English Life Table No. 3 was the Maryland standard for ages under 10. Mr. Clayton C. Hall adopted the death rate for each age from the male table as applying to the corresponding age next birthday. The result was therefore an empirical combination of the two mortality tables.

American Experience Select and Ultimate Table

This was an empirical modification of the American Experience Table proposed by Miles M. Dawson and incorporated into the Armstrong legislation in New York State in 1906. The assumption was that the American Experience Table was to be used as an ultimate table excluding the first five policy years and that the probabilities of death in the first year were 50% of the ultimate rates, in the second year 65%, in the third year 75%, in the fourth year 85% and in the fifth year 95%. Valuation by the select and ultimate method* on this table was made a permissive standard of valuation in New York State. The table was also used (before the amendment of 1929) as a criterion in the expense limitation law whereby commissions and certain other acquisition expenses within a given year had to be less than the loading and the so-called "mortality savings" according to this table on business paid for within the year.

* T. A. S. A., Vol. VII, p. 418 or Practical Lessons in Actuarial Science, Miles M. Dawson.

THIRTY AMERICAN OFFICES' TABLES*

Tables were compiled in 1881 by Levi W. Meech from the returns of 30 American Offices, dealing with 1,027,529 lives, of which 549,418 were existing at the close of the observations (1874), 46,543 having died and 431,568 having discontinued. The returns were furnished on cards, which gave:

Residence, Amount, Class, Calendar year of entry,
Calendar year of exit, Age at entry, Sex, Cause of death.

Male and female lives were investigated separately and each subdivided into the three classes—existing, discontinued, and died. Less than 5% of the lives were female. These classes were again subdivided into durations of policies, 0, 1, 2, 3, 4, . . . , and these again into still smaller groups according to the age at entry. Policies were assumed on the average to be taken in the middle of the initial year, and were treated as exposed for half of the first calendar year. Those which passed out of observation were also assumed to do so at uniform intervals. Average policy years thus begin and end with the middle of the calendar year, while the average birthday is assumed to be at the middle of the initial year, the general assumptions being similar to those made in forming the Institute of Actuaries' Tables. The office age at entry was "nearest" birthday; and the entry ages were diminished by $\frac{1}{2}$ for the period of exposure in the year of entry, i. e., as if the ages were changed from the middle to the beginning of the calendar year of duration. The average age at entry was 35.23 years, and the mean duration of the policies was 4.36 years. The mortality was investigated by lives, but the principal tables were based upon amounts insured and claims, with slight adjustments toward the extremes of the table. The rate of financial loss was greater than the rate of mortality, showing that the claim rate was higher amongst policies of large amount. The effect of medical selection was stated to disappear within $2\frac{1}{2}$ years.

At ages under 45, the mortality of females was found to exceed that of males, the excess from ages 20 to 35 being over 35%; from age 45 to age 65 the order is reversed,

* System and Tables of Life Insurance, by Levi W. Meech.

though by a much smaller difference relatively, and again at the older ages (ages 67 to 95 inclusive) the mortality amongst females exceeds that amongst males.

As 549,418 of the policies were existing at the close of observations, the record ceased before the real experience had developed. Had the investigations been deferred 15 or 20 years many more policies would have passed to their termination. The defect from having so many existing at the close of the observations was approximately corrected by *assuming* the existing to be carried forward until they should pass out of observation by discontinuance or death, the ratios of mortality and discontinuance for this purpose being taken as those actually experienced on select mortality and withdrawal bases down to 1874. This was called the method of "Final Series";* of course it is effective only when the real and the assumed statistics are combined in an aggregate table, since it increases the proportion which the non-select lives at any age bear to the total under observation at that age. A more complete description of this method is given in Actuarial Study No. 2, p. 61.

The figures from 10 to 90 in the male table were graduated by Makeham's Law of Mortality, taking the ungraduated values of l_{25} , l_{40} , l_{55} and l_{70} as data. Notwithstanding that previous tables showed considerable deviations from Makeham's Law, the results in this case showed close agreement with the ungraduated values, and this was explained by Mr. Meech as follows:

1. The observations were more numerous than those of any previous collection.
2. They were nearly homogeneous.
3. The statistics were accurately deduced.
4. The construction was, it was claimed, improved by "Final Series."
5. The observations were free from misstatements of age common to census returns.

But as more recent experience has shown that many tables can be graduated by Makeham's law, including even

* See J. I. A., Vol. XXXII, p. 9 et seq.

old tables like the "Carlisle," the suggestions as to the causes why this table should follow the law probably show a lack of familiarity with the adaptability and flexibility of the law as afterwards developed. The graduation of the table for female lives, on account of the small numbers above 70, was aided by comparison with other experiences, and then adjusted by Woolhouse's method. A very complete set of monetary values was published.

The statistics were also analyzed by states and territories of the United States, and tables were given of the actual and expected claims in each state or territory. These figures are less valuable than they would have been if the effect of selection had been taken into consideration, because in some states the business may have been relatively recent, whereas in others there may have been much old business. In some of the Southern states the expected and actual claims are given for each county in the state separately. There are also tables showing the causes of death from diseases which are grouped under the following heads: Zymotic, Constitutional, Nervous, Circulatory, Respiratory, Digestive, and Miscellaneous.

STANDARD INDUSTRIAL MORTALITY TABLE*

Prior to 1907 the policy liabilities in Industrial Insurance were based on the same tables as were used for Ordinary policies. Mortality tables had been compiled by individual companies from their own experiences and used in calculating premium rates; but as the statutes did not differentiate between the two classes the valuation of industrial policies was based upon the Actuaries', or Combined Experience, for policies issued prior to January 1, 1901, and upon the American Experience Table for policies issued thereafter. Early in 1907, after modification of the New York laws, the Superintendent of Insurance fixed as the legal basis for New York State a table based exclusively upon the experience of industrial policies, called the "Standard Industrial Mortality Table"; and, while permission has since been granted by the

* Mortality functions are given in United States Life Tables, 1890, 1901, 1910, and 1901-1910, Department of Commerce, Washington, 1921.

New York Legislature to use other tables, this one is still recognized as a standard.

The Standard Industrial Mortality Table was based upon the experience of the Metropolitan Life Insurance Company during the ten years 1896–1905. The policies at risk and the death claims were obtained from data contained in registers in which were kept a record of all Industrial premium-paying policies, classified according to year of issue and age next birthday at entry. The experience included 48,508,562 years of risk and 767,552 death claims.

The unadjusted data were first graduated by Woolhouse's formula for graduation from age 2 up to age 35 inclusive, also by Makeham's formula from age 20 to the end of life, using the groups of ages 27–39, 40–52, 53–65, 66–78 for obtaining the constants. The ages under 30 as obtained by the Woolhouse graduation were merged into the Makeham graduation by means of a second Woolhouse grading, the resulting probabilities being retained from ages 12 to 35 inclusive, the Makeham graduation being followed from age 36 upwards. The original figures from ages 2 to 6 inclusive were retained without graduation, while for ages 7 to 11 an interpolation formula was used.

The table starts at age 2 with radix of 100,000, and age 99 is the limit of life, there being 1 living at age 98. The rate of mortality is lower than that of the American Experience from ages 10 to 21, then higher to age 87 inclusive; and at the very old ages it of necessity becomes lower again since the limit of life in the American Experience is age 96. Medical selection is not such an important factor in case of industrial as of ordinary risks, since the applicants in most cases are subjected only to a medical inspection, and accordingly the tables were based upon the entire experience, thus being "aggregate" in form. The mortality curve does not agree at all closely with any known table, but it probably comes nearer the English Life Table No. 3 than any other.

As a result of the amendments in 1923 of the New York and New Jersey insurance laws permitting business to be written on lives aged one next birthday, the Standard Industrial Mortality Table was extended to age one. The computation of the rate of mortality at this age was based upon

the mortality as shown in the "United States Life Tables, 1910," modified to allow for the fact that the exposures are comparatively small during the first few weeks after birth.

SUB-STANDARD INDUSTRIAL MORTALITY TABLE

The Sub-Standard Industrial Mortality Table is specifically recognized by the New York insurance statute as a permissive legal valuation basis. The table was based upon the experience of the Metropolitan Life Insurance Company on certain classes of Industrial sub-standard risks during the years 1896 to 1905 inclusive. The experience included 5,275,163 years of life and 93,266 death claims. The records available for this experience were similar in form to those available for the Standard Industrial Mortality Table and the methods of construction and graduation followed those used in constructing the Standard Table. The original table started at age 2 with a radix of 100,000 and ended at age 100. The table was extended to age 1 in 1923 when the New York and New Jersey statutes were amended to permit insurance from birth.

The rates of mortality of the Sub-Standard Industrial Mortality Table are more than double those of the Standard Industrial Mortality Table at some of the younger ages. This difference, however, diminishes with advancing age; in fact, at the very high ages, the mortality rates are slightly lower than under the Standard Table.

INTERMEDIATE TABLES

The Intermediate Tables (1907 and 1912) are based on the experience of the Metropolitan Life Insurance Company (carried to 1905) on \$500 policies written from 1896 to 1905 in its Intermediate branch. These policies were issued with annual to quarterly premiums, but the risks covered were largely such as had formerly been restricted to Industrial Insurance.

The original table is an aggregate one, with certain adjustments to decrease the weight of the first two years' experience. After preliminary smoothing a Makeham graduation was made from age 30 to 87. The Combined Experience Table, with some adjustment, was used for the higher

ages. Special methods were adopted for ages 10 to 30. An extension to age zero has been made which merges with the Standard Industrial Table at age 5.

The death rates obtained as above, increased 10%, constitute the 1907 table, known as the New York Standard Intermediate Table. In the 1912 Intermediate Table the 10% loading was dropped. Both tables have been approved by the New York Insurance Department for the valuation of Intermediate business. The reserves throughout are higher than American Experience reserves.

JONES' TROPICAL TABLE

The Jones' Tropical Table was constructed by Mr. C. N. Jones and was based upon the experience to policy anniversaries in 1892, by amounts insured, of the New York Life Insurance Company within the American Tropics, the data on policies issued at the Semi-Tropical and Tropical rates being combined. The graduated rates of mortality were obtained by fitting a Makeham curve to the experience of the third and subsequent policy years combined. The table with a detailed description is given in T. A. S. A., Vol. III, p. 299.

TROPICAL AND SEMI-TROPICAL MORTALITY TABLES*

These tables were prepared by Dr. Arthur Hunter, of the New York Life Insurance Company, from the experience of that company in many tropical countries over the years 1873 to 1906. During that period two scales of premium rates for these countries were in effect—namely, tropical and semi-tropical; and for this investigation the data were classified separately according to these two scales—not according to a geographic distribution, as the names suggest.

Comparisons with the expected deaths on certain standard tables indicated that the ultimate rates of mortality were attained much earlier in the tropics and semi-tropics than in the temperate climates and that they were increasing percentages of the mortality in this country and Canada, up to about age 50.

* See T. A. S. A., Vol. X, p. 395.

An ultimate table was prepared from the Tropical experience based on lives for all policy years except the first and second by applying Makeham's formula. Inasmuch as the ratio of the actual to expected deaths for the sixth and succeeding policy years was 156% of the American Experience Table under the tropical experience and 129% of the American under the semi-tropical experience, and further since the period of medical selection under the semi-tropical was practically a mean between the periods for tropic and temperate climates, the q_x for the semi-tropical table was obtained by taking the mean between the q_x of the tropical table and the American Experience Table and regraduating by Makeham's formula. Select rates of mortality for the first two years under the Tropical table and the first five years under the Semi-tropical table were prepared by applying increasing percentages to the ultimate rates.

These tables are still used by some South American companies, but for American companies they are largely of historical interest only. A full description of the tables with commutation columns is given in T. A. S. A., Vol. X, p. 395.

NATIONAL FRATERNAL CONGRESS TABLE OF MORTALITY*

The fraternal organizations meet annually in convention, and at the National Fraternal Congress held in 1897 the feeling was expressed that the Actuaries' Table of Mortality which was then the standard table for old line life insurance companies showed a higher mortality rate than was necessary according to the past experience of the better class fraternalists. Accordingly a committee was appointed to look into the subject, and this committee made a report at Baltimore in 1898, accompanying their report by a table of mortality which was afterwards adopted as the National Fraternal Congress Table.

One of the principal objects in view was, as stated by the committee, to prepare a standard table which would bring out premium rates as low as practicable, apparently even

* Tables based upon the National Fraternal Congress Table of Mortality. By Abb Landis and G. D. Eldridge. New York. C. C. Hine's Sons Company, 1909. Mortality functions are also given in United States Life Tables, 1890, 1901, 1910, and 1901-1910.

taking into account the profit from lapsing, no surrender values being paid, since the committee reported that the problem in the preparation of the table "involved the basis of a minimum rate, *with the elimination of cash, paid-up, and extended values.*" The mortality rates are therefore low; yet a year later the committee stated that many fraternal orders had had an even more favorable experience, although others may have encountered a higher death experience than the tables indicate.

Details of the number of lives exposed to risk and of the number of years' exposure are not given, the statement made by the committee being:

"The experience examined and available embraced the mortality experience of the Old Line companies in the United States, England, Canada, and Australia; of the Fraternal, the experience of the two oldest and largest in this country."

Apparently, therefore, the table reflects the arbitrary judgment of the members of the committee; this judgment was confirmed by a continuing committee one year later, the report in 1899 stating that the committee "has had opportunities for extending its research somewhat, as new data have been compiled of the actual experience of the fraternal orders, members of this Congress, which were not available one year ago." After a careful investigation nothing was disclosed which would lead the committee to recommend a change.

The two "oldest and largest fraternal in this country" at that time were The Royal Arcanum and the Ancient Order of United Workmen, "and it is generally understood that the experience of the Royal Arcanum largely controlled in fixing the data agreed upon by the committee." The table was tested still further and a voluminous report submitted to the National Fraternal Congress in 1906, when the Congress considered that it was justified in continuing its endorsement as originally given.

The table was graduated by Mr. George D. Eldridge, who states that the material was placed in his hands "late on the afternoon of November 16, 1898, the day before the committee having the matter in charge was to report to the congress then in session. The work of graduating q_x was com-

pleted during that night." The table was graduated on the assumption that q_x could be represented by a function of the form $A + Bc^{x+1}$; and as a preliminary step the value of q_x was determined experimentally from the material at certain ages "taken at fifty or over, with a view to dealing first with that part of the table only which, under the general limitation of fraternal, is not affected by the admission of new lives." From these experimental values of q_x the three constants, A , B , and c , were obtained. These constants gave a rate of mortality at age 20 of .0052511; but it was the judgment of the actuaries that, excluding the savings from selection during the three years immediately succeeding admission, a rate at age 20 of .005 was a far more justifiable minimum. The latter was therefore adopted and a subtractive series formed from ages 20 to 45.

It will be observed generally with reference to the above table that while the results may be practical as a standard of measurement from the fraternal standpoint, representing *minimum mortality rates* for adoption by such orders, nevertheless the processes adopted in deducing the table were largely empirical.

THE FRATERNAL-AMERICAN MORTALITY TABLE

The Fraternal-American Mortality Table* is a combination table formed in 1912 by Fackler and Fackler to meet the special needs of fraternal issuing certificates which at the younger ages followed the step-rate plan with no surrender values but which merged into level-rate plans after middle life. It seemed expedient to use the National Fraternal Congress Table up to middle life but to use the American Experience Table during the level-rate period, so that there could be no doubt that surrender values might, under the New York Conference law, be granted on such certificates. The final table adopted to meet these objects and also to make a smooth bridge between the two sections was the National Fraternal Congress Table up to age 45 and the American Experience Table for ages 57 and over with graduated intermediate values. The result was a mortality curve very much lower than the American Experience at the young ages and not much higher at those ages than the AM⁽⁵⁾

* Proceedings of Fraternal Actuarial Association. No. 8, p. 17.

Table which, of course, had not then been derived. It also approximated the AM⁽⁵⁾ Table at the higher ages where the latter differed but slightly from the American Experience Table.

EXPERIENCE OF CANADA LIFE OFFICE

Published 1895, covering 46 years, 1847-93

One of the principal objects of this investigation, made by Mr. Frank Sanderson, was to determine the effects of selection in the northern half of North America. All lives rated-up, or charged an extra, as well as female lives, were excluded. The investigation was made according to lives assured. The policy year method of investigation was adopted, but some slight complication arose in regard to the ages on account of the next-birthday age having been inserted on the cards. The experience was reduced from fractional to even ages by means of the formula

$$l_x = l_{x-1} - \frac{1}{3}d_{x-1},$$

it having been found from actual experience that on the average the assurances were taken one-third of a year prior to the attainment of the age next birthday. But this assumption was ingeniously avoided in graduating the aggregate tables, constants having been determined from the facts at fractional ages.

In filling in the mode of exit four subdivisions were used: (1) Existing, (2) Matured, (3) Withdrawn, and (4) Died. The Matured contained expired term assurances and endowments. The existing were carried to the anniversaries in 1893, and the duration was found by subtracting the year of entry from 1893, thus giving an integral number of years in all such cases. The nearest integral duration of exposure was adopted for the Matured and Withdrawn, while those who died were carefully located in the policy year in which death took place and assumed to be under exposure to the end of that year. Duplicate lives were eliminated, it would appear, from the select experience as well as from the aggregate.

The total number of entrants was 35,287 covering 296,481 years of life. 55% of the entrants were "Existing" in 1893; 37% had "Withdrawn" and "Matured"; and 8% had

"Died." The average age at entry was 32; the average duration nearly $8\frac{1}{2}$ years. Of the total years of exposure, more than one-half relate to ages below 40 and four-fifths to ages below 50, a fact that probably accounts for the small percentage of deaths for so high an average exposure, although for the "Died" alone the average duration was 13.55 years.

The tables deduced include an aggregate table, a table excluding the experience of the first five years, some select tables, as well as full particulars under each age at entry of the exposed to risk and deaths for each year of duration. Statistics are given of the withdrawals and it is shown that nearly half of these took place at the end of the first year. Makeham's formula for graduation was used for the aggregate table, the method of determining the constants being that introduced by King and Hardy; $\log c = .0425$.

In forming select tables the experience after five years was first investigated and connecting mortality rates were formed for the first five years between the ages of 20 and 50 in the following manner: The first year's mortality was investigated by itself in three groups of ages and then graduated for each age. The third year's mortality was found by combining the second, third and fourth years, the resulting mortality rates being graduated by Makeham's formula; and lastly, the second, fourth and fifth years' mortality rates were obtained by interpolation.

In the published volume there were included comparative tables showing the death rates as compared with those of the more important mortality tables of individual offices and groups of offices. The mortality of the Canada Life appears to have been exceedingly favorable, being practically the same as that of the Australian Mutual Provident. The satisfactory experience may have been caused to a considerable extent by climatic influences, combined with strict medical selection and a goodly proportion of risks from rural communities. D. and N commutation columns and annuity values at 4% interest are given. Reserves by this table are high as compared with standard tables. The government of the Province of Ontario adopted the experience as the basis for the calculation of minimum rates for fraternal societies. A separate investigation was made into the

mortality amongst sub-standard lives, but the paucity of data and combination of various classes of risks render any conclusions from this class of doubtful practical value.

AMERICAN-CANADIAN MORTALITY INVESTIGATION*

American Men Table

The American Men Mortality Table was prepared by the Actuarial Society of America with the cooperation of the American Institute of Actuaries and of the National Convention of Insurance Commissioners. It reflected the normal distribution of business by states in the United States, and substantially the average distribution by plan of insurance of standard business issued on the legal reserve basis. The exposure was taken covering all years of issue through 1914, traced to the anniversary in 1915 from issue or, where issued prior to 1900, from the anniversary in 1900. The experience was based on sums insured except that, in order that the death of an individual with a very large amount of insurance might not have undue influence on the mortality at ages or durations where the exposure was not large, policies issued at the same age and on any one life for more than \$100,000 were considered throughout as \$100,000. This was possible because companies gave complete information on all policies for \$50,000 or more. The age in each case was taken at the nearest birthday.

To avoid giving undue weight to the experience of the large companies, all companies with \$50,000,000 or less insurance in force were requested to give their entire data called for by the instructions, and companies with more than \$50,000,000 insurance in force were to give data on \$50,000,000 of their business. In order to distribute their contributions thoroughly such companies gave their data on the proper proportion of the lower numbers in each series of one hundred policy numbers. In order to obtain sufficient data at the older ages all companies gave all their data on plans other than deferred dividend for policies issued prior to 1900

* American-Canadian Mortality Investigation. Conducted and published by the Actuarial Society of America with the cooperation of the American Institute of Actuaries and the National Convention of Insurance Commissioners, 1918.

in force after the anniversary in 1900. 59 companies took part in the general investigation and nearly all these contributed data for the American Men Mortality Table.

The exposures for this table, as well as for the corresponding sections on women, were as follows:

	American Men	American Women
Existing.....	\$1,894,044,000	\$68,257,900
Withdrawn.....	1,449,826,400	48,470,600
Dead.....	436,345,200	10,878,500
Total.....	3,780,215,600	127,607,000
Entrants prior to 1900.....	1,545,696,700	35,849,000
New Entrants.....	2,234,518,900	91,758,000
Total Exposures.....	26,003,321,900	787,819,500
Average Exposure.....	6.88 yrs.	6.18 yrs.

All policies on lives which were under-average or charged an extra premium at the outset were excluded.

As far as it was compatible with accuracy, refinements which would retard the work were avoided. Policies dated back more than three months were excluded. After determination by experiment that they would scarcely influence the experience, policies either increased or decreased in amount or changed in plan were considered as terminated at the date of change. Policies were included when running under extended term insurance after lapse only when the face value had not been decreased in amount. Lapses where policies were subsequently reinstated within six months were disregarded but reinstatements after longer lapse were excluded. Interim Term insurance for less than one year was disregarded, the date at issue being taken at the expiry of such term.

As it was desired to ascertain mortality rates for policies issued by legal reserve companies under modern conditions and subject to a regular medical examination, policies incontestable from date of issue, policies originally issued on the assessment plan or by fraternal organizations, policies reinsured in bulk and those issued on the industrial or group insurance plan were eliminated. Owing to the possible selection by policyholders at the end of the deferred dividend period, policies on the deferred dividend plan were terminated at the end of the deferred period.

As several companies had made a practice of maturing life policies as endowments at age 96, the limiting age under the American Experience Table, they were asked to obtain a history of the subsequent lifetime of such policyholders in order to have more material at ages above 96.

After experiments made by several companies the "mean duration" method was used to obtain the exposure on terminated policies, the error in this method having been found by several tests to be less than by the "nearest" duration method and to be only about one-fifth of one per cent as compared with the "exact" duration. (See Actuarial Study No. 2, p. 14.)

Every advantage was taken of electric punching, sorting, and tabulating machines. In addition to the usual methods of insuring accuracy, a tabulator-printer was of great value in detecting errors.

After the exposed to risk and the death losses were obtained, the crude death rate was deduced for each of the first five insurance years, for the sixth and succeeding years combined, and for the eleventh and succeeding years combined. The expected deaths for each of the first ten insurance years were then calculated by graded rates of mortality based upon the data for the sixth and succeeding insurance years in order that the number of years for which medical selection lasted could be determined. It was seen that the material for the sixth and succeeding insurance years could be safely combined according to attained age.

The ultimate mortality rate did not follow any recognized standards in the United States as is evident from the following table based on the rates finally determined.

RATIO OF $AM^{(6)} q_x$ TO THAT OF

Attained Age	American Experience Table	M. A. Ultimate	U. S. Males (White) 1910
25	53%	92%	78%
35	53	94	56
45	71	106	63
55	94	111	81
65	101	104	93
75	97	100	99

Graduations by recognized methods including Spencer's, Gompertz's, Makeham's, King's Osculatory, and the Graphic were not satisfactory. Owing to the flatness of the curve at the young ages, neither the Gompertz nor Makeham formula was suitable prior to age 50. The method finally adopted, proposed by Mr. Robert Henderson, was based upon Makeham's second modification of Gompertz's Law which may be expressed in the form

$$\text{colog } p_x = \alpha + \theta x + \beta c^x$$

The constants for the "ultimate" table were tentatively determined from the ungraduated quinquennial values of $\text{colog } {}_5p_x$, by differencing for fifteen year intervals the expression $\text{colog } {}_{10}p_x$ plus $\text{colog } {}_{10}p_{x+5}$, and then modifying as indicated by tests. The rates of mortality derived from such constants were found to fit satisfactorily down to age 30. The table was extended below that age by retaining unchanged the term involving c^x ($\log c = .034$) in the expression for $\text{colog } p_x$ and substituting for the remainder of the expression a constant for ages 20 and under and a function of the second degree in x for ages 20 to 30 inclusive. The following expressions for $\text{colog } p_x$ were finally adopted:

For ages 15 to 20—

$$\text{colog } p_x = .001087 + 10^{\bar{4}.1109 + .034x}$$

For ages 20 to 30—

$$\text{colog } p_x = .001087 - .000005 (x-20)^2 + 10^{\bar{4}.1109 + .034x}$$

For ages 30 and over—

$$\text{colog } p_x = .001087 - .0001 (x-25) + 10^{\bar{4}.1109 + .034x}$$

From the values of $\text{colog } p_x$ derived from these formulas the l_x column was calculated and the rates of mortality recalculated.

To the graduation of the experience during the first five insurance years the same general formula was applied, with modified values of the constants other than c for each duration. The formula was used in the following form, where $\log c = .034$.

$$\text{colog } p_{(x-\eta)+1} = \alpha_i - \theta_i \left\{ \begin{array}{c} 0 \\ (x-20)^2 \\ 20 \\ (x-25) \end{array} \right\} + \beta_i c^x \left\{ \begin{array}{l} \text{Ages 15-20} \\ \text{Ages 20-30} \\ \text{Ages 30 and over} \end{array} \right.$$

Values of α_0 , θ_0 and β_0 were first determined so as to reproduce very closely the mortality of the first insurance year. Values for the intermediate years were then determined so as to reproduce as closely as possible the mortality experience of those years and at the same time form as regular a series as possible. It was found necessary to increase α_t to the ultimate value in the second year. The values of θ_t were filled in according to the formula

$$\theta_t = \theta - \frac{(5-t)(6-t)}{30} (\theta - \theta_0) \text{ and those of } \log \beta_t \text{ by the formula}$$

$$\log \beta_t = \log \beta - \frac{(5-t)(6-t)(72-23t+9t^2)}{2160} (\log \beta - \log \beta_0)$$

By applying the values of $\log p_{[x]+t}$ derived from these constants to those of $\log l_{[x]+5}$ according to the ultimate table, the values of $\log l_{[x]+t}$ were constructed and those of $l_{[x]+t}$ taken out to the nearest unit. From these, values of $q_{[x]+t}$ were derived.

Not only were the deviations and accumulated deviations of the expected deaths from the actual studied thoroughly; but net premiums for the Ordinary Life form with 3% interest based on the Ultimate section of the new table were compared at quinquennial ages with net premiums worked from the ungraduated data.

Graduated rates of mortality for decennial ages are as follows:

AMERICAN MEN
Graduated Rates of Mortality

Age at Entry	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	$q_{[x]+3}$	$q_{[x]+4}$	q_{x+5}	Attained Age
15	.00247	.00324	.00341	.00355	.00372	.00392	20
25	.00293	.00384	.00402	.00412	.00427	.00446	30
35	.00316	.00429	.00457	.00480	.00523	.00584	40
45	.00482	.00690	.00782	.00866	.00992	.01158	50
55	.00994	.01447	.01712	.01951	.02271	.02668	60
65	.02252	.03267	.03931	.04517	.05263	.06147	70

Net premiums on the AM⁽⁵⁾ are considerably less than those on the American Experience Table at the young ages. The difference decreases until it is negligible above age 60.

COMPARISON OF NET PREMIUMS PER \$1,000 OF INSURANCE

3% Interest

Age at Entry	Ordinary Life		20 Payment Life		20 Year Endowment	
	Am. Exp.	AM ⁽⁶⁾	Am. Exp.	AM ⁽⁶⁾	Am. Exp.	AM ⁽⁶⁾
20	\$14.41	\$11.75	\$23.13	\$19.45	\$40.77	\$38.54
40	24.75	23.06	33.14	31.09	43.01	41.24
60	58.27	57.64	61.62	61.39	63.29	63.28
<i>3½% Interest</i>						
Age at Entry	Ordinary Life		20 Payment Life		20 Year Endowment	
	Am. Exp.	AM ⁽⁶⁾	Am. Exp.	AM ⁽⁶⁾	Am. Exp.	AM ⁽⁶⁾
20	\$13.48	\$10.72	\$20.72	\$16.96	\$38.90	\$36.63
40	23.50	21.75	30.75	28.64	41.18	39.36
60	56.83	56.23	59.85	59.61	61.65	61.65

The differences between mean reserves on the AM⁽⁶⁾ Table and the American Experience Table depend upon the age at entry, the plan, and duration of insurance, being positive in some cases and negative in others. The AM⁽⁶⁾ reserves are in general greater on the Ordinary Life plan. A test valuation of one of the oldest and largest companies showed a two per cent increase in reserve over the American Experience reserve. In the case of a typical younger company, a valuation in accordance with the Illinois Standard at three and one-half per cent resulted in a total reserve one and one-half per cent greater by the new table than by the American Experience Table.

It was pointed out at the time of publication, 1918, that the table represented average mortality experience during an era free from wars and epidemics, and that during the period there were marked changes in the insurance business in the United States. However, on the policies issued in the decade following the severe influenza epidemic of 1918 many companies have had an exceptionally favorable mortality experience, the actual deaths being considerably less than those expected by the American Men Select Table, particularly at the younger ages.

Several states have adopted the American Men Table (Ultimate), with interest at three and one-half per cent, as the minimum valuation standard for group insurance issued on the yearly renewable term plan. In about half the states

and in Canada the laws have been changed, amended or interpreted to permit the optional use of the new table for the valuation of Ordinary contracts.

A Makeham graduation of the $AM^{(6)}$ table, and joint life values for equal ages at 3% and $3\frac{1}{2}\%$, are given by Valentine Howell in T. A. S. A., Vol. XXI, p. 178. A graduation of the experience of Iowa, Kansas, Minnesota, Nebraska, North Dakota and South Dakota is to be found in the same paper.

Canadian Men Table

For the Canadian Men Table the rules under which the material was collected and the method of treatment of the data were the same as for the American Men. The extent of the data was as follows:

	Canadian Men	Canadian Women
Existing.....	\$258,738,200	\$6,466,800
Withdrawn.....	165,682,600	3,714,200
Dead.....	29,400,800	672,500
Total.....	453,821,600	10,853,500
Entrants prior to 1900.....	114,108,600	2,572,500
New Entrants.....	339,713,000	8,281,000
Total Exposures.....	2,743,665,100	69,144,100
Average Exposure.....	6.05 yrs.	6.37 yrs.

After a combination of summation formulæ had failed to remove major irregularities in a satisfactory manner, the ultimate section of the new table was regraduated in the same manner as the $AM^{(6)}$. The $C^{M^{(6)}}$ is thus a Makeham table up to age 20, a hybrid table thence to age 35, and is graduated by Makeham's second modification of Gompertz' law above age 35. The extension of this method of graduation to the select period follows the methods employed in constructing the American Men table. The formulæ for the ultimate rates of mortality finally arrived at were as follows:

For ages 15 to 20:

$$\text{colog } p_x = .001539 - .000041 (20 - x) + 10^{5.599 + .04x}$$

For ages 20 to 35:

$$\begin{aligned} \text{colog } p_x &= .001539 - .000040 (x - 20) \\ &+ .00000036 (x - 20) (35 - x)^2 + 10^{5.599 + .04x} \end{aligned}$$

For ages 35 and over:

$$\text{colog } p_x = .001539 - .000040 (x - 20) + 10^{5.599 + .04x}$$

From the following comparison, it will be seen that the rates of mortality are much lower than those given by the $O^{(u)}$ Table, the former standard of valuation in Canada. They are also lower than the $AM^{(u)}$ rates, except at the young ages, and the same relation holds generally for the first five policy years.

RATIO OF CANADIAN MEN $C^{(u)}$ q_x TO THAT OF

Attained Age	American Experience Table	$O^{(u)}$	$AM^{(u)}$
25	55%	64%	102%
35	50	53	93
45	63	58	88
55	79	71	84
65	88	83	87
75	91	93	94

Net premiums are lower on the $C^{(u)}$ Table than on the $O^{(u)}$; mean reserves are sometimes greater, sometimes less, depending upon the age at issue, and kind and duration of insurance.

Other Investigations Connected with the American-Canadian Mortality Investigation

The corresponding mortality experience of women, residents of the United States and Canada, was similarly investigated. Women who had been rated up or placed in a special dividend class only on account of sex were included, however, in this investigation, because women had been generally so treated.

Mortality among women, as revealed by the Medico-Actuarial Mortality Investigation, depends upon the marital status at the date of application for insurance. The principal value, therefore, of the new investigation, where the conjugal condition is not stated, lies in the comparison of the experience among women according to amounts insured with that among men.

Using the American Men Select Table as a measuring rod, the actual deaths among American women were 110.8 per

cent of the expected during the select period and 91.5 per cent for the sixth and subsequent years combined, the ratio for the total exposures at all ages and durations being 94.4 per cent. The deaths during the early insurance years were relatively greater than the deaths among American men, but above attained age 40 the experience on women after the fifth policy year was the more favorable.

The small amount of data on Canadian women did not justify extensive comparisons. The mortality experience, slightly more favorable in the aggregate than among Canadian men, was less favorable in the earlier insurance years and at the younger attained ages; on the whole, it was better than among American women. The actual deaths were 117.9 per cent of the expected by the Canadian Men table during the first five insurance years, 93.3 per cent for the sixth and succeeding years, and 97.9 per cent for the entire experience.

All the material previously described was given in Volume I of the report. In Volume II analysis was first made of the mortality among American men by plan of insurance. The ratio of actual to expected mortality according to the AM table on the Ordinary Life plan (including endowments maturing at ages 80 and over) was 107.6% during the select period, 101.8% during the ultimate period, and 102.2% for the total exposure. At the higher ages the bulk of the experience entering into the AM table itself was on the Ordinary Life plan. The mortality on the Twenty Payment Life plan (including Nineteen Payment Life and corresponding endowments maturing at age 80 or above) was correspondingly 92.0%, 94.9%, and 94.3%, and on the 19 and 20 Year Endowment, 99.4%, 94.7%, and 95.5%. There had been a tendency to limit some borderline risks to Endowment plans of insurance. The Ten Year Term mortality with a ratio of 102.2% reflected very strict selection.

The experience among policies converted from term policies or changed without a new medical examination within ten years of the original date of issue proved satisfactory, although, due to paucity of data, definite conclusions could not be reached. The mortality after renewal or change at the end of the original period indicated that it may be ad-

visible to limit the privilege of renewal or conversion without evidence of insurability to attained age 55 or thereabouts.

Mortality experience in the United States was investigated according to habitat in ten sections of the country grouped according to their geographical situations and economic conditions. The habitat was taken as of the date of issue. The lowest mortality was found in the western north-central states and in general in states situated in what is known as the corn or wheat belt. Two groups of Southern States showed, as anticipated, higher mortality than the average—to the extent of approximately 24 per cent in each case. It is probable, however, that the improvement in mortality in the Southern States since the AM investigation has been greater comparatively than in the rest of the country.

In Canada, as in the United States, the section where farming is the outstanding industry, i.e., the Prairie Provinces, showed the lowest mortality experience. Quebec, which has a large percentage of French-Canadian population, showed relatively high mortality in all insurance years, not only among men but, as a special investigation showed, among women.

An elaborate analysis was made of the causes of death in the cases of American men and of Canadian men. The deaths according to cause were tabulated showing (*a*) each cause as a percentage of total deaths from all causes, and (*b*) the death rate from each cause per \$10,000 exposed to risk. Both analyses were made by groups of attained ages and by insurance years 1, 2, 3-5 inclusive, and subsequent.

The mortality under policies for large amounts was investigated according to (1) insurances for \$50,000 or more on individual lives, and (2) insurances of \$100,000 or more on individual lives taken out in amounts of at least \$50,000 per policy. Unfortunately the statistics are limited, because a small part only of the data of the large companies was contributed and the investigation was confined to American men. As expected, the mortality was higher than normal—about 17 per cent in excess of the AM Table for the first group and 31 per cent for the second.

Extensions of American Men Table to Age 0

Mr. Frederick A. Draper* derived his extension of the American Men Ultimate table down to age 0 from the experience of the juvenile departments of seven fraternal societies covering seven years of operation from 1920 to 1926 inclusive. As over 80% of the deaths were due to either acute diseases or accident, select tables for the early policy years would have been very little lower than this ultimate section. As part of the experience of one of the large societies included showed some adverse selection after a large loss of membership and as the mortality rates were arbitrarily increased 10%, it was thought that the resulting rates would be high enough for use in conjunction with the American Men Ultimate table for ordinary standard business. Graduation was made by osculatory interpolation.

A 1929 law in Connecticut specified that the American Men Ultimate Table at 4% should be the official basis for computing succession and transfer taxes in Connecticut. To extend this to age 0, Mr. Russell Hooker consulted all the experiences available, especially that of the Industrial Department of the Metropolitan Life Insurance Company, and constructed the following empirical table which gave some weight to the recent low industrial mortality and which involved some other changes at ages above 6 in Mr. Draper's table. The latter's curve had an abnormally low minimum at age 10, .00175, followed by a peculiarly steep rise to its junction with the published AM⁽⁵⁾ curve at age 15. It was further thought that the published American Men rates from ages 15 to 19 did not originally represent true ultimate mortality experience. They were obtained by extrapolation and were so high that their retention would have meant too steep a gradient between ages 10 and 15. The death rates decided upon were graduated entirely by a graphic process which seemed most suitable on account of the large element of judgment involved. The resulting functions are as follows:

* American Men Super-standard and Sub-standard Tables and Extension of Table to Age 0—Frederick A. Draper, Des Moines.

MORTALITY TABLES

AMERICAN MEN ULTIMATE TABLE
Hooker's Extension to Age Zero

x	q_x	l_x	d_x
0	.08234	117,077	9,640
1	.01984	107,437	2,132
2	.01273	105,305	1,340
3	.00835	103,965	868
4	.00567	103,097	585
5	.00403	102,512	413
6	.00308	102,099	314
7	.00255	101,785	260
8	.00228	101,525	232
9	.00210	101,293	212
10	.00200	101,081	202
11	.00204	100,879	206
12	.00219	100,673	221
13	.00247	100,452	248
14	.00278	100,204	278
15	.00307	99,926	307
16	.00332	99,619	331
17	.00352	99,288	349
18	.00368	98,939	364
19	.00381	98,575	376
20	.00392	98,199	385

MORTALITY EXPERIENCE UNDER GROUP INSURANCE*

For several years annual investigations have been made, under the auspices of a Committee on Group Mortality Investigations, of the Group Insurance experience on the One Year Renewable Term plan issued by six of the companies which transact a large proportion of this business. The results, which show actual claims compared with the expected according to the American Men Ultimate force of mortality, have been published by Mr. E. E. Cammack, at first in papers in the Transactions of the Actuarial Society and later in separate pamphlets giving detailed analyses of the experience. Disability claims payable six months after proof of disability have been treated as death claims. The experience has been compiled on the calendar year basis, the exposure being the mean of the insurance in force at the beginning and at the end of the year on individual group policies, distributed in quinquennial age groups according to

*T. A. S. A., Vol. XIX, p. 29; Vol. XXII, p. 222; Vol. XXIV, p. 334; Vol. XXVI, p. 332.

Combined Group Mortality Experience [E. E. Cammack, chairman] 1913-26, 1913-28 and each year thereafter. 1931 pamphlet gives 1922-26 and 1927-30.

the age of the insured on the anniversary. Thus exposures at ages $20\frac{1}{2}$ to $25\frac{1}{2}$ were considered as at age 23 and the corresponding deaths were taken as those at the nearest age in the calendar year at ages 21 to 25. Exposures on new issues and on cancelations were prorated. This method gave on the average only one-half a year of exposure in the year of death, the exposure at age x corresponding approximately to the year of life $x - \frac{1}{2}$ to $x + \frac{1}{2}$. The experience in the early investigations was studied both by lives and amounts, but as there was not much difference in the incidence of mortality, most of the published data has included that based on lives only. In the earlier experiences a separate analysis was made of cases where the employer paid only part of the premium. For several years the experience on the contributory plan was found to be higher than on the non-contributory plan, but the excess on the contributory plan was due almost entirely to the high mortality rate in the electric and street railway classification and in the later investigations the contributory and non-contributory experience has been combined.

Most attention has been given to the analysis of the experience by industry and industrial groups. The industrial classification used was an adaptation of one taken from the thirteenth census of the United States. The five general classifications adopted were: (1) Mining, (2) Iron, Steel and Other Metal Industries, Lead Industry and Construction, (3) Transportation and Public Service, (4) Clerical, and (5) All Others.

Elaborate analyses of claims have been made by cause of death, by industrial groups, and by attained age. The earlier investigations showed the analysis by policy year, but it was found that the loss ratio varied very little after the first year.

Each investigation has summarized the disability experience separately. An almost constantly increasing percentage of actual to expected disability claims, according to Hunter's Disability table, was found through 1931.

The mortality experience under conversions from Group Insurance to Regular Insurance has usually been included, showing the excessively high mortality in the early years after conversion.

The first such group mortality investigation published included the experience from 1913 to 1924. The investigations have been published annually since then. The ratio of actual to expected mortality has ranged slightly above 90% of the AM Ultimate Table.

Cammack's Group Tables

After studying the experience through 1926, Mr. Cammack constructed two mortality tables*, one on the Clerical classification (Group 4), which might be considered as a super-standard class, and one on the All Others or Miscellaneous occupation class (Group 5), which embraced two-thirds of the total experience and might be considered as the standard class representing Non-hazardous Industries. The first three occupation groups were sub-standard classes.

The total experience for the years 1922-26 was studied, the following being a summary:

General Industrial Classification	Years Exposed	Actual Claims	Ratio of Actual to Expected A.M. Ult. Table
Mining.....	167,892	1,895	140.8%
Iron, Steel and Other Metal Industries, Lead Industry and Construction.....	465,217	4,602	107.6
Transportation and Public Service...	1,504,558	14,066	103.3
Clerical.....	771,589	4,302	76.1
All Others.....	6,232,386	45,072	85.6
Total.....	9,141,642	69,937	90.2%

While the combined rate of death and disability had varied only slightly from year to year, being slightly higher in 1925 and 1926, the total and permanent disability claims increased steadily from a ratio of 54% of Hunter's expected in 1922 to a ratio of 104% in 1925, and then fell to 94% in 1926. On the assumption that the disability experience of 1925 and 1926 was a fair indication of what was to be expected in the future, the *General Mortality Table* was based

* T. A. S. A., Vol. XXVIII, p. 247.

on the 1925 and 1926 experience only. There were 3,046,298 life years exposed. The death and disability claims were 20,021 and 2,536 respectively.

The study of the Clerical experience showed that the combined rate of death and disability had varied only slightly from year to year and that the rate of total and permanent disability had been about constant during the years 1923 to 1926. For this reason and also because the exposure was so much smaller than in the non-hazardous industry classes the *Clerical Mortality Table* was based on the 1923 to 1926 experience. The exposed to risk totaled 663,511 life years. The death and disability claims were 3,359 and 372 respectively.

In the construction of each table the data were taken by quinquennial age groups. Both the exposed and the number of claims at the central age of each group were obtained by George King's method of graduation given in J. I. A., Vol. XLIII, p. 109. Only slight adjustment was thereafter necessary in the force of mortality. The rate of disability shown is the rate of becoming disabled during any year and of remaining so at the end of the year, which is a fair assumption under the conditions of Group Insurance. There being no total and permanent disability experience above age 60, the assumption was made that all are totally and permanently disabled at age 80, the rate of disability being extended to age 80 by interpolation. The General Table was constructed so that it could be used either on the assumption that disability benefits cease at age 60 or so that it could be used in connection with pension plans where the contingencies involve disability above age 60. As there is no mortality experience available on disabled lives under Group Insurance, Hunter's rates were used in the construction of this part of the table. The best approximation for mortality at the high ages where the exposure was small appeared to be to use 80% of the American Men Table. The graduated tables were merged into these values above age 80.

The Clerical Mortality Table was constructed similarly to the General Mortality Table except that slight adjustments were made to make it more consistent with the latter. Representative probabilities are as follows:

MORTALITY TABLES

	Non-hazardous Industries		Clerical	
	1000 p_x^{di}	1000 q_x	1000 p_x^{di}	1000 q_x
20	.41	2.56	.38	2.01
30	.64	3.15	.49	2.44
40	.79	5.48	.46	4.42
50	1.60	10.54	1.13	10.38
60	4.42	26.26	3.16	24.89
70	27.86	50.76	27.86	55.65
80		108.59		108.59

A comparison of the two tables shows the following: Up to age 45 the rate of mortality of the Clerical Table is about 80% of the General Table; from age 45 to age 65 it follows closely the General Table; above age 65 the mortality of the Clerical Table is distinctly higher. However, the rates at the extreme end of the Clerical Table were based upon a small experience. The rate of disability in the Clerical Table equals the rate in the General Table at ages 21-25, falls to nearly one-half the rate of the General Table at age 40, and then increases to about two-thirds of the General rate.

The tables are valuable inasmuch as they represent the true experience under group policies. The general table was used as a basis for determining gross premium rates in order to test the adequacy and reasonableness of the premium rates prescribed by the New York State Insurance Department, which were based upon the American Men Ultimate Table. In addition the Clerical Table, which showed a low mortality during the early and intermediate ages (the working period), was used by Mr. Craig and Mr. Henderson as the basis of the construction of the Combined Annuity Table for the younger ages. However, more recent studies of the group experience show that while the rate of mortality is about the same as that shown by the 1926 study, the rate of disability has increased steadily from the 1925 and 1926 high and is still rising. Consequently the disability portion of these two tables is of little value at present.

CHAPTER VI.

GENERAL MORTALITY INVESTIGATIONS AMERICA

(Special Mortality Tables Constructed)

SPECIALIZED MORTALITY INVESTIGATION*

THIS is not strictly a mortality table but rather an investigation into the relative mortality as affected by certain special features, such as: nationalities resident in the United States, occupations, personal and family histories, and localities. The work was undertaken in 1901, under the direction of the Actuarial Society of America, from thirty years' experience (1870-1899) of thirty-four life insurance companies in the United States and Canada. Its object was to supply those officers who have to decide upon the acceptance of risks with material for ascertaining whether a particular class is better or worse than normal.

The investigation was based upon policies, except that if several policies were issued on one application in the same year only one, the longest in force, was reported. The duration of policies cancelled was computed by subtracting the calendar year of issue from that of termination, and of policies in force by taking the exact duration at the anniversaries in 1900.

The mortality experience in the different classes was compared with a table adopted to represent standard mortality amongst healthy lives. The expected deaths were computed by this standard table, which was based upon Farr's Healthy English Male Table, modified for ages 15 to 21, inclusive, and for 52 to 61, inclusive. For ages 15 to 21, eleven-

* Volume published by Actuarial Society of America, 1903, 479 pp., folio.

tenths of the annual mortality of the then new male experience $O^{(5)}$ was taken, while at ages 52 to 61, five-sixths of the $O^{(5)}$ mortality was adopted. In order to allow for the effect of selection during the early years, the following percentages of mortality by the standard table were used:

Ages at Entry	Percentages of Standard Mortality to Allow for Selection					
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6 and After
15 to 28	45	64	79	90	97	100
29 to 42	50	68	82	92	98	100
43 to 56	55	72	85	94	99	100
57 to 70	60	76	88	96	100	100

The final results were published in 1903, with the actual and expected deaths for each class in the four age groups above shown as well as in total. The detailed facts were also published to permit of individual study. From them more accurate conclusions may be deduced.

The classes of risks may be briefly described as follows:

1. *Policies for large amounts*; one class over \$20,000.
2. *Policies granted on terms other than applied for*; 2 classes.
3. *Nationality*; divided into 4 classes.
4. *Occupation*; divided into 35 classes and covering army, navy and marine service; the more important hazardous trades; liquor dealing; and railway service.
5. *Personal Disability*; covering 32 classes, including past history of diseases such as gout, blood-spitting, etc., unusual weights and unusual heights.
6. *Family history unsatisfactory*; covering 2 classes, dealing respectively with cancer and insanity.
7. *Place of Residence*; 22 classes, each relating to a different county in the United States, principally in the South.

Care must be exercised in using the results for the following reasons: All the lives investigated were *accepted* by companies and therefore may not show *average* results in some of the specialized classes. While those who were charged an extra solely on account of occupation were included in the

experience, no risks were included if they had been treated as under average on account of personal or family history. Accordingly all the classes in the latter category may be viewed as the better selected risks of each class. Again, the standard of measurement (Farr's Table Modified) has not been universally accepted as representing the normal risk. If the standard mortality were too low for any group of ages the result would be to give apparently unfavorable results for those ages and vice versa.

MEDICO-ACTUARIAL MORTALITY INVESTIGATION*

The Association of Life Insurance Medical Directors and the Actuarial Society of America felt that the work of the Specialized Mortality Investigation should be extended, and a joint committee was formed for the purpose, October, 1909; memorandum of instructions issued May, 1910. Forty-three companies supplied the basic information and the work of compiling, tabulating, and recording the data was conducted at a central bureau in New York City.

The experience covered about 93 per cent of the policies issued in the United States and Canada by all legal reserve companies during the years 1885–1909, traced to policy anniversaries in 1909. It dealt with the following:

- a.* 68 groups involving occupational hazard;
- b.* 76 groups of medical impairments;
- c.* 4 groups of women;
- d.* 3 groups of colored risks;
- e.* 4 groups of joint life policies;
- f.* A study of the influence of build on longevity.

Policies which had been treated as sub-standard for medical reasons were excluded; policies rated as sub-standard for family history alone, or for occupation, were included. The experience was based on policy years, and mean durations. Each death was placed in the proper policy year.

To investigate the influence of build on longevity, the

* Medico-Actuarial Mortality Investigation, Vols. I-V, by the Association of Life Insurance Medical Directors and The Actuarial Society of America, New York, 1912.

companies furnished records of their business on standard lives for one month in each of the sixteen years, 1885-1900, January being taken in the odd years and July in the even years, by this means securing an average of summer and winter weight. The mortality rates from this data were much lower than the standard table assumed in the Specialized Investigation. They were, however, confirmed by the experience of two large companies, and were therefore used for purposes of comparison. The rates of mortality were low at the younger ages, the ultimate mortality for the eleventh and succeeding policy years being less than 60 per cent of the American Experience for attained ages below 40. After the first year the mortality by policy years for entrants under 50 showed the effects of selection to be relatively slight.

The committee asked the companies to furnish additional data to examine further the improvement in mortality and a standard table was thereafter formed. The following shows the extent of statistics:

Years of Issue	Number of Entrants	Total Number of Years' Exposure	Deaths	Expected Deaths	Ratio
1885-1892	80,976	781,852	7,180	6,850.03	105%
1893-1900	148,995	1,106,216	8,000	7,911.45	101%
1901-1908	270,404	926,108	5,042	5,441.91	93%
Totals	500,375	2,814,176	20,222	20,203.39	100.3%

The final table shows the mortality rates for the first four years of duration and thereafter is merged into a general table excluding these years—now commonly mentioned as the "M. A. Table." Only the first two years showed a distinctly lower mortality than the ultimate. The Committee was satisfied that the table might with confidence be used for the particular purpose for which it was constructed.

Subdivision of the data according to years of issue showed a continuous improvement in mortality. The committee issued a warning against the general use of this table for any purpose other than that for which it was prepared. It is based upon policies and should not be applied to the solution of financial problems, since mortality rates are higher when based upon amounts insured.

Five volumes of statistics were published from 1912 to 1914, the first volume containing details of the investigation and basic facts for the investigation of build, also the adjusted mortality table. Tables of average height and weight were prepared and in Volume II are published mortality ratios in groups of lives according to the extent of departure from the average build.*

These tables should be used with caution, as the preponderance of recent business has the effect of indicating a lower mortality than normal in some cases, in others a higher. In addition to showing the effect of build on mortality Volume II contains an analysis of the causes of death amongst men at different ages; also tables of mortality amongst women† divided into four classes—

- a.* Spinsters;
- b.* Married women, beneficiary husband;
- c.* Married women, beneficiary other than husband;
- d.* Widows and divorcees.

Classification by plan shows that endowments were favored by unmarried women, whereas life and limited payment forms were more often taken by married women, including widows and divorcees. The volume closes with statistics of mortality amongst North American Indians and certain colored groups. The mortality amongst Negroes was shown to be about 40 per cent in excess of the expected in spite of the careful selection of such risks.

Vol. III deals with the "Effect of Occupation on Mortality," the most important divisions being the Liquor Industry, Railroad, Metal Trades, and Mining. The causes of death are not given.

Vol. IV contains statistics of cases showing medical impairments, including the effects of alcohol, also diseases affecting the various physical organs, the arteries, etc., including certain surgical conditions. It must be remembered that cases accepted without rating, although coming within these classes must have been unusually good in every respect

* T. A. S. A., Vol. XV, p. 315, and XVII, p. 17.

† See also T. A. S. A., Vol. XVIII pp. 318-326.

except for the particular impairment. High mortality appeared among those who had suffered from syphilis, even after two years' continuous treatment and one year's freedom from symptoms; also in cases in which albumen or sugar were found, cases of alcoholic habits, high pulse rate, abdominal girth greater than chest expanded, etc.

Questions of family history (especially history of tuberculosis), habitat mostly in southern states, malaria, and joint life insurance, were covered in Vol. V. It was developed that under joint policies to men and women, there was a high mortality among women at the younger ages, but at the middle and older ages they appeared better than men.

Students should read with close attention the warnings contained in "An Interpretation of the Results of the M. A. Investigation," T. A. S. A., Vol. XV, pp. 62-76.

JOINT OCCUPATION STUDY*

On March 1, 1928, the Joint Committee on Mortality of the Actuarial Society of America and the Association of Life Insurance Medical Directors invited the companies represented in either of the two bodies to contribute data for an investigation of mortality in various occupations. It had been pointed out that the material of the Medico-Actuarial Investigation was from 19 to 42 years old, that industry had undergone many changes since the World War, and that disability and accidental death benefits had been introduced during recent years.

Owing to the belief that working conditions had changed in many fields of employment since the advent of Group and Workmen's Compensation Insurance, in that safety devices had been improved and that more attention had been paid to health and welfare work, it was decided to go back to the year 1915 only. For some industries in which it was anticipated that extra mortality would be due to accidents only, duration is less important and issues prior to 1920 were not included. Thus there were two groups of classes by years of

* Volume published by Actuarial Society of America and the Association of Life Insurance Medical Directors, July, 1929.

issue, 1915 to 1926, and 1920 to 1926, all carried to anniversaries in 1927.

The investigation was based on observations by policies, no distinction between sexes being made. Only Ordinary policies rated standard except for occupation were included. Group, Industrial, Intermediate, and Non-Medical insurance were not considered. Cases issued standard and sub-standard were included together, but the committee knew that certain companies did not furnish any standard data and that others gave only a proportion of their standard material coming within the required occupational classes. In a few cases the results were somewhat distorted by these facts and a section of the report endeavors to make allowance for these conditions.

The exposure was terminated upon reduction in rating from sub-standard to standard or from a higher to a lower extra premium for a hazardous occupation, the procedure thus differing from that in the M. A. Mortality Investigation.

The occupational code followed was that published by the Joint Committee in 1926 in the hope of securing uniformity. It provides for more than 1,000 classes in conformity with modern distinctions. A very complete index of occupations had been issued in connection with it.

The data for 430 groups of occupations were called for, but it was necessary to combine a number of small classes to get significant results. The total number of entrants in the occupational classes was over 1,300,000, the average duration 3.6 years, the average age at entry 36.6, and the number of deaths 22,600. The average duration of the older group of policies (1915-1926) was 4.1 years.

Twelve companies were in a position to contribute the data requested; these companies had three-fifths of the total insurance in force in the United States and Canada, and probably a greater proportion of the sub-standard business.

The experience of each company for a given occupation, before being sent to the Central Bureau, was summarized in quinary groups of ages from 15 to 64 inclusive, with one group for ages 65 and over, by policy years, under the headings "Existing," "Lapsed," "Dead," and "Exposed." The number of deaths according to the principal causes was

shown in two groups of ages at issue, 15 to 39 and 40 and over.

The actual deaths might have been compared with the deaths expected by the American Men Select Table, but it was deemed preferable to use specially prepared standards. From the material of the ten largest contributing companies, new basic tables of mortality among standard lives were prepared, one for each of the two groups of years of issue. As it would have been very laborious for some of the companies to have obtained the exposures and deaths by policies, the data by amounts insured were used. Summary sheets similar to those for the occupation study were submitted. The comparative extent of the data was:

	Death Losses	Exposed to Risk
1915-1926 Basic Table.....	\$546,357,000	\$115,792,035,000
1920-1926 Basic Table.....	284,884,000	72,338,796,000
American Men Table.....	436,345,000	26,003,322,000

All data were furnished in quinquennial age groups, but it was proven that the central ages were approximately correct for comparison with other tables.

Mention may be made of a peculiar feature in the basic data of the 1920-1926 table. The mortality rate for the youngest age-group was at several durations and in the aggregate greater than the corresponding rates of the next two five-age groups. A similar characteristic was found in the ungraded data of the AM Select Table (using seven years of duration only) but to a less pronounced extent. For the 1920-1926 Basic Table this peculiarity was eliminated and the adjusted death rate in each age-group is never less than for the same duration in the preceding age-groups. On the other hand, the rate for any duration among entrants in any age-group does not exceed the rate for a duration five years greater among those entering the next previous age-group.

In the case of the 1915-1926 Basic Table, however, the effect of the influenza epidemic was of more importance than the effect of selection at the young and middle ages. To follow the same principles as in the other table would have sacrificed too much in the way of accuracy or fidelity to the original. The characteristics of the ungraded data were, therefore, more closely preserved in this table.

The formula used for a preliminary graduation of the first five years of duration was a modification of Henderson's Formula A* which was applied separately to the first five years of duration. The resulting percentages were afterwards modified by inspection so as to give a smoother progression by duration and were graded into a constant percentage (78%) of the AM Table at the older attained ages. In both tables the function graduated was the ratio of the rate of mortality to that of the AM Select Table.

Since the durations of the tables—twelve years and seven years—are short, it was not advisable to make an ultimate rate of mortality for the purposes in hand. In both tables the mortality at every age and duration is less than in the American Men Table of 1900–1915, although the influenza epidemic of 1918–1919 was included in one. In general, the ratios run from 60% to 80% of the American Men Select mortality. There is a tendency for these ratios to be slightly higher in the first few years of insurance than in the later

BASIC TABLE 1915-1926						BASIC TABLE 1920-1926				
Policy Year	Ages at Entry									
	20-24	30-34	40-44	50-54	60-64	20-24	30-34	40-44	50-54	60-64
	Graded Rates of Mortality per 1,000									
1	2.26	2.36	3.26	6.41	14.28	1.68	1.90	3.00	6.70	14.29
2	2.86	3.08	4.57	9.19	19.64	1.98	2.41	4.23	9.49	18.84
3	3.10	3.37	5.24	11.11	22.66	2.20	2.63	5.16	10.59	21.71
4	3.07	3.45	5.64	11.62	24.31	2.35	2.80	5.51	11.40	24.24
5	2.65	3.19	5.87	12.82	26.56	2.49	3.08	5.82	12.89	27.87
6	2.41	3.41	7.08	15.75	35.37	2.55	3.38	6.56	15.14	33.55
9	2.50	4.01	8.96	20.30	45.31					
12	2.76	5.07	11.43	26.15	57.84					
Ratios to A M Select Graded Rates of Mortality										
1	.80	.78	.80	.83	.86	.59	.63	.74	.87	.86
2	.77	.77	.80	.82	.81	.53	.60	.74	.85	.78
3	.79	.80	.82	.85	.78	.56	.62	.81	.81	.75
4	.76	.79	.81	.78	.73	.58	.64	.79	.77	.73
5	.63	.68	.75	.75	.69	.59	.66	.74	.75	.72
6	.55	.67	.78	.78	.78	.58	.66	.72	.75	.74
9	.56	.69	.78	.78	.78					
12	.60	.73	.78	.78	.78					

* Transactions of the Actuarial Society, Vol. XXV, p. 29.

years, and at the older as compared with the younger ages. When the two new tables are compared with each other the principal difference is evidently due to the influenza epidemic of 1918, which had its effect chiefly at the ages under 45.

To throw light upon the underwriting of accidental death benefits in connection with life insurance policies, tables were prepared for 87 occupational classes. The death rate from all accidents, excluding those of war, was divided in each class by the corresponding rate in the appropriate basic table, and after the resulting ratio had been adjusted to correct anomalies due to age distribution it was compared with the average rating of the three largest companies.

The death rate from accidents, other than war casualties, was .56 per 1,000 for the 1920-1926 basic table and .52 per 1,000 for the 1915-1926 basic table.

The Report of the Joint Committee comments upon the results in some 200 occupations or combinations of occupations. Detailed tables pertaining to 176 of these are given. To indicate the significance of the data, approximate values of the statistical function known as the "probable deviation" follow the mortality ratios in the report. The average of the ratings of the three largest companies, which contributed two-thirds of the data in the investigation, was stated for certain of the principal occupational classes for comparative purposes.

The Committee directed attention to twenty-nine groups in which there were marked differences between the mortality experienced and that indicated by the average rating of the three companies. Several occupations connected with the extraction of minerals, five classes of laborers, and employment involving police duties, had unexpectedly high death rates.

In only about 100 of the classes did the causes of death require comment. Of these, two-thirds showed occupational accidents as an outstanding factor, while non-occupational accidents were found excessive in 20 classes. Tuberculosis of the lungs stands high in 25 classes, chiefly among the groups of unskilled laborers and occupations in which there is a dust hazard. Pneumonia appears important where there is exposure to abnormal temperatures. Cirrhosis of the liver was significantly high among bartenders and saloon-keepers,

and also among the large group of underground miners.

The Committee pointed out that, since the average duration of the exposure in this investigation is distinctly shorter than the average life of a policy on the books of the companies, the period of lowest relative extra mortality may be unduly emphasized in some occupations while the period of greatest extra cost may be stressed in others. Other factors, such as the average age at entry and average age at death, must be given due weight.

Soon after the publication of this occupation report the committee published its "Occupational Mortality Ratings,"* which gave ratings for each of the occupational groups as percentages of the standard mortality, representing the opinion of the Joint Committee as to ratings which might reasonably be used by the life insurance companies. These ratings are intended to represent in each class not only the new experience but also the old figures which might be company ratings or previous experience. This is closely analogous to the problem of experience-rating in compensation and group insurance. Allowance had to be made, however, for classes where conditions had changed—for example, in those occupations where safety devices had been introduced or those which were affected by the Prohibition Amendment. Where little or no material was available the ratings were deduced by analogy. As a control over the entire set of ratings a comparison was made of the actual deaths in each class with those which would be found if the mortality were that indicated by the proposed ratings. In the aggregate and in representative sections the results were in satisfactory agreement. In preparing this pamphlet* it was found convenient to make a practical adaptation of the probable errors.

MEDICAL IMPAIRMENT STUDY (1929)†

This publication, a large part of which appeared in the year 1931, was designed to deal with most of the matters

* Occupational Mortality Ratings. Compiled and published by Actuarial Society of America and the Association of Life Insurance Medical Directors. New York, Dec., 1929.

† Medical Impairment Study, 1929. Compiled and published by the Actuarial Society of America and the Association of Life Insurance Medical Directors. New York, 1931.

taken up in the Medico-Actuarial report which had not already been covered in the Joint Occupation Study (1928). The medical code of 1925, which had been prepared by the same committee, was used as a basis for classifying the various medical impairments. The medical impairment study was more complex than the occupational one because subdivisions were made according to the time elapsed since the operation or the attack. Furthermore, the standard data were separated from the sub-standard and when they were combined it was necessary to allow for the standard material which had not been contributed by some of the companies. The investigation included not only medical impairments, but also policies for large amounts of insurance, the consideration of a table of height and weight, and mortality according to build. The mortality summary sheets were again used in the respective classes unless the contribution of any company was less than 50 entrants, in which case it furnished perforated cards to be combined at the central bureau. The total number of entrants in the medical impairment classes including those with a tuberculous family history, was nearly 1,100,000 with 41,000 deaths. One-third of the material contributed was classed as sub-standard. In the investigation of mortality according to build there were about 970,000 cards. Adding to both of these the data in the investigation dealing with large sized policies, the total number of entrants was more than 2,100,000.

There were 39 contributing companies representing four-fifths of the insurance in force in the United States and Canada. A basic table of mortality was again prepared dealing with the years of issue 1909-1927. The statistics forming this basic table were more extensive than those of any other based upon insured lives. The death losses amounted to more than one billion dollars and the exposed to risk to 190 billions. An ultimate table was not formed. The formula used in graduating the ratio of the rate of mortality to that of the AM Select table was the one which had been used in the occupational report. The characteristics of this table were rather unusual. The aggregate of death claims was 78% of that expected by the American Men Select table. The ratios run in general from 70% to 90%.

The peculiar characteristics of this table were partly caused by the influenza epidemic of 1918 and by the great improvement in mortality which followed during several years after that time and affected particularly the younger and middle ages.

As in the case of the occupational report, company ratings for insurances were compared with the actual experience. No material was included in this report relative to disability and double accidental death benefits. The method of obtaining the probable deviation due to insufficient numbers was refined and made somewhat more accurate than the formula used in the occupational report. The material in the text dealing with causes of death in the medical impairment report shows the ratio of death rates per thousand exposed to risk in each case as compared with the corresponding death rates in the basic table, which was also the basis used in the M. A. Mortality Investigation, whereas the occupational report compares the percentage of deaths from a given cause in each occupational group with the corresponding percentage in the general experience. This point should be borne in mind in making comparisons.

The material dealing with height and weight included ages at entry as low as ten and this made available new material among boys and girls. At ages above 20, however, the results of the M. A. Mortality Investigation were confirmed and it seemed unnecessary to prepare a new table. This applied to average height as well as to average weight. In the case of men there was a slight reduction in the average height, although other investigators had found a tendency to increase in recent years,—for example, college graduates, as compared to the height of their fathers when they were the same age. The possible reasons for this discrepancy were carefully analyzed in the report.

This outline, of course, cannot attempt to give any real conception of the results of this investigation for which the original report must be consulted. A brief review was given by Dr. Hunter in T. A. S. A., Vol. XXXII, p. 414.

It is planned to publish at an early date a pamphlet giving medical impairment mortality ratings along the same lines as the corresponding pamphlet which has already been pub-

lished for occupational mortality ratings. The medical impairment report deals with policies, and occasionally where there were a number of policies on a single life which died, adjustments were made for that fact. The probable deviations apply theoretically to variations in mortality of lives rather than of policies. An investigation was also made of how the mortality by amounts insured compares in various medical impairment investigations with the corresponding mortality by policies. It was found that in the case of overweight and heart murmurs, particularly, the mortality by amounts was considerably higher than by policies. This was to be accounted for by the fact that a large proportion of persons who take the largest policies of insurance are inclined to suffer from these two principal impairments—overweight and heart murmurs. The section which deals with large amounts insured is at the time of this writing being carefully analyzed to indicate the mortality according to the total amount insured in all companies on each life. There will be 50,000 or 60,000 policies entered in this section of the work.

The section of the report which deals with mortality by build is to be published at an early date. It is planned to follow, in general, the lines of the M. A. Mortality Investigation, Volume II. The investigation of mortality under lives with a family history of tuberculosis depends greatly on the build of the applicant. Consequently, this section of the report has been deferred until after the mortality standards for ordinary lives have been determined. It is planned to compare the mortality experienced in the family history groups with the mortality experienced among the general groups of insured lives, according to the same deviations from average build.

CHAPTER VII.

MORTALITY OF ANNUITANTS GREAT BRITAIN

GOVERNMENT ANNUITANTS—No. 1, 1829

IN 1808 the National Debt Commissioners of Great Britain commenced to grant life annuities, and they applied to Mr. William Morgan, the Actuary of the Old Equitable, to compute tables for this purpose; these tables he based on the Northampton Table. In 1819 Mr. John Finlaison, who had been appointed Government Actuary, pointed out that the prices charged for annuities were too small, and that the Government was losing heavily. Speculators had discovered the excellent values in such annuities, and had been buying annuities on carefully selected lives, sometimes also protecting themselves against loss by life insurance on the same lives. The loss to the government in the eleven years from 1808 amounted to nearly \$10,000,000, yet a period of about ten more years elapsed before the error was corrected. Mr. Finlaison had in the meantime been taking steps to ascertain the rate of mortality amongst annuitants. He took for his materials the nominees of

1. Various English and Irish Tontines from 1693 to 1789;
2. Life annuities issued at the Exchequer from 1745 to 1779, and
3. Life annuities chargeable on Sinking Fund from 1808 (as above).

In each case the facts were carefully tabulated, and in extracting them duplicate lives were eliminated. The ages last birthday were given in each case, and on an average the lives were assumed to be half a year older. Those stated to be 30 were assumed on the average to be $30\frac{1}{2}$, and the sum of

the numbers at 29 and 30 was assumed to give double the number exposed to risk at age 30.

Mr. Finlaison drew up twenty-one tables, two of which he adjusted by the formula:

$$\text{Graduated } p_x = \frac{1}{25} [p'_{x-4} + 2p'_{x-3} + 3p'_{x-2} + 4p'_{x-1} + 5p'_x + 4p'_{x+1} + 3p'_{x+2} + 2p'_{x+3} + p'_{x+4}].$$

This was called Finlaison's Method of Graduation.

There is reason to believe, however, that he subjected his data to some modification before applying his method of adjustment. One of the most important features of this investigation was the marked superiority of female life over male. Values of annuities were deduced from his results and employed by the government.

GOVERNMENT ANNUITANTS—No. 2, 1860

This table was formed by Mr. A. G. Finlaison (son of Mr. John Finlaison), the Actuary to the National Debt Commissioners, the materials used being the nominees of:

1. The Irish Tontines of 1773-1777.....	3,384 persons
2. The English Tontines of 1789.....	8,171 persons
3. The annuities granted by National Debt Commissioners 1808-1850.....	16,812 persons
	<u>28,367</u>

Of these 28,367 persons, 11,829 were males and 16,538 females; 19,434 had died during the period over which the observations extended; while amongst the males were 675 carefully selected lives nominated by speculators who had purchased the annuities. This practice has been stopped by law. The Tables were never extensively used, and are now of but little interest.

GOVERNMENT ANNUITANTS—No. 3, 1883

Formed by Mr. A. J. Finlaison (grandson of Mr. John Finlaison), Actuary to the National Debt Commissioners, exclusively from the records of the annuities issued from the National Debt Office during the period from 1808 to 1875.

The number included was 30,788, of whom 10,929 were

males and 19,859 females. The 675 selected nominees previously referred to were also included amongst the males. 22,998 had died during the period and 7,790 were alive at the close of the observations. The tables published give the following particulars for each age:

1. Number of entrants.
2. Number alive at close of observations.
3. Deaths, and
4. Exposed to risk.

In the case of those alive at the close of the observations—1875—they were each kept under observation until their birthdays that year in order to avoid fractional years of age. The following is an extract from the table of elementary facts, age at purchase 60:

NUMBER OF ENTRANTS 475

Age	Alive at Close of Observations	Deaths	Number Exposed to Risk
60	—	4	316.7
61	4	16	467
62	6	11	445
63	7	14	427
etc.	etc.	etc.	etc.

The ages at purchase of the annuities were stated as those last birthday, and it was found that, on an average, the annuitants were actually four months older. Accordingly, the true average age of the 475 entrants above mentioned was $60\frac{1}{3}$; and up to the age of 61 these were exposed to risk for two-thirds of a year. Mr. Finlaison, to get the mortality for the first year, and for age 60 at entry, divided the four deaths above mentioned by two-thirds of 475, i. e., 316.7, this being equivalent to an assumption that the rate of mortality during the four months from the birthday to purchase was the same as for the eight months immediately following. A similar assumption was of course made for other ages. For the second year's mortality at age 61, the four who died, and the four alive at close of observations must be deducted from the entrants, thus giving 467 exposed to risk at age 61, among whom 16 deaths occurred, which showed a rate of

mortality of .0343; similarly for subsequent years. The formula for deducing the exposed to risk would therefore be:

$$\begin{aligned} \text{or } E_{[x]+n} &= E_{[x]+n-1} - d_{[x]+n-1} - e_{[x]+n} \\ &= E_{[x]} - \sum_1^n e_{[x]+n} - \sum_1^n d_{[x]+n-1}, \end{aligned}$$

and for the general mortality table where the entrants at all ages were combined:

$$\begin{aligned} E_x &= \sum n_{x-1} + \frac{2}{8}n_x - \sum e_x - \sum d_{x-1} \\ &= E_{x-1} + \frac{1}{8}n_{x-1} + \frac{2}{8}n_x - e_x - d_{x-1}, \end{aligned}$$

where n_x represents the new entrants.

An analysis of the facts was made to ascertain the effect of the selection which the annuitants exercise, and it was found to be unimportant after four years. The entrants at each age were accordingly traced separately for the first four years of their existence (for ages 40 to 80) and thereafter combined with the entrants at all previous ages, thus giving Select Mortality Tables.

The treatment was equivalent to a "life-year" method, the annuitants being traced on the average from birthday to birthday. It has been suggested that the method of graduation by Woolhouse's formula distorted the table, and resulted in higher mortality rates than the facts justified; but this is now only of academic interest.

GOVERNMENT ANNUITANTS—No. 4, 1910*

The Actuary to the National Debt Commissioners of Great Britain, Mr. J. Blakey, submitted a report dated 12th October, 1910, in which he gave particulars of an investigation into the mortality of government annuitants from 1st January, 1875, to 31st December, 1903, including new annuities issued during that time and annuitants living on the anniversary of purchase date in 1875. Where two or more annuities were granted, only the experience of the first was

* J. I. A., Vol. XLVII, p. 66.

included. The following table gives a summary of the data:

GOVERNMENT ANNUITY EXPERIENCE, 1875-1903. SUMMARY OF DATA

	Males	Females	Total
Number of lives under observation.....	5,504	13,863	19,367
Number of years of risk.....	57,652	163,378	221,030
Number of deaths.....	4,168	9,333	13,501
Number of lives existing at the close of the observation.....	1,336	4,530	5,866

The investigation proceeded on the policy year plan, tracing the mortality from the date of admission to the next anniversary, and so forth, from anniversary to anniversary of the purchase of the annuity, following the method of the British Offices' Experience. The ages at entry were taken as the nearest age at date of purchase. An investigation as to the effect of using this showed that the assumption had the effect of understating the tabular ages by about 20 days in the case of male lives and about 15 days in the case of female lives—a trivial difference.

A considerable number of annuities were purchased at the National Debt Office under wills, etc., where the nominees did not exercise any option, but in most cases the nominees themselves were the purchasers.

The new tables were based on the assumption that the effects of selection are practically exhausted after the expiration of four years and the rates of mortality were, therefore, merged into the aggregate table after that period. The tables were graduated by Makeham's method, satisfactorily in the case of aggregate tables, but modified for select tables in a manner similar to that employed in the graduation of the British Offices' Annuitants' Tables.

GOVERNMENT LIFE ANNUITANTS, 1900 TO 1920*

This investigation covered the National Debt annuities and the Savings Bank annuities issued by the Government of Great Britain. Contracts were studied instead of lives after

* Mortality Experience of Government Life Annuitants, 1900-20. His Majesty's Stationery Office, London, 1924. See also J. I. A., Vol. LV, p. 144 and p. 160.

a special investigation showed that the differences in the experiences were immaterial.

Twenty-three per cent of the National Debt annuities and thirteen per cent of the Savings Bank annuities were on joint-life and survivor contracts. Special investigation showed that the experience on these was essentially the same as on single life annuities.

The experience was taken from the policy anniversary in 1900 to that in 1920. The nearest age was taken. The exposure was:

	ENTRANTS		DEATHS	
	National Debt	Savings Bank	National Debt	Savings Bank
Males.....	6,349	16,881	3,740	8,486
Females.....	14,285	43,266	7,874	18,583

The expected deaths were calculated on the basis of the Government Life Annuitants' experience, 1875-1904, with the following results:

	Males	Females
National Debt.....	94%	84%
Savings Bank.....	86	80
All.....	88	81

There were evidences of selection at least during the first five policy years, but it decreases rapidly and is of little monetary importance after the first year except at the highest ages. The first year mortality rates were found to be approximately 63% of the ultimate for males and 59% for females. However, in the tables finally adopted, a one year period of selection was employed with the rates of mortality for both males and females uniformly 60% of the ultimate. In order to allow more accurately for the selection persisting more than one year a small addition, dependent on age but not on sex or interest rate, was recommended to be added to the annuity value at ages 61 to 80.

These results, based on actual experience, would increase the cost of annuities in varying proportions rising to a maximum of about 7%, but it was recommended that annuity premiums be further increased 4% in the case of females and 3% in the case of males to allow for future improvement in mortality. These additions were derived after considerable investigation and extrapolation taking into consideration not only the 1875-1904 experience but also an analysis of the

present experience in the three periods, 1900–1907, 1907–1914, and 1914–1920. This analysis disclosed a steadily declining mortality rate, although the rate of decline was checked in the third period, probably as a result of war privations. The methods of extrapolation employed were similar to those followed in the preparation of the British Offices' Life Annuitants' Experience, 1900–20* with two important differences. Firstly, the minimum rates toward which it was thought mortality would tend in the future were not assumed but were fixed by the analysis into periods above referred to. Secondly, it was assumed that the future rate of mortality to be experienced at any attained age is a variable dependent not on the year of entry but upon the year passed through.

The graduation of the male rates was by two overlapping curves, a Makeham curve based on the data from 44 to 70, and a single Gompertz curve based on the data from 64 to 89. In the former $\log c = .052$ and in the latter $\log c = .04$. These curves crossed each other at about age 65, and from ages 61 to 69 a blend of the overlapping values of $\text{colog } p_x$ was obtained by multiplying one set of values by the function

$$\frac{1}{2}(1 + \cos \frac{n\pi}{10})$$

in which n was given successive integral values from 1 to 9, while the other set was multiplied by these factors reversed and the sum of the two sets of products taken. The female rates were correspondingly graduated by a Makeham curve based on the data from 40 to 67 for which $\log c = .046$ and a Gompertz curve based on the data from 65 to 86 with the same value for $\log c$. These curves crossed between ages 66 and 67, and the overlapping values of $\text{colog } p_x$ for four years on each side of 67 were blended in the same way as in the male table. Values above age 90 were specially adjusted with a limiting age of 103 for males and 105 for females.

To obtain annuity values for use on Savings Bank Annuities taken as low as age 5, the female English Life Table No. 8 was used as the basis for females. The actual rates from 5 to 44 were obtained by a third difference curve, the

* See page 113.

constants being derived from q_5 , q_{25} , q_{45} and q_{40} , the latter two values being taken from the figures already obtained from this experience. Constant values of q were used from 5 to 10 to avoid a decreasing mortality rate. A similar procedure was used in the case of males.

BRITISH OFFICES' LIFE ANNUITY TABLES, 1893*

This investigation related entirely to the experience of annuitants under contracts purchased from 43 life assurance and annuity institutions, including the British annuity experience of three American companies. Particulars of each annuity current at the commencement of the observations in 1863 and of those subsequently granted up to 1893 were supplied upon cards, a separate card being written for each annuity. The total of such cards was 9,700 for male and 24,300 for female lives, reduced by elimination of duplicates, etc., to 8,641 and 23,056, respectively. In the select male section 4,214 were existing; there had been 4,427 deaths and 67,250 years of exposure. The corresponding figures in the female section were 11,956, 11,100, and 207,324. The tables were published in the form of select tables, the entrants at each age having been traced separately during the first five years, the rates of mortality thereafter being those of a modified ultimate experience formed by omitting the observations of the select period.

It had long been felt that the government tables were not applicable to the annuitants' experience of life assurance companies, the rates of mortality in the latter being lighter than the rates in the government tables. The fact that the observations for the government experience extended from 1808 to 1875 was in itself a deterrent influence, since it was believed that mortality had continued to improve. Moreover, the selection exercised against insurance companies was probably more intelligent, because government annuities were often bought for small amounts under the provisions of a will to pension old servants. Still further, life office annuitants were doubtless of a better social class than the govern-

* See "An Account of the Principles and Methods adopted in the Compilation of the Data, etc.," "Combined Experience of Life Annuitants, etc.," and "British Offices' Life Annuity Tables." C. & E. Layton, London, 1903.

ment nominees. In the case of females, forming the great proportion of the annuitants, the British Offices' annuity values approximate to those of the government at an age one year younger. In the case of the males at the older ages the difference is about half a year. The difference in reserves in case of a "model office" is roughly $3\frac{1}{2}\%$.

The notation was:

Male Annuitants' Table aggregate, O^{am} Female O^{af}
 " " " select, O^{asm} " O^{af1}

The graduation was performed by Mr. George F. Hardy. A preliminary graduation of the aggregate table excluding the first five years, showed that the male mortality from age 40 upward could be well represented by Makeham's formula. The female mortality could only be thus represented at ages above 65, and a supplementary curve had to be introduced below that age. The constants for these curves were obtained by equating to zero the sum of the deviations of the adjusted and unadjusted deaths, and also the sum of the accumulated deviations.

Three trial graduations of the male table were made, for assumed values of $\log c$ equal to .040, .038, and .036; and the final value adopted was .038 as giving on the whole the most practical results, while following closely the ungraduated facts. An arbitrary adjustment was made in one of the constants, so as to make the male annuitants' mortality at the younger ages nearly the same as the mortality amongst assured lives. The facts at these ages were meager. But for this adjustment the annuity values of the annuitants would have been appreciably smaller (i. e., would have been based upon higher mortality) than those of assured lives.

The graduation of the select tables for the first five years was made by introducing into Makeham's formula a function dependent upon duration after entry. (See Actuarial Study No. 4 and T. A. S. A., Vol. XXI, p. 548.)

BRITISH OFFICES' ANNUITANTS, 1900-1920*

This investigation by the Institute of Actuaries and the Faculty of Actuaries jointly, covered the experience amongst

* Mortality of Life Annuitants, 1900-1920. C. & E. Layton, London, 1923. See also J. I. A., Vol. LIV, p. 43 and Vol. LV, p. 160.

annuitants under Single Premium Immediate Life Annuities on lives resident in Great Britain at the time the annuities were granted. It included exposure during the period 1900 to 1920 on all years of issue up to and including 1919. In order that a study might be made of the trend of mortality rates during this period, in addition to making a study for the whole period, the experience was also taken out according to the three periods: I, 1900-1907; II, 1907-1914; III, 1914-1920. The investigation was made according to policy anniversary throughout. Forty-nine offices, including five colonial and one American, contributed their experience. The number of annuities included was:

	Period I Up to 1907	Period II 1907-1914	Period III 1914-1920	Total
Females.....	28,393	14,331	9,168	51,892
Males.....	10,607	5,547	3,792	19,946

Aggregate tables were constructed for each period excluding the first five years after issue and a select table for the first five years with ages grouped as under 40, 40-49, then by quinquennial age groups to 85 and then above 85. The age at last birthday was used and a curtate duration for deaths, exact duration for existing and curtate for withdrawals with half a year in the year of withdrawal. The age at last birthday means that the exact age is half a year older than recorded so allowance must be made for this in comparison with the British Offices' Table, 1893. Such allowance is made in the comparisons given on page 116 and in the tables and graphs at the end of this book.

The mortality on females was far lighter than in the British Offices' Table, 1893. It has been progressively lighter since 1900 in the ultimate section and to a less extent in the select section. There was a tendency for annuities to be taken out at older ages recently, particularly in Period III. When first studied it appeared as if selection against the companies might have lasted five years or longer, but on further study this appeared to be spurious selection, due partly to the decreasing rate of mortality by periods and partly to class selection, that is, the change in the type of annuitant from a class with poor mortality to one with better mortality. After especially investigating Period I, a one year

select period was decided upon for the estimate of future mortality with a first year mortality rate of a constant 63% of the ultimate, thus necessitating use of only the ultimate table for valuation.

There was a great improvement over the British Offices', 1893 male annuitants but less improvement than in the case of females. There was little difference in the mortality between periods, that in the second being a little the heaviest. Period I was less satisfactory for studying. Evidently spurious selection was involved. The average age of entrants at most important age groups was appreciably less than the average age of the exposed to risk in the ultimate table. Fewer annuities were taken out in Period III. Other possible explanations were discussed—the drop of security prices after the South African war, the greater number of persons covered by pension schemes, the fact that the British Office Annuitants' table had been available in 1903 and resulted in high premium rates thereafter, the fact that many annuitants reassumed work during the war period, and the possibility of a different class selection at different ages such as might arise from a comparatively higher rate being charged at some ages as compared with others.

The first year mortality rate was again taken as 63% of the ultimate for the estimate of future mortality as in the case of the females.

In constructing the tables, Mr. W. Palin Elderton, chairman of the Investigating Committee, introduced a new principle by endeavoring to forecast the mortality likely to be experienced in the future on the assumption that the improvement in mortality shown by the 1900–1920 experience over the corresponding experience of 1863–1893 would continue into the future. The fundamental basis of this forecast was the theory that the mortality to be expected in the future is, apart from temporary selection, a function of the year of entry. To give effect to this theory in the calculations it was necessary to relate the results of the two investigations 1863–93 and 1900–20 to approximate years of entry. For each attained age an average duration, based on the earlier experience 1863–93, was assumed. The experiences of 1863–93 and 1900–20 were assumed to apply respectively to the

years 1880 and 1910, whence by deduction of the assumed average duration the approximate average years of entry for the various attained ages were obtained. Taking, for example, the age group 80-84, for which the average duration was taken as 16 years, the average years of entry into the two experiences 1863-93 and 1900-20 were respectively 1864 and 1894, a period of 30 years. To obtain the future mortality rate applicable to entrants of, say, 1924 on attaining ages 80-84 it was only necessary to extrapolate for a further period of 30 years from the rates already known and applicable, as pointed out, to the years of entry 1864 and 1894. A logarithmic process of extrapolation was employed, with the further condition that the lowest level to which mortality would tend in the future would be 63% of the 1900-20 rates, i. e., the percentage that the rates of mortality for the first year bore to the aggregate excluding that year in the preliminary investigation.

The effect of the forecast upon annuity values may be seen by the following table, the proposed values being those appropriate to entrants in 1925.

ANNUITY VALUES 4%

Age	FEMALES			MALES		
	1863-93	1900-20 Old Basis	1900-20 Proposed	1863-93	1900-20 Old Basis	1900-20 Proposed
50	13.9	14.9	15.4	13.0	13.3	14.0
60	11.2	11.9	12.4	10.0	10.2	10.8
70	7.9	8.5	8.9	7.0	7.3	7.6
80	4.8	5.3	5.4	4.4	4.6	4.6

The above annuity values are based on the forecast of future mortality without other graduation than that inherent in the process of operating upon quinquennial groups of ages. However, once the ungraduated results had been compared, as above, with the 1863-93 experience, it was felt that some graduation would be desirable.

The female table was graduated by using two Makeham formulæ:

- (1) Covering the age range 40 to 85 with the following constants,

$$\begin{aligned}\log c &= .052964 \\ \log B &= \bar{3}.85696 \\ A &= .00288\end{aligned}$$

- (2) Covering the age range 80 to 104 with the following constants,

$$\begin{aligned}\log c &= .029967 \\ \log B &= \bar{2}.828366 \\ A &= -.03144\end{aligned}$$

The male table was graduated by using two Gompertz formulæ:

- (1) Covering the age range 50–70 with the following constants,

$$\begin{aligned}\log c &= .03370 \\ \log B &= \bar{3}.97260\end{aligned}$$

- (2) Covering the age range 81 and above with the following constants,

$$\begin{aligned}\log c &= .03512 \\ \log B &= \bar{1}.06480\end{aligned}$$

For the female table the values for ages 80 to 85 and for the male table the values for ages 71 to 80 were so adjusted as to produce a smooth junction between the pairs of curves employed.

The symbols $a(f)$ and $a(m)$ are used to designate these tables.

BRITISH OFFICES' LIFE ANNUITANTS, 1921–1925*

To replace the former sporadic investigations into mortality experience the British Offices have developed machinery for the continuous collection of data. At the end of each calendar year they return to a central bureau (*a*) the number of annuitants subsisting at each age, and (*b*) the deaths during that calendar year at each age. The subsisting and deaths are given separately by durations 0, 1, 2, 3, 4, and "5 and over," so that the effect of duration can be studied in

* J. I. A., Vol. LIX, p. 387.

spite of the substitution of a "Census" method for a strict "Policy Year" method.

January 1, 1921 was taken as a starting point and the offices returned as of that date the number subsisting at each age. This account, based on the returns for calendar years 1921-1925, is the first formal report on the results of the continuous investigation.

All returns are given or adjusted to the "nearest age." To the mean of the business in force at nearest age x at the beginning and end of each year are added one-half of the year's deaths at nearest age x . The resulting figures are then regarded as the exposed to risk for age $x - \frac{1}{2}$. The ratio of deaths at nearest age x (being age $x - \frac{1}{2}$ on the average at the beginning of the year of life in which they die) to such exposed to risk, gives $q_{x-\frac{1}{2}}$, compared with the age at last birthday, that is the age $x + \frac{1}{2}$, in the 1900-1920 experience. Comparison with the 1900-1920 experience was made in quinquennial age groups including the same exact ages. Mean rates of mortality were applied to obtain the expected deaths so as to be on a comparable basis.

In the case of the female experience the new mortality was lighter than for 1900-1920. The mortality excluding the first year was slightly lighter than that excluding the first five years, but the difference was so small that however "selection" may be interpreted it will be seen that it was easier to run into the ultimate table after the first year than after any other period. The rates of mortality at durations 1, 2, 3 and 4 did not join either the ultimate "five and over" or the ultimate "one and over." There was thus the same phenomenon as in the 1900-1920 experience. Evidence was produced that this difference was not due to selection in the sense that the mortality at duration 4 was different from the mortality at duration 1 for the same attained age.

For the five years 1921-1925 the actual deaths (excluding duration 0) were in the case of females 107.1 per cent and in the case of males 109 per cent of those expected by the forecast ultimate tables $a(f)$ and $a(m)$. These ratios may be considered to afford a margin of safety in current valuations, but if the improvement, evident from the present and the previous investigations, continues so that in the course of

another two or three quinquennia these ratios drop to par, it will be obvious that, looking into what will then be the immediate future, the $a(f)$ and $a(m)$ tables, if unadjusted, will form a low standard for valuation purposes and may prove actually dangerous for the granting of new annuities. Actuaries should, therefore, be warned to make allowance for these changing conditions. Mr. Fraser suggested the simple rule of setting back ages one-tenth of a year for each calendar year after 1925. (See J.I.A., Vol. LV, p. 160.)

Special attention should be drawn to the low rates of mortality for durations 1-4, which may be taken as constant during those durations. No reason is seen in the new experience to go back on the assumption made after the 1900-1920 experience that the most serviceable explanation is that the persons who bought annuities many years ago are showing a different mortality from those buying annuities today, and that those buying annuities today at the early ages will exhibit when they attain age 70-74 (for instance), a mortality like that for durations 1-4 now shown in this experience against that age group; this means a single table for all practical work and much simplification.

In the case of male lives there was a strange break in the mortality as compared with the past at age 70. Up to age 70 the new mortality was lighter and after 70 it was heavier than the old. This brought doubt about statistical accuracy, but no errors were found. Whatever the interpretation of the experience may be, the type of selection which was accepted as an explanation at the time of the 1863-93 experience should not be held to apply to the present experience.

As in the 1900-1920 experience, it was difficult to interpret the facts regarding male annuitants. The mortality has decreased little, if at all. The rates of mortality for durations 1-4 were approximately constant and yet were less than those for the continued experience at all longer durations. It is possible that the War may have left an influence in the type of annuitant or by increasing temporarily the mortality of pensioners; or we may have had a more or less permanent feature due to annuities being bought for males in an entirely different set of circumstances from those which prevailed in the case for females. In the meantime the conclusion was

reached that no useful purpose could be served by any alteration of the principles assumed in reaching the forecast mortality from the 1900-1920 experience.

RUTHERFORD'S ANNUITY TABLES*

The 1927 Amendment to the Insurance Act of the Dominion of Canada required that life annuities be valued on tables of mortality deduced from the *actual* mortality experience covering the period from 1900 to 1920 published in 1924 on behalf of the Institute of Actuaries and the Faculty of Actuaries in Scotland. The only published tables covered the ultimate experience with estimates for the first five policy years for ages at entry 50, 60, 70, and 80. Very little information has been given as to the construction of the ultimate table and the construction of the select portion was admittedly arbitrary. Mr. C. D. Rutherford† decided that considerable adjustment would be necessary to comply with the requirements of the Act and felt it desirable to return to the original data. These consisted of exposed to risk and deaths for a period of five policy years after issue tabulated in age groups by age at issue for each policy year separately, together with the experience after five years giving the exposed to risk tabulated by attained age for each age from 40 to 105. All figures were tabulated according to age last birthday. The age groups for the select portion were: under 40, 40-49, 50-54 and thereafter proceeding by intervals of five years to age 84 with a final group including age 85 and over. These ages, it will be recollected, are ages at entry so that the last group may be accepted for all practical purposes as giving entry ages 85 to 89.

As there had been considerable doubt as to the true length of the select period, the weight of opinion apparently indicating one year, and as this table was intended for use in valuation, it was decided to construct an aggregate table after excluding the data for the first policy year. The data for the first four policy years had been shown in grouped

* Rutherford's Annuity Tables. By C. D. Rutherford and G. W. Bourke. Published for the Sun Life Assurance Company of Canada by C. R. Corneil, Ltd., Montreal, 1929.

† T. A. S. A., Vol. XXVIII, p. 54.

ages at entry and for each policy year separately. The exposed to risk and deaths for each policy year were broken up into individual ages at entry by King's census method with special treatment at the younger ages. A slight adjustment was made where necessary so that the original totals were reproduced on combining the individual ages. The data were then combined according to attained age. After it was found that a Makeham graduation was not applicable, the data for age 40 and above were graduated by the Henderson-Whittaker formula B, thus taking into account the exposed to risk at the different ages. As the mortality rates thus determined were according to age at last birthday, interpolation by a third difference formula was made to obtain rates at the exact age on the assumption that the age last birthday differs from the exact age by half a year. This was on the conservative side as the original data indicated a difference of four and one-half months. The rates thus obtained showed diminishing mortality up to age 50 in the male table and up to age 44 in the female table. Although this peculiarity was undoubtedly characteristic of the data, it was eliminated for the sake of simplicity because the effect on the monetary values was relatively small. A slight adjustment was also made at the end of the tables.

In order to extend the male table backwards to age 20 it was decided after considering various possibilities to use the English Life Table No. 8 at ages 24 and under down to age 0, and the $C^{(6)}$ table for ages 34 to 49, with an arbitrary value of .00425 at age 29 equal to the minimum rate shown at any age under the $C^{(6)}$ table. The remaining portions of the male table were completed by third difference osculatory interpolation merging with the graduated table at age 54.

For the female table the English Life Table No. 8 was assumed for age 26 and under down to age 0 and the rate of mortality according to the $C^{(6)}$ was assumed for ages 31 to 36. The rates of mortality for ages 46 and over were taken from the graduated table. The value for age 41 was interpolated based on the nearest six quinquennial age values and the rates for the remaining ages were then completed by third difference osculatory interpolation, thus securing a smooth junction of the various portions of the table. A slight ad-

justment was also made at the end of the tables, making both tables terminate at age 104. It was suggested that if allowance was desired for the first year mortality it could be sufficiently well represented as being uniformly 59% of the values shown in the final table.

This table has been approved by the superintendent of insurance in Canada as complying with the requirements of the 1927 law.

CHAPTER VIII.

MORTALITY OF ANNUITANTS AMERICA

McCLINTOCK ANNUITY TABLES

IN 1896 a paper appeared in the Transactions of the Actuarial Society of America* giving particulars of the annuity experience of fifteen American companies collected by Mr. Rufus W. Weeks, Actuary of the New York Life. At first all the annuities issued prior to 1890 were used, and this material was afterwards carried forward to 1892 with additional new issues in the meantime. A feature of the experience was the preponderance of foreign business; about three fourths of the lives were European and one fourth American. The experience was based on lives; duplicates were eliminated. The age was taken at the birthday nearest to the date of first exposure; observation began on entry into any one of the companies and continued until death or the anniversary in 1892 when the experience closed. The exposure of a few deferred annuities in the experience was taken from the date at which the first annuity payment became due—at which date there is usually an option to take a cash settlement. Men and women were investigated separately, and complete select data are given for each age at entry. The number of lives included in the experience was 4,365 men, and 4,821 women.

Mr. McClintock took these data and derived aggregate mortality tables† for male and female annuitants respectively. They were graduated by Makeham's Formula and have a common value for the constant c —namely: $\log c = .04$. Mr. McClintock stated that the tables differ somewhat from

* T. A. S. A., Vol. IV, p. 275.

† T. A. S. A., Vol. VI, pp. 13 and 137.

those which might be formed from a mere adjustment of Mr. Weeks' data but that he had endeavored to admit only such differences as seem reasonable. His female table, for example, shows a somewhat higher mortality at ages under 70, because he considered that the experience at those ages had been materially affected by the large proportion of recent entries. With reference to the male table Mr. McClintock stated that he thought it best, if any departure from Mr. Weeks' experience should be permitted, that such departure should be on the safe side—that is to say, the mortality in Mr. McClintock's adjustment would be lighter than that actually shown by a direct application of the experience.

This table was adopted by the State of New York and by several other states as the standard for annuity valuations.

AMERICAN ANNUITANTS' MORTALITY TABLES

This investigation was made by Dr. Arthur Hunter* covering immediate annuities on single lives resident in the United States issued by twenty American life insurance companies up to and including the year 1917. It was estimated that this included 95% of such annuities issued in the United States. Annuities providing for the return of the balance of the consideration at death or for some other substantial payment at death were excluded, as were also annuities guaranteed for a term of years and all cases in which an annuity became payable at the death of an insured through life income policies, trust agreements, or other forms of supplementary contracts. Temporary, deferred, joint and survivorship annuities, and also annuities issued in exchange for dividends were excluded. The experience was carried to the annuity anniversary in 1918. The age was taken at nearest integral age at purchase as this is approximately correct where companies base the purchase price on completed quarters of an age. All duplicates were eliminated. The investigation was made both by lives and by amounts of annuity. The extent of the data may be illustrated by the following:

* T. A. S. A., Vol. XXI, p. 157.

	Number of Entrants	Deaths	Annual Amount of Annuities	Annuities Released by Death
Men.....	5,301	2,463	\$2,907,120	\$1,456,220
Women.....	8,201	3,211	3,447,470	1,326,910

The average age at entry among men was 62.6 and among women 63.6. The average duration of annuities by amounts was 8.7 years for men and 8.9 years for women. It was 8.9 years for men and 9.3 years for women on the basis of lives after the duplicate annuities for the same age at entry had been combined.

Dr. Hunter had previously investigated the life annuity experience in the United States and Canada in 1905 and again in 1911, but the exposure was so small that those investigations are of interest now principally in an historic sense.

The new experience showed that neither the McClintock nor the British Offices' 1893 tables were sufficiently stringent for the calculation of either premiums or reserves. A new table was therefore constructed. This was based on lives instead of amounts of annuity because the volume of statistics was not sufficient to prevent substantial fluctuations by amounts, especially in the early years, and because the difference as a whole between the mortality by lives and amounts was not large and, where there was a difference, experience by lives was on the side of conservatism. The lower mortality exhibited as compared with the British Offices' Annuitants' Select Table is illustrated by the following ratios of actual to expected mortality by that table:

NEW EXPERIENCE—RATIO TO BRITISH OFFICES' ANNUITANTS (SELECT), 1893

Year	LIVES		AMOUNTS	
	Males	Females	Males	Females
1- 5	87%	82%	100%	81%
6-10	86	82	86	87
11 and up	98	86	103	88
Total	90%	83%	96%	85%

Considerable difficulties were met in the graduation in obtaining formulæ applicable to both males and females and to the experience at both young and old ages. After many recognized modes of procedure had been found to be inappli-

cable, experimental methods were adopted and graduation effected by Makeham's formula, first on the ultimate data, then on the select male data and finally on the select female data, using $\log c = .035$ throughout and making the ratio of B for the female table to B for the male table constant for each annuity year and for the ultimate tables, thus preserving the principle of uniform seniority. For convenience tables were prepared in the form of select rates for the first five annuity years with ultimate rates for the sixth and succeeding years. This selection was made because the experience was not sufficiently large to determine accurately whether or not a longer period should be assumed. Mortality rates on women were adjusted above age 100 so as to terminate the tables at age 106. Various tests of the graduation were made comparing the actual with the expected deaths, but even greater evidence of the success of the graduation was shown in the comparison of typical values of life annuities calculated from the graduated tables with those resulting from the ungraduated data.

The following table illustrates the ratios of the values at 4% interest of life annuities of \$1.00 per annum on the American Annuitants' Table to those on McClintock's and on the British Offices' Select Tables:

PERCENTAGES OF AMERICAN ANNUITANTS' VALUES OF LIFE ANNUITIES
TO THOSE DERIVED FROM OTHER TABLES

Ages	MALES		FEMALES	
	McClintock	O[am]	McClintock	O[af]
20	106	104	100	105
40	104	103	100	105
60	106	104	103	104
80	124	112	121	120

Although the extent of the data was limited, the resulting experience was consistent with the results shown under similar recent experiences and showed lighter mortality than any previous existing table. The new tables were, therefore, the best available at that time for rates and reserves on annuity business.

COMBINED ANNUITY MORTALITY TABLE

When various companies became interested in deferred annuities for Group pensions and similar benefits issued in conjunction with Group insurance, no suitable mortality table was available for premium rates and reserves. A committee of actuaries*, at the suggestion of the Superintendent of Insurance of the State of New York, studied the problem. Practically all the experience in the American Annuitants' table was at the older ages, where it was considered reasonable. Mr. Cammack's Clerical Mortality table appeared most appropriate to reflect the general mortality at the younger ages. An empirical table was therefore constructed on this foundation up to age 50 and approaching the American Annuitants' table at the higher ages. The graduation for ages 60 and over was made so that the resulting immediate annuity value at 4% for age 60 would be substantially equal to that of the American Annuitants' Ultimate table. The changes from the Select to the Ultimate values were made gradually from age 60 to age 70. The gap between 50 and 60 was bridged by interpolation. After an attempted Makeham graduation had been unsuccessful, the following formula was used:

$$\text{Colog } p_x = a_x + 10^{.035(x-117)}$$

Where $a_x = .000487$ for ages 10 to 20 inclusive

$$a_x = .000487 \frac{(35-x)^2 (2x-25)}{3375} \text{ for ages 21 to}$$

34 inclusive, and

$$a_x = 0 \text{ for ages 35 and over.}$$

It will be observed that this really resulted in a Gompertz graduation for ages 35 and over, a Makeham graduation for ages 20 and under, and a special graduation for the intermediate ages. The resulting probabilities of death were used for male lives. It was found that a practical and advantageous assumption was that the same table should be used for females after decreasing the age four years. Thus the

*T. A. S. A., Vol. XXIX, Page 120.

probability of death for a female aged 25 was taken as the same as for a male aged 21.

The Combined Annuity table was prepared originally as a valuation basis for Group Annuities but subsequent investigation of individual annuity experiences has indicated that it could also be safely adopted as a valuation standard for individual Annuity contracts. This table at 4% interest was made the minimum basis of valuation by New York State for all annuity contracts, both individual and Group, issued on and after January 1, 1930.

UNITED STATES ANNUITIES*

This was based on the experience of twenty-five American and six Canadian companies. At first all duplicate annuities issued at the same age nearest birthday on the same life were eliminated so as to reduce the experience practically to selection units. There remained the following cards:

	UNITED STATES		CANADA	
Males	8,686	annuities on 5,340 lives	458	annuities on 341 lives
Females	12,753	" " 8,895 "	346	" " 292 "
	21,439	" " 14,235 "	804	" " 633 "

The first five years were separated from the subsequent years and duplicate annuities on the same life were eliminated from the latter group. The comparison of the actual to the expected by the American Annuitants' table was:

TOTAL EXPERIENCE—UNITED STATES		
	Male	Female
First 5 Years.....	93%	91%
Subsequent.....	104%	105%
Total.....	99.5%	100.4%
Both Sexes.....	100.0%	

EXPERIENCE 1918-1927—UNITED STATES		
	Male	Female
First 5 Years.....	85%	93%
Subsequent.....	100%	105%
Total.....	94.3%	101.6%
Both Sexes.....	98.7%	

The Canadian experience was very scanty. The ratios in the total experience were 89% on males and 78% on females.

* T. A. S. A., Vol. XXIX, p. 297; Vol. XXX, p. 237.

After various investigations it was decided to confine investigation to the experience since 1918. A table of ungraduated rates of mortality based on the data for the years 1902-1927 was prepared. The expected deaths from 1918-1927 were then calculated on the basis of this table and the actual compared with the expected. The data for each sex were divided into three groups by duration, consisting of the experience of the first five contract years, of the 6th to 10th years, and of the 11th and subsequent years, and each of these groups was again subdivided by attained ages, the youngest group covering ages up to 67, the middle from 68 to 82 inclusive and the oldest group ages 83 and over, thus making 9 subdivisions in all.

The results obtained for the two sexes differed radically. Among male lives the actual deaths were in every subdivision lower than the expected, the general average being 93%, which reflects fairly well the ratios in the nine subdivisions analyzed, thus indicating a very significant and general trend for improvement in mortality in the later period as compared to the earlier. On female lives, on the other hand, of the nine sub-groups five were in one direction and four in the other, thus indicating not a decisive and uniform trend but a variation by attained age.

A comparison was then made of the actual deaths during this period with the expected according to the American Annuity Table. The result of this comparison indicates for the female experience an agreement with the American Annuity Table which is probably about as close as in the original data on which that table was constructed. On the male lives, on the other hand, the actual deaths aggregated about 90% of the expected, the percentage being fairly uniform for different attained ages, thus indicating roughly that a deduction of one year from the attained age under the American Annuity Table would approximately represent the more recent experience.

All of these calculations included the exposures and deaths on additional annuities purchased at older ages on lives already covered. In the tables which were given in the preliminary report these exposures and deaths had been excluded after the expiration of five years of duration. A com-

parison of the aggregate results in the male experience for 6th and subsequent years shows a ratio of actual to expected of 92% here as compared with 100% by lives. Inclusion of the duplicates made practically no difference in the experience among female lives.

The persistence of selection was investigated by calculating an ungraduated aggregate mortality table on the experience of the sixth and subsequent years and comparing the actual to the expected deaths on this basis for each of the first ten years of duration and for subsequent years. A noticeable amount of selection persisted up to the 10th year, but annuity values based on graduation of the experience, retaining the ten year period of selection, differed very little from those based on the experience of the sixth and subsequent years.

Although the American Annuitants' Table for female lives represents fairly the experience since 1918 on such lives and the corresponding male table, with one year deducted from the age, that on male lives, it was thought desirable to present a table not forced into conformity with Makeham's Law which might more nearly represent the experience of the nine year period from 1918 to 1927. A modification of King's method was accordingly applied to the construction of ultimate rates of mortality based on the combined experience of the 6th and subsequent years and for quinquennial ages at issue from 60 to 85 inclusive for each of the first five years of duration. These latter rates were then smoothed by duration in such a way as to make as little change as possible in the initial annuity values.

CHAPTER IX.

MORTALITY TABLES OF OTHER COUNTRIES

No attempt is made to enumerate all the mortality tables that have been compiled but it is intended to mention enough of them to indicate that there has been great activity along these lines in many countries.

Practically no mention has been made of the mortality experiences of individual insurance or annuity companies, of associations or fraternal societies, of widows' and pension funds, of certain special classes like the peerage, nor for various occupations, though there are numerous such tables published in several countries.

Mortality experience among disabled lives has not been covered whether included in tables used in America (like Hunter's Disability Table or the Manchester Unity Table or in modifications of these) or in the similar tables illustrating such experience in other countries.

It is surprising and illuminating to compile a list of published mortality tables even with the restrictions of the previous paragraphs in mind. For instance we note that Harald Westergaard attempted to construct a mortality table for Greenland despite the difficulties in the statistics there. Mention is found of a Polish table for 1927.

In "United States Life Tables 1890, 1901, 1910, and 1901-1910" various mortality tables are given accompanied by graphs and references to the original sources. Of especial interest are the population tables of many countries. These tables correspond as closely as possible to the period from 1900 to 1910. Some of the most interesting tables not often found elsewhere in English are given in Dawson's "Practical Lessons in Actuarial Science." The principal sources of information as to mortality tables in foreign languages

(other than the actuarial and census references given herewith in the footnotes) are "Versicherungsglossikon," by Alfred Manes and "Die Lehre von der Mortalität und Morbilität," by Harald Westergaard, Jena, 1901. In the latter the development of mortality tables up to that time and especially of the earliest tables is covered in great detail. Many of the earliest attempts at tables are not reviewed in this book.

AUSTRALIA

Life Insurance Experience

In his paper on Australian Mortality, Mr. Chas. H. Wickens¹ summarizes the various published mortality experiences of the Australian Mutual Provident Society, of the Friendly Societies and of Public Service. In each of the first three investigations of the Australian Mutual Provident Society the usual tables of elementary values were compiled and published, accompanied by somewhat extensive reports and numerous graphs. The most comprehensive study was the third investigation covering 1849-1903 and published in 1911 by Mr. R. Teece. Most of the experience was on male lives insured at ordinary rates but there were also data for rated-up lives, for female lives, and on endowment insurance. The data were shown only for aggregate tables and only for age groups. Mr. George King² used the original data for male lives under whole life policies at ordinary rates in illustrating his method of graduating mortality tables. He later gave more details of the data and the methods employed³. He first constructed the ultimate table, (A.M.P.)^u by reference to the British Offices' O^u Table. As an afterthought he constructed the Australian Aggregate Table, (A.M.P.)^a, in which the graduation was more smooth than that in the previous report where the graduation had been by Woolhouse's formula. He adjusted the select tables by reference to the ultimate table. He made two complete constructions of these tables in one of which the first year was treated by itself with the other four years

¹ J. I. A., Vol. LXI, p. 165.

² J. I. A., Vol. XLIII, p. 109; Vol. LII, p. 286; Vol. LIII, p. 104.

³ Mortality Tables, Aggregate, Select and Ultimate, George King (Actuarial Society of Australasia), Bloxham and Chambers, 1924, Sydney.

graded and in the second of which all five years were graded. In a third construction of select tables he used uniform factors for each column.

Population Tables

Prior to 1911 there had been numerous intermittent investigations of mortality experience among the general populations in sections of Australia¹. They include those of Prof. M. B. Pell² on the mortality from 1856–1866 in New South Wales; a similar further investigation by Prof. Pell covering 1860–1875³; an investigation by Mr. A. F. BurrIDGE⁴ based on deaths in Victoria in 1871 and the census taken in that year; a similar paper by him covering New South Wales, Victoria, and Queensland in combination during 1870–1881 with also a separate table for Victorian males⁵; a paper by Mr. T. W. Bremner in 1889 including separate tables for New South Wales, Victoria, Queensland and Tasmania⁶; a paper in 1893 by Mr. W. R. Dovey⁷ containing life tables for each sex for New South Wales and Victoria combined; life tables in 1894 by Mr. A. Duckworth⁸ in respect of sexes combined for New South Wales and Victoria combined and also for Melbourne and suburbs and for Sydney and suburbs; the first recorded officially constructed table in Australia—that of New South Wales in 1891 by Sir T. A. Coghlan⁹, Government Statistician of that State; a paper in 1901 by Messrs. E. M. Moors and W. R. Day¹⁰ on mortality in New South Wales and Victoria; and the first life table constructed for Western Australia, based on the 1901 census by Mr. Chas. H. Wickens¹¹.

The investigations mentioned, although numerous, were irregular and fragmentary, relating in no case to all the

¹ J. I. A., Vol. LXI, p. 165.

² Transactions of the Royal Society of New South Wales, Vol. I, p. 66.

³ J. I. A., Vol. XXI, p. 257.

⁴ J. I. A., Vol. XXIII, p. 309.

⁵ J. I. A., Vol. XXIV, p. 333.

⁶ Australasian Insurance and Banking Record, Vol. XIII, p. 618.

⁷ Australasian Insurance and Banking Record, Vol. XVII, p. 832.

⁸ Australasian Insurance and Banking Record, Vol. XVIII, p. 656.

⁹ Statistician's Report on Census of New South Wales for 1891, p. 146.

¹⁰ J. I. A., Vol. XXXVI, p. 151.

¹¹ Report on Census (Superintendent's), Western Australia, for 1901, p. 208.

States separately nor to a combination of all the States. In connection with the 1911 census Mr. Chas. H. Wickens constructed complete life tables on the basis of the population and deaths for the three decennia, 1881-90, 1891-1900, and 1901-10, in respect of each sex for each decennium for each State and also for Australia as a whole. Thus there were 42 separate life tables published in the Census Report¹. Monetary values including the usual commutation columns, insurance and annuity functions at several rates of interest, were prepared as the Australian Life Tables, 1901-1910, the A^m tables covering male lives and the A^f female lives. Supplementary joint life annuity tables based on the same experience were published as the Australian Joint-Life Tables, 1901-1910.

Because of the war, the abnormal pneumonic influenza, and an expected improvement in mortality, tables were not compiled for 1911-20 but a table in connection with the 1921 census was based on the triennium 1920-1922 for Australia as a whole. Further tables for each state are in course of construction. These Australian life tables², A^{m21} and A^{f21} comprise tables of insurance and annuity functions at rates of interest proceeding by $\frac{1}{2}\%$ from $2\frac{1}{2}\%$ to 7%. A brief description of data, methods and results is given in the Statistician's Report bound up with the Australian Census, Vol. II.

AUSTRIA-HUNGARY

Viktor Kitz published the first Austrian mortality experience from the results of the census of the year 1869 and the birth and death registers of the years 1865-1875. Then he built up a mortality table based on the censuses of the years 1869 and 1880 and the birth and death registers of the years 1870 to 1880. Two tables were derived from the results of the census as of December 31, 1900,—one with the help of the death registers of the years 1895-1900 for the probabilities of death in quinquennial age groups, and the other for the mortality of the year 1901. The Austrian population life

¹ Census of the Commonwealth of Australia, 1911, Vol. I, p. 293; Vol. III, pp. 1207, 2149.

² Census of the Commonwealth of Australia, 1921, Vol. II, p. 1835, Census Bureau, Melbourne, 1925, and Statistician's Report, p. 326.

tables, 1901-1910, described in detail by Mr. George W. Richmond in *J. I. A.*, Vol. LIII, p. 209, embraced general Austrian mortality tables for the decennium 1901-1910 and the quinquennia 1901-1905 and 1906-1910. The latter, for which monetary functions were given, embraced both male and female tables, graduated both according to the Gompertz-Makeham formula and by the Method of the Bruns series explained in *J. I. A.*, Vol. LIII, p. 229.

A Hungarian population mortality table was published in 1906 by the Central Statistical Bureau. Its basis was the 1900 census and other data relating to the movement of the population during 1900 and 1901.

The principal Austrian insurance mortality tables were the result of a comprehensive investigation of twenty-eight companies published in four volumes by the Mathematisch-statistische Vereinigung des österreichisch-ungarischen Verbandes der Privatversicherungs-Anstalten. They covered the exposures from 1876 to 1900 inclusive. Numerous tables both for males and females were compiled both by lives and by units of medical selection, that is, where all policies issued to an individual on one medical examination are treated as a unit. The latter method thus gives results intermediate between experiences by lives and by policies. Aggregate, ultimate and select tables are given for all business, for old business contracted before 1876, for new business issued after 1876 and for whole-life and endowment insurances separately. Tables on annuitants are also included.

Corresponding mortality tables based on Hungarian insured lives were published in four volumes by the Central office for Hungarian Mortality Investigation. Eighteen companies contributed their data from 1876 to 1900 inclusive.

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Eine Untersuchung über die aus den österreichischen und ungarischen Sterblichkeitsmessungen resultierenden Sterblichkeitsverhältnisse—E. L. Vermeeren. Proc. Seventh Int. Cong. Act., I, 469.

Die Änderung der Sterblichkeitsverhältnisse der Versicherten in Ungarn—Dr. Karl Goldziher und Dr. Ernst Sós. Proc. Seventh Int. Cong. Act., I, 579.

Über den Unterschied in der Sterbeintensität der Versicherten des "Alten" und des "Neuen Bestandes" nach Beobachtungen der österreichisch-ungarischen Versicherungsgesellschaften—B. Iastremsky. Proc. Seventh Int. Cong. Act., I, 601.

BELGIUM

Ducpetiaux constructed a table for Brussels covering 1840–1842 based on the census of 1842 and Quetelet one on the census of 1846 and on the death lists of 1841–1850. Undoubtedly there have been many later Belgian tables. The 1880–1900 table is given in Dawson's "Practical Lessons in Actuarial Science."

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Tables de Mortalité ou de survie et Table de Population pour la Belgique, dressées au moyen des Statistiques Annuelles de 1880 à 1890, J. M. J. Leclerc, Bruxelles, 1893.

CEYLON

At least two life tables of Ceylon have been published by the Registrar-General of that Colony, one in 1888, in his Annual Report on the Vital Statistics of Ceylon for 1887, and the other in 1902 in his report on the 1901 census.

DENMARK

Mortality tables for Denmark are mentioned or described covering 1835–1839, 1840–1844, 1840–1849, 1845–1854, 1860–1869, 1870–1879, 1880–1889, 1890–1894, 1895–1900, 1901–1905 and 1906–1910.

The official Danish statistics have included several mortality tables partly for the whole kingdom and partly for single sections of the population, especially Copenhagen, the

smaller cities, and the rural districts of the country. In 1903 for the first time official Danish statistics included mortality tables according to marital condition.

Mortality of the Statsanstalten for Livsforsikring has been investigated many times. It was their mortality under reversionary annuities on which the Danish Survivorship Annuity table was based.

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Danmark, Statistisk Tabelværk, Femte Række, Litra A. Nr. 8, Aegteskaber, Fødte og Døde, 1 Aarene 1906–10, p. 35, København, 1913.

*Danish Survivorship Annuity Table**

This table formed by Christian Jensen is based upon data derived from the experiences of voluntary survivorship or reversionary annuities, granted by "Statsanstalten for Livsforsikring," the Danish State's Life Insurance Institution. The investigation deals with female beneficiaries—Danish women only. No tables were computed showing death rates of the insured lives. Obligatory annuities on Civil Servants were not included.

The period of observation is 1842 to 1900 and the experience is by lives. Exact ages from birthday to birthday were used, and the observations ceased on the birthday of the beneficiary in 1900, or with the first birthday after death of the insured prior to 1900. The graduation was by Makeham's formula. The constant $\log c = .04579609$ is the same as that used in Hunter's Makehamized American experience, thus facilitating the use of the two tables for joint lives. The other constants were derived by the "Method of Least Squares" from the table at age 40 upwards, involving 4,041 deaths.

The experience showed an improvement in the death rate towards the end of the period. The mortality of the Danish population is said to be as good as that in the healthier parts

* T. A. S. A., Vol. X, p. 253; *Survivorship Annuity Tables*, Dawson, p. vi.

of the United States, so that the tables may be used until American tables for Survivorship Annuities are available. In the absence of other authoritative tables showing the rates of mortality of annuitants under Survivorship Annuity contracts, this table has been adopted as the standard for workmen's compensation allowances in the State of New York.

FRANCE

In 1746 M. Déparcieux* published his "Essai sur les probabilités de la Durée de la Vie Humaine" in which several tables of mortality were given. One table was drawn from the experience of the tontine funds of 1689, 1696 and 1734 and one from the mortuary registers of fourteen monasteries and twenty-six convents covering 8700 male lives and 1,519 female lives. The latter table, somewhat adapted, became the French standard for life insurances, and is known as the Déparcieux Table practically to the exclusion of the other. This table is given in Dawson's "Practical Lessons in Actuarial Science." The mortality shown by this table is generally higher than that of the Carlisle Table, with which in other respects it agrees quite closely. This table was never freely used by English-speaking people, although it was more accurate than any of the other tables published in the eighteenth century. It has now only an historical interest.

The Duvillard Table of Mortality was published in 1806 in a work entitled "Analyse de l'influence de la petite vérole sur la Mortalité." It was drawn from population statistics embracing 101,542 deaths in a population of 2,920,672. This table for many years was the French Standard for annuities. It is given in Dawson's "Practical Lessons in Actuarial Science."

The A. F. (Assurances Français or French Insurance) tables were based on the experience on insured lives in four French companies, the observations extending from 1819 to the end of 1887. Although the original data were separated by sex (even the unadjusted mortality rates published in J. I. A., Vol. XXXIII, p. 485 were so separated), the final

*Walford's Insurance Cyclopædia, Vol. II, p. 261.

table included both males and females. The graduation finally adopted was by Makeham's formula. The table runs very near the $O^{M(5)}$ table from age 17 to about age 45 after which it shows higher mortality rates. The tables were published in complete form in "Tables de Mortalité" in 1895. Several mortality functions for all ages are given in "United States Life Tables 1890, 1901, 1910, and 1901-1910" and commutation columns are given in Dawson's "Practical Lessons in Actuarial Science." The R. F. (Rentiers Français or French Annuitants) is a similar table for life annuitants, including both males and females, based on the French annuitants' experience from 1819 to 1889.

The material in the R. F. table was brought up to 1898 by three French companies and new French annuity R. F. tables constructed. These tables covered all kinds of annuity contracts and pure endowments. Separate tables were constructed for males and females. Four tables were constructed as hereinafter set forth, each for male and female lives separately and for the sexes combined: (1) 1900-A, a select table arranged according to age of observation, really consisting of 108 tables corresponding to ages 0 to 107, distinguished by duration since issue, the greatest duration being 74 years, (2) Table 1900-B, select, consisting of 75 tables arranged according to duration, (3) Table 1900-C, also a select table arranged according to attained age in quinquennial durations, and (4) Table 1900-D, consisting of aggregate tables. The original tables were not graduated but Table 1900-D has been graduated by Karup's method.

The French population mortality table, 1898-1903, based on the census of Jan. 1, 1901, and deaths of 1898-1903, is given in the "United States Life Tables, 1890, 1901, 1910, and 1901-1910." A previous table was constructed from the experience of 1880. M. Huber's French tables, based on the mortality from 1920-1923, appeared in "Tables de mortalité pour la population de la France, 1920-23"—Bull. Stat. Gen. de France, July-Sept., 1928.

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GERMANY

The Twenty-three German Offices' Life Tables (23 D.G.M.u.W.I.) were based upon the experience on insured lives of twenty-three German life insurance companies (including one Austrian and one Swiss company). The experience closed in 1875 and included tables for male lives, female lives, and for both sexes combined. The table used for many years by German life insurance companies was the one based on the combined experience on men and women. Several mortality functions for all ages are given in "United States Life Tables 1890, 1901, 1910, and 1901-1910" and commutation columns are given in Dawson's "Practical Lessons in Actuarial Science." It shows rates of mortality considerably above those in the American Experience, the $O^{M(5)}$ and French Offices' Tables. The principal peculiarity in the construction of this table was that exposures throughout were taken between the exact birthdays, the fractions of the year in the year of issue and termination being treated separately.

The Table LM of the Leipziger Lebensversicherungs-Gesellschaft, 1830-1899, from the number of references to it, appears to have been considered important in Germany.

Ten German annuity tables are described and the probabilities of death given in Die Gebräuchlichsten Sterblichkeitstafeln, Volume XI of the Veröffentlichungen des Deutschen Vereins für Versicherungs-Wissenschaft. In five of these the sexes are combined. In the rest separate tables were made. These tables were all of individual offices except the German annuity mortality tables of 1891 and 1893, the first of which embraced, without separation of the sexes,

annuities of various kinds issued by 24 German, 11 Austrian, and 3 Swiss life insurance companies up to the end of 1889. In the latter table, two more years' experience was added and separate tables shown for men and women. One other similar table and various comparisons between the tables in which the sexes are separated are given in *Blätter für Versicherungs-Mathematik*, I, 378.

German population mortality tables for each sex for 1871/72—1880/81; 1881—1890; and 1891—1900 have been published, apparently with comparisons of tables covering various German states. The 1901—10 table is given in "United States Life Tables, 1890, 1901, 1910, and 1901—1910." A similar mortality table, based on the census of 1925 and the registered deaths of 1924—26, was given in "Wirtschaft und Statistik." The probabilities of death both for men and women for the latter table, together with the ratios of these probabilities to those in the tables for the last three decades mentioned, are given with the comparison of annuity tables previously referred to. Corresponding tables for Bavaria, Saxony, and Wurtemberg have been prepared.

The first mortality table for Prussia was the 1859—1869 table by Becker. Another was a table for 1865 by Boeckh. A table by von Fircks covered 1867—1868, 1872, 1875, 1876, and 1877, eliminating the war mortality and other years of increased mortality. Several other mortality tables were constructed and finally in 1905—1906 the Prussian statistical office prepared about 50 mortality tables for the period 1891—1900, especially for 1896—1900 and 1900—1901. Abstracts of the 1891—1900 tables on the whole kingdom of Prussia, on the cities and on the rural districts, on the 22 largest cities combined, on cities of 20,000 to 100,000 inhabitants, and on smaller cities are given in the Proceedings of the Fifth International Congress of Actuaries. Twenty-five mortality tables for each sex were calculated for Berlin for the period 1876 to 1900. Mortality tables have been made for other large German cities, like Breslau.

A population mortality table for Saxony was based on censuses of 1840, 1843, 1846, 1849, and the deaths from 1838—1849. Tables were also constructed for 1880—81, 1885—86, 1890—91, 1895—96, 1900—01 and 1900—10.

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*Mortality Experience of the Gotha Life Office, 1829-1896**

This investigation, made under the supervision of Dr. Johannes Karup, embraces the experience of the Gotha Life Office on all classes of life and endowment insurances from 1829 to 1895, the observations ceasing on the policy anniversaries in 1896. The policy year method was used. There were 150,594 lives with 2,255,813 years of exposure, and 46,480 deaths. All joint life, survivorship, and short term insurances were excluded. The experience was analyzed by lives, and by amounts; also according to sex and classes of insurance (life, endowment, etc.); all according to the duration of policies, thus producing select tables.

* This account has been taken from T. F. A., Vol. V, p. 87 et seq.

The nearest age at entry was taken, and the nearest duration method used. Duplicate lives were excluded whether of the same class or not, except for ascertaining the rate of claim, i. e., mortality by amounts. The treatment of withdrawals followed the British Offices' Experience, except that exact calculations were made for the first year.

An investigation of the manner in which the mortality changed during the period was made, by tracing the new business of different periods separately, and dividing 1829-1896 into four parts, namely: (1) 1829-1852, (2) 1852-1867, (3) 1867-1881, and (4) 1881-1896. The mortality of each period was compared with (1) the whole experience 1829-1896, and (2) the partial experience 1852-1896 by convenient age groupings for (a) the first five years, and (b) the sixth and subsequent years. The results showed that the mortality diminished from period to period, especially during the first five insurance years, indicating an improvement in methods of selection.

The final graduated Select Tables are joined to the ultimate table after 7 years. The first year's mortality declines from age 15 to age 28, and until age 40 continues less than for age 15; this characteristic is observed in a less pronounced form in subsequent durations.

The practice of the company had been to accept many risks on endowment plans where the excess mortality might be expected to fall mainly after maturity of the endowment; indeed this seems to be the only method used for treatment of substandard lives. Endowment policies were divided into three classes: (1) "Voluntaries," i. e., those who apply for such of their own accord, (2) "Involuntaries," i. e., those upon whom such policies were imposed, and (3) a class of policies issued as collateral for loans. These three divisions taken together show a mortality lighter than that for whole life policies, being 84% during the first five years and 96.3% thereafter. The voluntary endowment class shows an unusually low mortality and each of the other classes a mortality considerably in excess of the whole life, the involuntary class being slightly the worst, the heavier mortality of this last named class being also distinctly observed during the first five years after entry.

The female experience was not extensive, there being 2,735 deaths. During the first year women appear as favorably as men, but during the second and subsequent years for ages under forty there is a noticeably higher mortality amongst females. From about age 40 onward the excess of male mortality over female is quite appreciable.

*The Life Tables, 1926, of the Association of German Life Assurance Companies**

These tables are based upon the experience under all "normal" whole-life policies, both participating and non-participating, issued prior to 1906 by the companies concerned. Medically examined male lives only were included. No distinction was made as regards occupation, personal or family history, physical condition or characteristics, on the ground that the secular variations in mortality during the period studied far overshadowed any lack of homogeneity in the data used.

In the course of the investigations (which extended from 1910 to the middle of 1925), the Central Office analyzed the material in four distinct periods of entry: before 1876, 1876-85, 1886-95, and 1896-1905. The total years exposed were nearly 19,000,000, with 272,316 deaths. Some thirty tables in all were prepared. Those relating to the entry periods 1876-85 and 1896-1905 were reviewed in Mr. Joffe's paper in T. A. S. A., Vol. XXI, p. 207 (see also discussion, p. 561). The final tables published as Vol. 40 of the Publications of the German Society for Insurance Science were based upon a consolidation of all four entry periods—the business before 1876, however, being included only from the renewal dates in 1876. This amalgamation would appear to be a retrograde movement when compared with our present tendency towards obtaining recent experience and even towards forecasted mortality, but the reason is that these tables were to be employed in the building up afresh of German Life Assurance after the period of currency difficulties was past. They represented a deliberate bid for

* Publications of the German Society for Insurance Science, Vol. XXXX. English Translation, E. S. Mittler and Son, Publishers, Berlin, 1926.

security on the lines of what happened by chance in the early days of insurance in this country. The material for an investigation into the secular change in the rate of mortality is, however, available.

The unit of observation was the policy, as that diminished the work of tabulation to a certain extent, and since, as the result of several recent investigations, it had been found that the choice of unit between lives and policies had little effect on the rates of mortality produced.

The tables given include the original exposed to risk and deaths, for the consolidated period 1876 to 1906, for each age at entry and also for every duration (up to 73), a graduated aggregate table, graduated select and ultimate tables based on a ten-year select period, commutation columns and values of a_x , A_x , and P_x , and temporary annuities, at 4%. The technical methods employed were determined by a "Mathematical Committee." Details were given in the Reports of that Committee, in Part XXII of Publications of the German Society for Insurance Science and in "Part 2 of the Documents of the Central Office." The mode of graduating the rates of mortality is stated to be "a development of the graphic method with a system of numerical checks on the closeness of the adjustments." In the case of the aggregate tables the fit to the data was tested both cumulatively over the whole curve and over suitable short sections; while smoothness was ensured by adjustment of the first and second differences. For the select tables the rates of mortality were first graduated for the ultimate curve and also for the first year of duration; and then the data for each separate year of duration were divided into five sections, each embracing nine ages attained, from age 20 to age 64, and "factors for interpolation" were calculated, these factors then being graduated horizontally and vertically and finally calculated for each age, after the average age to which they applied had been determined by weighting. This method is similar to King's Short Method of Constructing Select Tables, except that King worked with factors on only one limiting curve, the ultimate values. It is stated that although the period of selection could be limited to 10 years without difficulty from about age 40 onwards, "this could not be

done for the younger ages without forcing the graduation, and thus impairing the validity either of the select or of the ultimate curves. This was due to the rapid diminution in the rate of mortality at the younger and middle ages during the period of observation. The inclusion of non-profit assurances proved likewise to be another disturbing feature in this respect." Thus, as in the investigations of annuity mortality in Great Britain, it was recognized that the classical theory of selection was not sufficient to explain the observations. A 10-year select period throughout was finally adopted, however, for practical reasons.

The results of the investigation for the entire period 1876-1905 may be seen from the following table:

COMBINED EXPERIENCE OF GERMAN LIFE ASSURANCE COMPANIES—1876-1905
Graduated Rates of Mortality per 1,000

Duration	SELECT (Ultimate for Durations 10 and Over)				AGGREGATE			
	Age at Entry							
	25	35	45	55	25	35	45	55
0	2.13	2.71	5.24	11.70	3.34	5.17	10.61	22.67
1	3.30	4.17	7.92	15.41	3.37	5.62	11.43	24.51
2	3.93	5.32	9.74	18.11	3.41	6.10	12.34	26.51
3	4.37	6.23	11.10	21.26	3.47	6.58	13.33	28.68
4	4.67	6.98	12.46	24.74	3.56	7.08	14.39	31.04
5	4.88	7.67	13.95	28.28	3.69	7.59	15.52	33.63
8	5.17	9.74	19.20	40.32	4.39	9.23	19.45	42.59
10	5.49	11.46	23.19	49.27	5.17	10.61	22.67	49.22
15	8.29	16.25	34.38	71.60	7.59	15.52	33.63	71.59
20	11.46	23.19	49.27	105.10	10.61	22.67	49.22	105.10

INDIA

Life Insurance Experience

Dr. Arthur Hunter* constructed a mortality table covering the experience by amounts of insurance on standard business of the New York Life on native males of India insured from 1885 carried to the end of the year of 1921. This table was graduated by interpolation of the ratios of the actual mortality rates to the $O^{(5)}$ rates of mortality for an age eight years greater. Selection was found to last such a short period that an aggregate table only was made.

* J. I. A., Vol. LVII, p. 235.

Mr. L. S. Vaidyanathan† constructed a similar mortality table based on male lives of Hindus insured in the Oriental Government Security Life Assurance Company, covering the period 1905 to 1925. Graduation was by King's method, in which the ultimate table was graduated by comparison with the $O^{(u)}$ with an addition of seven years to the age. A one year select period only was used. This paper also attempted to study the mortality by Whole Life, Limited Payment Life, and Endowment insurances; by major races—Hindus, Parsis, Europeans, Eurasians, and Mohammedans; by periods from 1874; and by cause of death.

Mr. James Bacon‡ gave a mortality table based on the State insurance fund of the State of Mysore on male officials from 1912 to 1928. A one year select period only was used. For various reasons the mortality in this experience was comparatively light.

The principal characteristic of the mortality in India is the extremely high mortality rates. Other characteristics in the life insurance experience are the very short select period and the fact that the excess of the mortality rate as compared with the northern rate of mortality rises to a maximum (from age 45 to age 55 in different experiences) and then declines.

The papers referred to include references to the previous corresponding mortality tables.

Census Experience

Mr. G. F. Hardy studied the rates of mortality of the male and female population, recorded by the General Census for India, in connection with the separate censuses of 1881, 1891, and 1901, these investigations being included in his reports of these Censuses to the Indian Government, the 1881 Report being summarized in J. I. A., Vol. XXV, p. 217. The difficulties as to defective and inaccurate census and registration figures are immensely magnified in the case of India. He sought to deduce the rates of mortality which might be assumed to hold, in each Province, and in all India,

† J. I. A., Vol. LX, p. 180.

‡ J. I. A., Vol. LX, p. 161.

on the average, irrespective of exceptional periods of stress and strain, arising from famine, plague, etc. Each time he made use of all available previous Census figures and made special allowances for exceptional and severe general visitations of plague, famine or malaria.

Mr. Thomas G. Ackland* made a similar report on the 1901-1911 rates of mortality, based on a comparison of the census returns of 1901 and 1911. Mortality tables were constructed for male lives for all India and for six subdivisions, and similarly for female lives for all India and for five subdivisions. These rates were intended to represent the mortality in a decennium which, while by no means free from such abnormal disturbances, was not on the whole affected very seriously by them. The life table for all India for male lives was formed by combining and weighting the data in the provincial tables. It was then graduated by Makeham's second law.

Mr. H. G. W. Meikle†, in order to reduce as much as possible the effects of the serious epidemics which so frequently occur in India and especially of the influenza epidemic, which in the decade to 1921 is estimated to have caused the death of 7% of the total population, based the 1921 tables on the four previous decennial censuses as well as that of 1921. "Since the results were admitted to be merely a careful approximation of more academic than practical value the commendable policy was adopted of dispensing with the presentation of life-tables and expectations of life—the comparative results being expressed quite satisfactorily by showing simply the estimated values of $100 q_x$ at each quinquennial age after each of the censuses of 1901, 1911, and 1921 for over thirty combinations of the Hindu and Mohammedan populations of Bengal, Bombay, the United Provinces, the Punjab, Madras, and Burma. . . . The method of graduation adopted was the unquestionably suitable one of graduating the logarithm of the ratio of the T_x to be graduated to the T_x of a standard table, the stand-

* J. I. A., Vol. XLVII, p. 315.

† "Report on the Age Distribution and Rates of Mortality deduced from the Indian Census Returns of 1921 and Previous Enumerations." Reviewed in J. I. A., Vol. LVII, p. 271 and T. A. S. A., Vol. XXVII, p. 467 (q.v. for quotation below).

ards employed being in most cases empirical modifications of Mr. Ackland's 1911 All-India Males Table. . . . After the populations living at two different censuses had been determined by the above methods of correction and graduation, the rates of increase over the intercensal period were calculated and thence the numbers six months before and after the middle point of that period were found, from which p_{x+t} followed". (See Actuarial Study No. 3, p. 88.) On account of the unreliability of the general census and registration data at ages under 5, Mr. Meikle concluded that no rates of mortality should be given at those ages.

ITALY

Italian population mortality tables based on the censuses of 1881, 1901, 1911 and 1921 are given in the first volume of *Giornale dell'Istituto Italiano degli Attuari*. The last table was based on the census of December 1, 1921, and the deaths of 1921-1922. Another table which is given in "United States Life Tables 1890, 1901, 1910, and 1901-1910" was based on the experience of the years 1901-1910.

REFERENCES

Movimento della Popolazione nell' anno 1902, Roma, 1904.

Tavole di Mortalità della Popolazione Italiana, Movimento della Popolazione secondo gli atti dello stato civile, nell' anno 1914, Roma, 1917.

Le Tavole di mortalità della Popolazione Italiana, Corrado Gini, *Giornale dell'Istituto Italiano degli Attuari*, I, 16.

JAPAN

The first Japanese population table was constructed by Tsuneta Yano covering the period 1891-1898 for males and females separately. A table based on data for the years 1898-1903 is given in "United States Life Tables, 1890, 1901, 1910, and 1901-1910." In "United States Abridged Life Tables, 1919-1920," comparison is made of the mortality of Japanese in Hawaii with that shown by the 1908-1913 life tables of Japan.

REFERENCES

On the Mortality of the Japanese—Tsuneta Yano. *Proc. Fourth Int. Cong. Act.*, I, 123.

Cabinet Impérial, Bureau de la Statistique Générale, *Table de Mortalité de Japonais, 1899-1903*, Tokio, 1912.

*Three Japanese Offices' Life Tables**

These tables were the first, scientifically deduced from life insurance records, to give the mortality of an Oriental race. The tables, formed by Mr. K. Ebihara, F. I. A., were published in 1912. The material, furnished by three Japanese life companies, was of recent character, the oldest company having been founded in 1881. The experience closed in 1905 and to avoid special risks incurred during the Russo-Japanese war, all emigrants and lives in military service effecting policies in 1903 or after, were excluded.

The system of investigation followed closely that of the British Offices' Life Tables. 484,815 cards were sent in, of which 203,143 related to endowments. Special features were:

1. Very heavy mortality at young ages and marked improvement between ages 20 and 30—generally heavier mortality than British or American standard.
2. The method of handling selection. The actual deaths in the first year were 62 per cent of the expected by the Ultimate Japanese Table. This ratio of 62 per cent was used for all ages; similarly for the second, third, and fourth years, the ratios were 87 per cent, 95 per cent and 97 per cent.
3. The use of the function $B_{(x)+1}$, meaning "Brought down" or "net movement."

Japanese Experience Life Tables, 1912-1927

These tables,† painstakingly compiled under the Minister of Commerce and Industry of Japan with the co-operation of the Japanese actuaries, include all the exposure on standard business of 19 Japanese companies from 1912 to 1927 on all years of issue. The methods used in assembling the data were practically the same as those used in the case of the American Men Mortality Table. Results were compiled both by policies and amounts of insurance. Separate tables

* J. I. A., Vol. XLVII, pp. 100-105. The mortality functions for males are also given in United States Life Tables, 1890, 1901, 1910, and 1901-10.

† Japanese Experience Life Tables, 1912-1927—Department of Commerce and Industry, 1931.

were compiled for males and females. Abstracts of the data from which mortality tables could be constructed were given for 20 tables, that is, select tables, three, five, and ten years truncated aggregate tables, and full aggregate tables, each for males by policies, females by policies, males by amounts, and females by amounts. Of these twenty tables, eight, considered the most useful, were graduated as follows: J^{PM} , J^{PF} , J^{AM} , J^{AF} full aggregate tables, males and females, by policies and amounts; $J^{PM(3)}$, $J^{AM(3)}$, the 3 years truncated aggregate tables for males by policies and amounts, and the J^{PM} , and the J^{AM} , select tables for males by policies and amounts. It was considered that for practical purposes the select period should be treated as three years. A variety of graduation methods including Spencer's twenty-one term summation formula, Makeham's formula, an integral function of the fifth degree and another of the third degree with the coefficients in each case chosen by the method of least squares and other methods, were chosen for the different sections of these tables as seemed most appropriate for each. In general, the death rate by amounts was less than by numbers. Another peculiarity of the experience was that the death rate increased rapidly up to about age 20 after which it decreased to about age 32 both by numbers and amounts and for both males and females. The latter feature may have been partly due to the fact that the experience during the influenza epidemic of 1918 was included because it was considered impossible to separate satisfactorily the deaths entirely due to influenza from other deaths. Deaths resulting from the earthquake of 1923 were included because earthquakes are recurrent in Japan.

NETHERLANDS

Population mortality tables for males and females for 1816-25, 1840-51, 1850-59 and for each decade to and including 1890-99, together with references and brief descriptions of the sources, are given in the Proceedings of the Fourth International Congress of Actuaries. Further description of these tables except the last, also a table relating to the twenty principal cities of the Netherlands for 1880-89, and summaries of several tables based on officials residing

in the Dutch East Indies are given in the Proceedings of the Third Congress. The 1900-1909 table is given in "United States Life Tables 1890, 1901, 1910, and 1901-1910."

Eight annuity mortality tables are given and briefly described in the Proceedings of the Fifth International Congress of Actuaries. These begin with the table of Johan de Witt, 1671, and lead up to Tables on the Widows of the State Pension Fund and Composite Annuity Tables of the life insurance offices from 1874 to 1898 inclusive. Other tables, including two tables based on the experience of a certain Industrial Insurance Company and four tables based on the experience among annuitants under the Government Pension Fund, are given in the Proceedings of the Seventh Congress.

REFERENCES

- Waardije van Lijfrenten, naer proportie van Losrenten. Jan de Witt. 1671.
 On the Improvement in Longevity during the Nineteenth Century in the Netherlands—M. C. Paraira and Corneille L. Landré, Proc. Fourth Int. Cong. Act., I, 10.
 Mortalité en Hollande et dans les Indes Orientales hollandaises—R. H. van Dorsten. Proc. Third Int. Cong. Act., 366.
 Holländische Rentner-Sterbetafeln—R. H. van Dorsten. Proc. Fifth Int. Cong. Act., I, 369.
 Abnahme der Sterblichkeit unter versicherten Personen in den Niederlanden—R. H. van Dorsten. Proc. Seventh Int. Cong. Act., I, 587.
 Annexe au Neuvième Recensement Décennal du Royaume des Pays-Bas, Tables de Mortalité, 1900-1909, La Haye, 1912.

NEW ZEALAND

An extensive Table, based on the census results of 1881, 1886 and 1891, and the death records for the 13 years, 1880-92, prepared by Mr. Geo. Leslie of the New Zealand Government Life Insurance Department, was published in the New Zealand Journal of Insurance, Mining, and Finance, in 1895. It gives male and female experience separately, graduated by the graphic system. These tables are given in Dawson's "Practical Lessons in Actuarial Science."

The New Zealand Official Year Book, 1915, contains tables based on five censuses and deaths for the intervening periods 1891-95, 1896-1900, 1901-05, and 1906-10, which were prepared with the assistance of Mr. A. T. Traversi. He also constructed tables for 1911-15 and mortality rates for each year from 1916-20, which are published in the

Government Statistician's General Report on the Census of 1921. Tables prepared by Mr. L. S. Polden for 1921-22 are to be found in the New Zealand Official Year Book for 1927. Their construction is described in a paper read before the Actuarial Society of Australasia in 1926.

NORWAY

Investigation of the experience of 19 Scandinavian and Finnish life insurance companies was carried up to 1900 with various studies made of the mortality in Norway, Sweden, Denmark and Finland. Graduations of these combined Scandinavian tables, for males and females separately, as published in 1904 by the Scandinavian Life Insurance Companies' Congress are given in Dawson's "Practical Lessons in Actuarial Science."

There has been a series of population tables also. The 1901-10 table is given in "United States Life Tables 1890, 1901, 1910, and 1901-1910."

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Bij Livs-og Dødstabeller for det norske Folk. Norges officielle Statistik. 4 R, No. 118, Kristiania, 1905.

Norges Officielle Statistik. VI. 45, Dødelighetstabeller for det Norske Folk 1901/02—1910/11, Kristiania, 1915.

RUSSIA

The first mortality table based on insured lives covered the experience of nine Russian companies and was published in 1916 in a large volume under the title "Russian Mortality Tables of Nine Insurance Companies."¹ Graduated tables were included for males on whole life insurance for each of three periods—1835-1860, 1861-1885, 1886-1910—for endowment insurance for the second and third periods separately, and one table for pure endowments for all periods combined. The tables were constructed (1) on the basis of lives, (2) on the basis of selection units, and (3) ultimate tables eliminating 8 years. Tables for female lives were published for both whole life insurance and for endowment insurance based on the third period (ungraduated) and for pure endowments and annuities on the basis of all three

¹Russische Sterbetafeln—W. Hochmann. Zeitschrift für die gesamte Versicherungs-Wissenschaft, XXVII, 56.

periods together, these tables being on the selection unit basis.

Graduation was made by the Iastremsky dispersion method which consists of the graduation with the help of parabolic curves of the third, fourth, etc., order by the method of least squares in which the test of the fitness of the graduation is made with the help of the dispersion coefficients.

SWEDEN

The first Swedish mortality table was constructed by P. W. Wargentin in 1755 in a manner similar to Halley's. New tables which covered the years 1755-63 were published by him in 1766. Shortly before his death he sent Dr. Richard Price his investigations of mortality in Sweden during the years 1763-1776. Dr. Price published them in 1783 in his fourth edition of "Observations on Reversionary Payments."

The Swedish Life Table was prepared by Dr. Price in 1783 on the basis of abstracts of living and dying in Sweden during a period of 21 years. This was claimed to be the first national life table ever constructed. It was apparently the first table ever compiled by means of a comparison of the living and the dying.

Another Swedish table, mixed lives, graduated by Dr. Price and published in 1812, covered nine years' statistics of deaths, population and births in the city of Stockholm.

H. Nikander published the results of a study of the mortality of Sweden and Finland for the years 1767-1795. In his "Treatise on the Valuation of Annuities and Construction of Tables of Mortality" in 1815 Milne constructed from these figures two mortality tables, for males and females separately. With the help of mortality statistics of 1801-1805 Milne published another mortality table in the seventh edition of the *Encyclopedia Britannica* and also in the "Treatise on the Law of Mortality and on Annuities" in 1837. Commutation columns of these tables are given in Dawson's "Practical Lessons in Actuarial Science." Other Swedish tables covered the period 1830-1835 and each quinquennium from 1816 to 1860. Mortality tables for males from 1816 to 1840 and for each decade

thereafter to 1900 are given in the Proceedings of the Fifth International Congress of Actuaries.

The first combined experience of Scandinavian insurance companies was mentioned under "Norway."

The Seventeen Swedish Offices' Life Tables were based on standard insured lives exposed from 1895 to 1906 in seventeen Swedish life insurance companies. Numerous tables were constructed from this experience, some based on the medical examination as the unit of observation, some on the amount insured as the unit, and some on the life insured. Aggregate, ultimate and select tables were given for both men and women. Both the aggregate and ultimate tables for men on the basis of the medical selection unit were further divided between Life and Endowment insurances.

Apparently population tables have been constructed for several decades. The 1901-1910 table is given in "United States Life Tables, 1890, 1901, 1910, 1901-1910."

REFERENCES

- Memoirs of the Academy of Sciences at Stockholm, 1776.
Handlungen der Kgl. Wissenschaftsakademie vom Jahre 1801.
Sveriges Officiella Statistik, Dödlighets-och Livslängdstabeller för årtiondet 1901-1910, Stockholm, 1916.
Über die Bedeutung der Verlängerung der Lebensdauer für die Berechnung der Leibrenten—N. V. E. Nordenmark. Proc. Fifth Int. Cong. Act., I, 421.
Untersuchung der Sterblichkeit unter normalen Leben nach den Erfahrungen von neunzehn skandinavischen und finnischen Lebensversicherungsanstalten—Samuel af Ugglas. Proc. Fifth Int. Cong. Act., II, 143.
Sur la prolongation de la vie des personnes assurées d'après les données fournies par les sociétés suédoises d'assurance sur la vie. N. V. E. Nordenmark. Proc. Seventh Int. Cong. Act., I, 611.
Untersuchung über die Sterblichkeit laut Erfahrung von 17 schwedischen Lebensversicherungsgesellschaften—Svenska Aktuarieföreningens Tidskrift, II, 93.

*Swedish Offices' Life Tables of 1928**

These tables, presented by a Committee of Swedish Actuaries, were constructed to serve as a common rate basis for Ordinary Life Insurance business in Sweden as no other existing tables seemed to be suitable. Approved by the Insurance Department as a minimum for premiums, they are now adopted by all companies but one.

An ultimate table following Makeham's law was con-

* Skandinavisk Aktuarietidskrift, 1929, Vol. XII, p. 43.

structed so as to produce rates of mortality falling between the mortality curves of the two existing tables based on the experience of Thule and Svenska Lif. A table with a five year select period was also constructed based on an inter-company investigation in 1917 of the experience on insurance written without medical examination.

Although this latter table was expected to give slightly conservative results over a long period of time, another table developed from it, but with a "loaded rate of mortality" $\mu'_{[x]+1} = \frac{\mu_{[x]+1} + .0006}{.95}$, is the one actually used in the calculation of the premiums and reserve values. This "loaded mortality" is not a loading for expenses or even for chance fluctuations, since they are being taken care of by further loadings.

For comparison the ratios of q_x in the new tables to the $AM^{(6)} q_x$ are given:

Age	On the Net Mortality	On the "Loaded Mortality"
20.....	90%	110%
30.....	96	115
40.....	107	123
50.....	96	106
60.....	87	94

A paper describing the calculation of the tables, approximation formulæ, the rates, etc., is published in English by Reinh. Palmquist in the "Skandinavisk Aktuarietidskrift" 1929.

SWITZERLAND

A summary of Swiss mortality tables gives various population mortality tables. The graduated population tables are those of 1876-1880, 1881-1888, 1889-1900 and 1901-1910, all graduated by Woolhouse's method, and the tables of 1910-1911 and 1920-1921 graduated by Makeham's formula, the constants being determined by the method of least squares. In all these tables the sexes are shown separately. The last three tables as modified for use for insurance purposes are also given. The probability of death is also

given for 15 mortality tables, based on the population in different cantons or groups of cantons. In most cases ages and sexes are dealt with separately. The first three of these tables are summaries of tables during the 16th century (from 1560 to 1600), the 17th century and the 18th century up to 1760. The other tables cover various periods up to tables for Basle 1918-24, Berne 1919-22 and Zurich 1919-25.

REFERENCES

De la mortalité dans la population suisse; Dr. Marcel Ney, Bulletin de l'association des Actuaires suisses, XXII, 11.

Table Suisse de Survie pour les années 1901-1910, Resultats Statistiques du Recensement Général de la Population du 1er Decembre, 1910, XII, 52. Bümpliz-Berne, 1917.

APPENDIX

TABLE SHOWING THE RATE OF MORTALITY (q_x)
ACCORDING TO CERTAIN POPULATION TABLES

Age	United States Life 1919-20		English Life No. 9		India 1901-10		Age
	Male	Female	Male	Female	Male	Female	
20	.0045	.0047	.0035	.0031	.0169	.0170	20
25	.0054	.0061	.0040	.0035	.0203	.0200	25
30	.0065	.0069	.0043	.0039	.0235	.0231	30
35	.0073	.0070	.0055	.0045	.0278	.0266	35
40	.0082	.0073	.0069	.0053	.0323	.0308	40
45	.0096	.0085	.0088	.0067	.0372	.0353	45
50	.0121	.0109	.0118	.0092	.0428	.0406	50
55	.0174	.0154	.0176	.0132	.0499	.0474	55
60	.0246	.0218	.0256	.0190	.0598	.0578	60
65	.0364	.0330	.0398	.0299	.0755	.0744	65
70	.0555	.0509	.0600	.0465	.1017	.1012	70
75	.0823	.0766	.0938	.0759	.1480	.1460	75
80	.1246	.1160	.1400	.1177	.2287	.2249	80

Age	United States Life 1901-10 Male	Healthy English No. 1 Males	English Life No. 3 Males	English Life No. 6 Males	Carlisle Table	North- ampton Table	Age
20	.0055	.0070	.0083	.0046	.0071	.0140	20
25	.0062	.0078	.0092	.0057	.0073	.0158	25
30	.0073	.0082	.0101	.0067	.0101	.0171	30
35	.0091	.0086	.0113	.0090	.0103	.0187	35
40	.0104	.0094	.0130	.0119	.0130	.0209	40
45	.0131	.0108	.0154	.0148	.0148	.0240	45
50	.0153	.0130	.0188	.0194	.0134	.0284	50
55	.0214	.0166	.0246	.0257	.0179	.0335	55
60	.0299	.0237	.0325	.0360	.0335	.0402	60
65	.0429	.0368	.0459	.0497	.0411	.0490	65
70	.0599	.0556	.0673	.0721	.0516	.0649	70
75	.0902	.0840	.0988	.1054	.0955	.0962	75
80	.1337	.1249	.1418	.1520	.1217	.1343	80

TABLE SHOWING THE COMPLETE EXPECTATION OF LIFE
(e_x) ACCORDING TO CERTAIN POPULATION TABLES

Age	United States Life 1919-20		English Life No. 9		India 1901-10		Age
	Male	Female	Male	Female	Male	Female	
20	44.9	45.8	45.8	48.7	27.5	28.0	20
25	41.0	41.9	41.6	44.5	24.9	25.4	25
30	37.1	38.2	37.4	40.3	22.4	23.0	30
35	33.3	34.5	33.3	36.1	20.2	20.7	35
40	29.5	30.7	29.2	31.9	18.0	18.5	40
45	25.7	26.8	25.2	27.7	16.0	16.4	45
50	22.0	22.9	21.4	23.7	14.0	14.3	50
55	18.4	19.2	17.7	19.9	12.0	12.2	55
60	15.1	15.8	14.4	16.2	10.0	10.1	60
65	12.1	12.6	11.4	12.9	8.0	8.1	65
70	9.4	9.9	8.8	10.0	6.2	6.2	70
75	7.2	7.6	6.6	7.5	4.5	4.5	75
80	5.4	5.7	4.9	5.6	3.0	3.1	80

Age	United States Life 1901-10 Male	Healthy English No. 1 Males	English Life No. 3 Males	English Life No. 6 Males	Carlisle Table	North- ampton Table	Age
20	42.4	43.4	39.5	41.0	41.5	33.4	20
25	38.6	39.9	36.1	37.0	37.9	30.8	25
30	34.8	36.4	32.8	33.1	34.3	28.3	30
35	31.1	32.9	29.4	29.2	31.0	25.7	35
40	27.6	29.3	26.1	25.6	27.6	23.1	40
45	24.0	25.6	22.8	22.2	24.5	20.5	45
50	20.6	22.0	19.5	18.9	21.1	18.0	50
55	17.2	18.5	16.4	15.8	17.6	15.6	55
60	14.2	15.1	13.5	12.9	14.3	13.2	60
65	11.4	12.0	10.8	10.3	11.8	10.9	65
70	9.0	9.4	8.4	8.0	9.2	8.6	70
75	6.8	7.2	6.5	6.2	7.0	6.5	75
80	5.1	5.4	4.9	4.6	5.5	4.8	80

TABLE SHOWING THE RATE OF MORTALITY (q_x)
ACCORDING TO CERTAIN INSURANCE TABLES

Age	American Ex- perience Table	American Men AM ⁽⁵⁾	Canadian Men CM ⁽⁵⁾	Cam- mack's Group (Clerical)	OM ⁽⁶⁾	Standard Indus- trial Table	Age
20	.0078	.0039	.0041	.0020	.0065	.0069	20
25	.0081	.0043	.0044	.0022	.0069	.0095	25
30	.0084	.0045	.0043	.0024	.0075	.0116	30
35	.0090	.0048	.0045	.0031	.0084	.0130	35
40	.0098	.0058	.0053	.0044	.0098	.0147	40
45	.0112	.0079	.0070	.0067	.0120	.0174	45
50	.0138	.0116	.0099	.0104	.0154	.0216	50
55	.0186	.0175	.0147	.0157	.0208	.0285	55
60	.0267	.0267	.0226	.0249	.0292	.0392	60
65	.0401	.0407	.0352	.0369	.0422	.0561	65
70	.0620	.0615	.0551	.0557	.0622	.0825	70
75	.0944	.0919	.0860	.0801	.0927	.1230	75
80	.1445	.1357	.1332	.1086	.1385	.1838	80
Age	OM	HM	N.F.C.	Actuaries' Table	Thirty American Offices Males	Gotha 1852-95 Whole Life Males	Age
20	.0040	.0063	.0050	.0073	.0068	.0053	20
25	.0048	.0066	.0052	.0078	.0070	.0047	25
30	.0060	.0077	.0056	.0084	.0075	.0045	30
35	.0074	.0088	.0062	.0093	.0082	.0058	35
40	.0092	.0103	.0072	.0104	.0094	.0078	40
45	.0115	.0122	.0089	.0122	.0112	.0104	45
50	.0150	.0160	.0114	.0159	.0142	.0144	50
55	.0204	.0210	.0157	.0217	.0189	.0213	55
60	.0289	.0297	.0228	.0303	.0265	.0320	60
65	.0420	.0434	.0344	.0441	.0386	.0467	65
70	.0621	.0622	.0537	.0649	.0578	.0709	70
75	.0926	.0984	.0855	.0956	.0878	.1061	75
80	.1384	.1447	.1381	.1404	.1341	.1619	80

TABLE SHOWING THE COMPLETE EXPECTATION OF LIFE
(e_x^0) ACCORDING TO CERTAIN INSURANCE TABLES

Age	American Ex- perience Table	American Men AM ⁽⁵⁾	O ^(M)	O ^M	O ^{M(5)}	Standard Indus- trial Table	Age
20	42.2	45.9	42.9	43.7	42.4	37.5	20
25	38.8	41.8	39.2	39.6	38.7	33.9	25
30	35.3	37.7	35.6	35.6	35.1	30.6	30
35	31.8	33.5	31.9	31.7	31.4	27.4	35
40	28.2	29.3	28.2	27.9	27.7	24.2	40
45	24.5	25.2	24.7	24.2	24.0	20.9	45
50	20.9	21.3	21.2	20.6	20.5	17.8	50
55	17.4	17.6	17.9	17.2	17.2	14.8	55
60	14.1	14.3	14.9	14.1	14.0	11.9	60
65	11.1	11.3	12.1	11.2	11.2	9.4	65
70	8.5	8.8	9.7	8.7	8.7	7.2	70
75	6.3	6.7	7.6	6.6	6.6	5.3	75
80	4.4	5.0		4.8	4.8	3.8	80

Age	Gotha		N.F.C.	Actuaries' Table	Thirty American Offices Males	H ^M	Age
	Select	Aggre- gate					
20	43.0	43.6	45.6	41.5	43.1	42.1	20
25	39.2	39.7	41.8	38.0	39.5	38.4	25
30	35.3	35.6	37.8	34.4	35.8	34.7	30
35	31.3	31.4	33.9	30.9	32.2	31.0	35
40	27.4	27.4	29.9	27.3	28.5	27.4	40
45	23.6	23.5	26.0	23.7	24.8	23.8	45
50	20.0	19.8	22.2	20.2	21.2	20.3	50
55	16.8	16.3	18.5	16.9	17.8	17.0	55
60	13.9	13.1	15.0	13.8	14.6	13.8	60
65	11.4	10.4	11.8	11.0	11.6	11.0	65
70		7.9	9.0	8.5	9.0	8.5	70
75		5.9	6.6	6.5	6.7	6.4	75
80		4.3	4.6	4.8	4.9	4.7	80

MORTALITY TABLES

TABLE SHOWING THE RATE OF MORTALITY (q_x)
ACCORDING TO CERTAIN ANNUITY TABLES

Age	American Annuity Table, 1900-20*		United States Annuity Table, 1918-27*		British Offices, 1900-20†		British Gov't Annuity Table, 1900-20†		Age
	Male	Female	Male	Female	Male	Female	Male	Female	
20							.0034	.0026	20
25	.0043	.0040					.0040	.0033	25
30	.0050	.0045					.0047	.0042	30
35	.0060	.0053					.0055	.0052	35
40	.0075	.0064			.0080	.0072	.0066	.0063	40
45	.0098	.0081			.0085	.0076	.0080	.0074	45
50	.0132	.0106			.0097	.0085	.0101	.0085	50
55	.0182	.0143			.0142	.0100	.0140	.0105	55
60	.0257	.0198	.0186	.0112	.0208	.0129	.0209	.0138	60
65	.0367	.0281	.0318	.0185	.0305	.0181	.0332	.0195	65
70	.0531	.0403	.0511	.0360	.0446	.0277	.0524	.0325	70
75	.0770	.0583	.0739	.0641	.0686	.0451	.0817	.0555	75
80	.1117	.0846	.1068	.0977	.1090	.0762	.1264	.0923	80

Age	Combined Annuity Table		Canadian Standard Rutherford		McClintock's Table		British Offices, 1863-93 Oam (a) Oa (b)		Age
	Male	Female	Male	Female	Male	Female	Male	Female	
20	.0021	.0018	.0035	.0030	.0079	.0038			20
25	.0022	.0021	.0040	.0034	.0082	.0041	.0070	.0068	25
30	.0024	.0022	.0043	.0041	.0086	.0044	.0076	.0075	30
35	.0031	.0024	.0045	.0045	.0094	.0050	.0085	.0087	35
40	.0046	.0034	.0053	.0048	.0106	.0059	.0099	.0104	40
45	.0069	.0050	.0070	.0056	.0125	.0073	.0121	.0127	45
50	.0104	.0075	.0099	.0073	.0154	.0096	.0154	.0153	50
55	.0155	.0112	.0146	.0094	.0201	.0132	.0206	.0181	55
60	.0230	.0167	.0230	.0134	.0275	.0188	.0285	.0218	60
65	.0343	.0249	.0349	.0212	.0391	.0277	.0407	.0290	65
70	.0508	.0371	.0499	.0318	.0572	.0417	.0593	.0436	70
75	.0751	.0550	.0751	.0527	.0852	.0633	.0873	.0689	75
80	.1102	.0811	.1159	.0898	.1279	.0967	.1290	.1079	80

*Ultimate.

†Excluding experience of 1st policy year.

TABLE SHOWING ANNUITY VALUES (a_x) AT 4%
ACCORDING TO CERTAIN ANNUITY TABLES

Age	American Annuityants, 1900-20 (Select)		United States Annuityants, 1918-27 (Select)		British Offices 1900-20* (Select)		British Gov't Annuityants, 1900-20* (Select)		Age
	Male	Female	Male	Female	Male	Female	Male	Female	
20	19.6	20.1							20
25	18.9	19.4							25
30	18.1	18.7					18.9	19.8	30
35	17.1	17.8					18.0	18.9	35
40	16.0	16.8			16.5	17.5	16.9	18.0	40
45	14.7	15.7			15.4	16.5	15.6	16.9	45
50	13.4	14.4			14.0	15.4	14.2	15.7	50
55	12.0	13.0			12.5	14.1	12.5	14.2	55
60	10.5	11.6	10.9	12.0	10.9	12.6	10.8	12.6	60
65	9.0	10.1	9.3	10.1	9.3	10.9	9.1	10.8	65
70	7.5	8.6	7.9	8.5	7.7	9.1	7.4	8.9	70
75	6.1	7.2	6.6	6.9	6.1	7.2	5.9	7.1	75
80	4.9	5.8	5.6	5.7	4.7	5.5	4.6	5.4	80

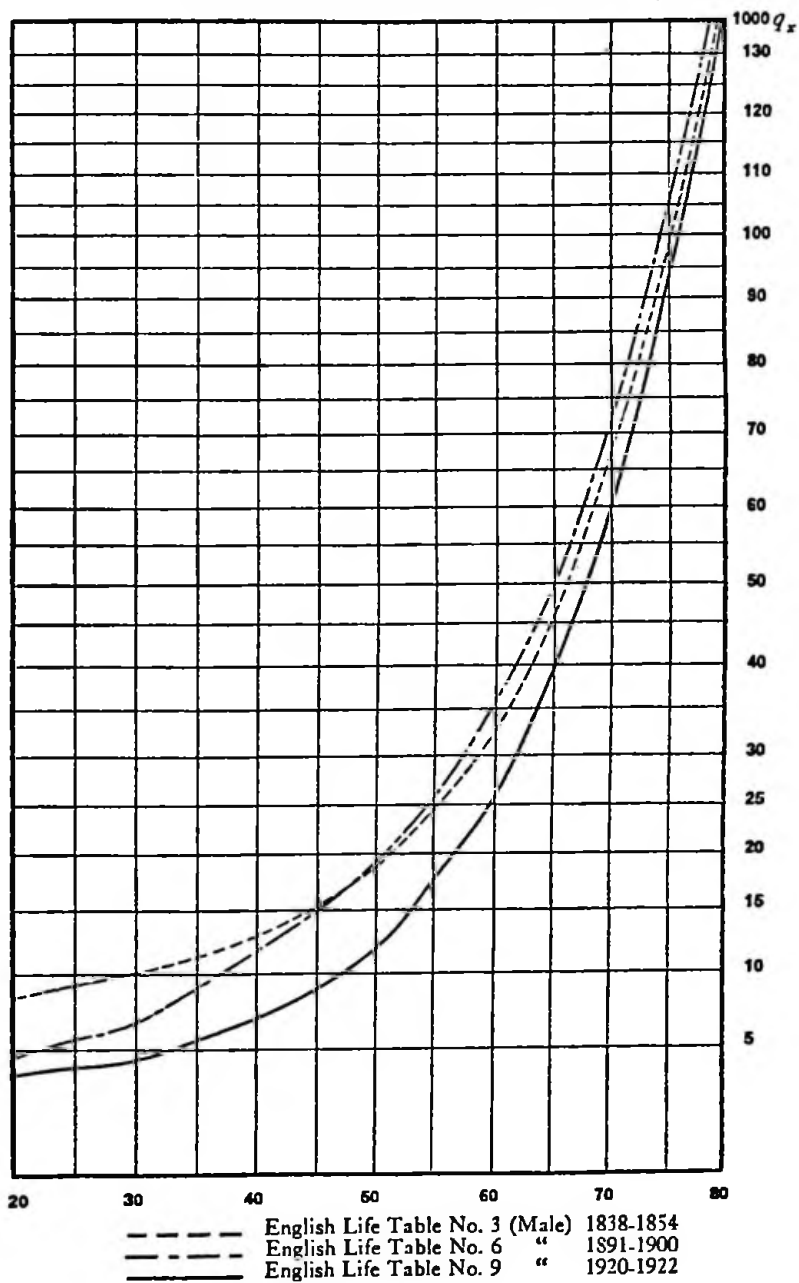
Age	Combined Annuity Table		Canadian Standard Rutherford		McClintock's Table		British Offices, 1863-93 (Select) Oam Oof		Age
	Male	Female	Male	Female	Male	Female	Male	Female	
20	20.4	20.9	20.0	20.6	18.5	20.1	18.9	19.1	20
25	19.7	20.3	19.3	20.0	17.9	19.5	18.2	18.4	25
30	18.8	19.5	18.6	19.3	17.1	18.7	17.4	17.7	30
35	17.7	18.6	17.6	18.5	16.3	17.8	16.5	16.9	35
40	16.5	17.5	16.5	17.6	15.3	16.8	15.5	16.0	40
45	15.1	16.2	15.2	16.4	14.1	15.6	14.3	15.0	45
50	13.6	14.8	13.7	15.1	12.8	14.3	13.0	13.9	50
55	12.1	13.3	12.1	13.7	11.4	12.8	11.5	12.6	55
60	10.5	11.8	10.4	12.0	9.9	11.2	10.0	11.2	60
65	8.8	10.1	8.8	10.3	8.3	9.5	8.5	9.6	65
70	7.3	8.5	7.2	8.4	6.7	7.9	7.0	7.9	70
75	5.8	7.0	5.7	6.6	5.3	6.3	5.6	6.3	75
80	4.5	5.5	4.3	4.9	3.9	4.8	4.4	4.8	80

*Based on the forecast of future mortality.

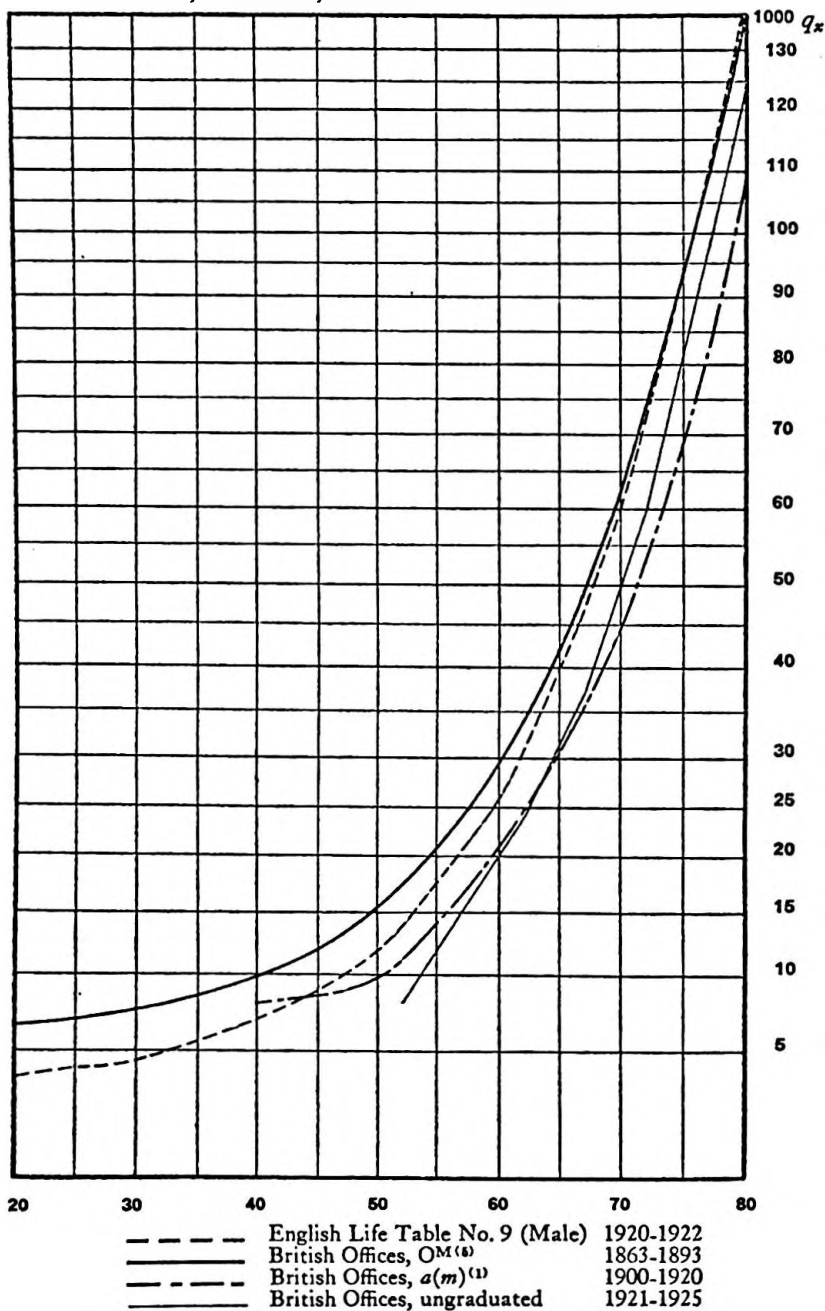
MORTALITY GRAPHS

THE rates of mortality according to some of the more important tables just described are compared from various standpoints in the graphs which follow. The treatment of the vertical scale in these graphs may be of interest. In order to keep the course of the q_x curve within bound at the higher ages, without rendering the representation microscopic at the younger ages, the graphs show not q_x but $kq_x^{\frac{2}{3}}$. This particular transformation while rendering the curves fully as manageable as the use of a logarithmic scale has the additional property of transforming a Gompertz curve approximately into another Gompertz curve, thus avoiding the serious distortion produced by the use of a logarithmic scale. The familiar, typical form of the resulting curves testifies to the suitability of the device employed.

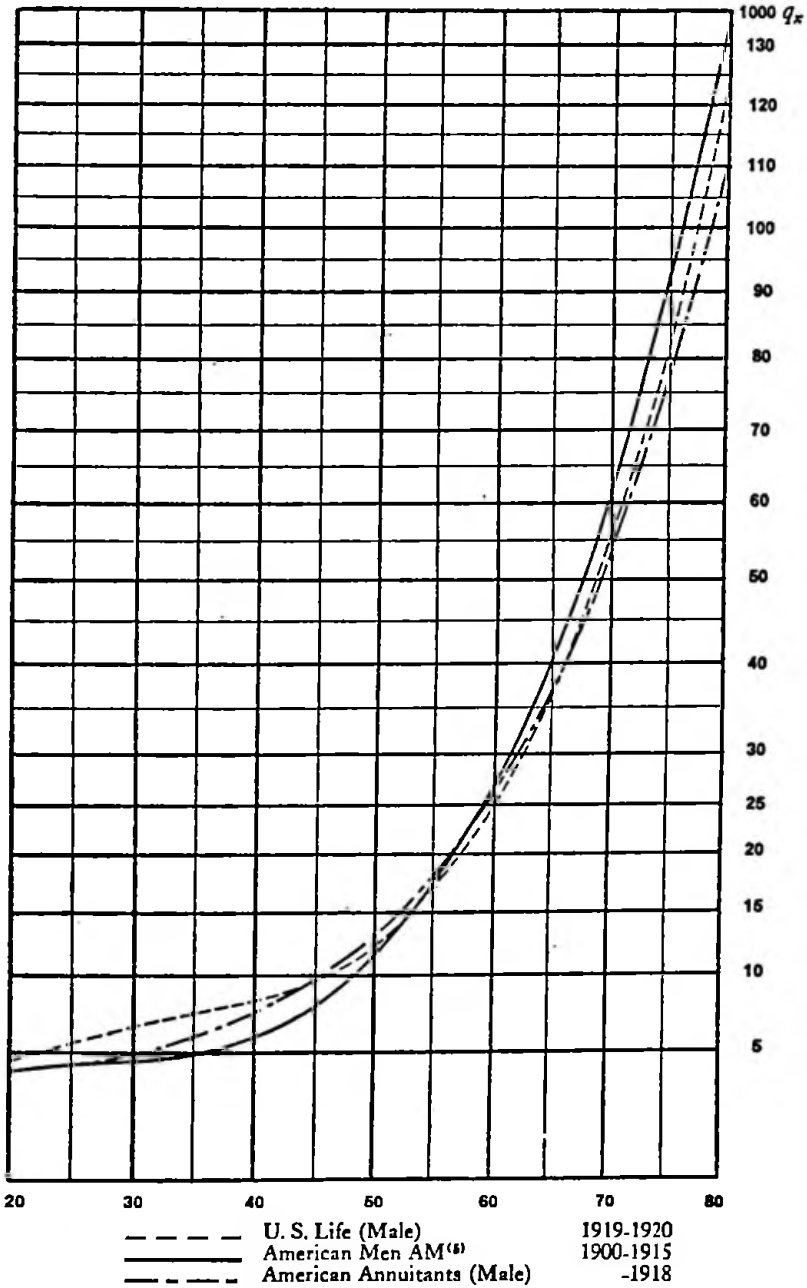
CHANGE IN POPULATION MORTALITY OVER A LONG PERIOD (ENGLAND)



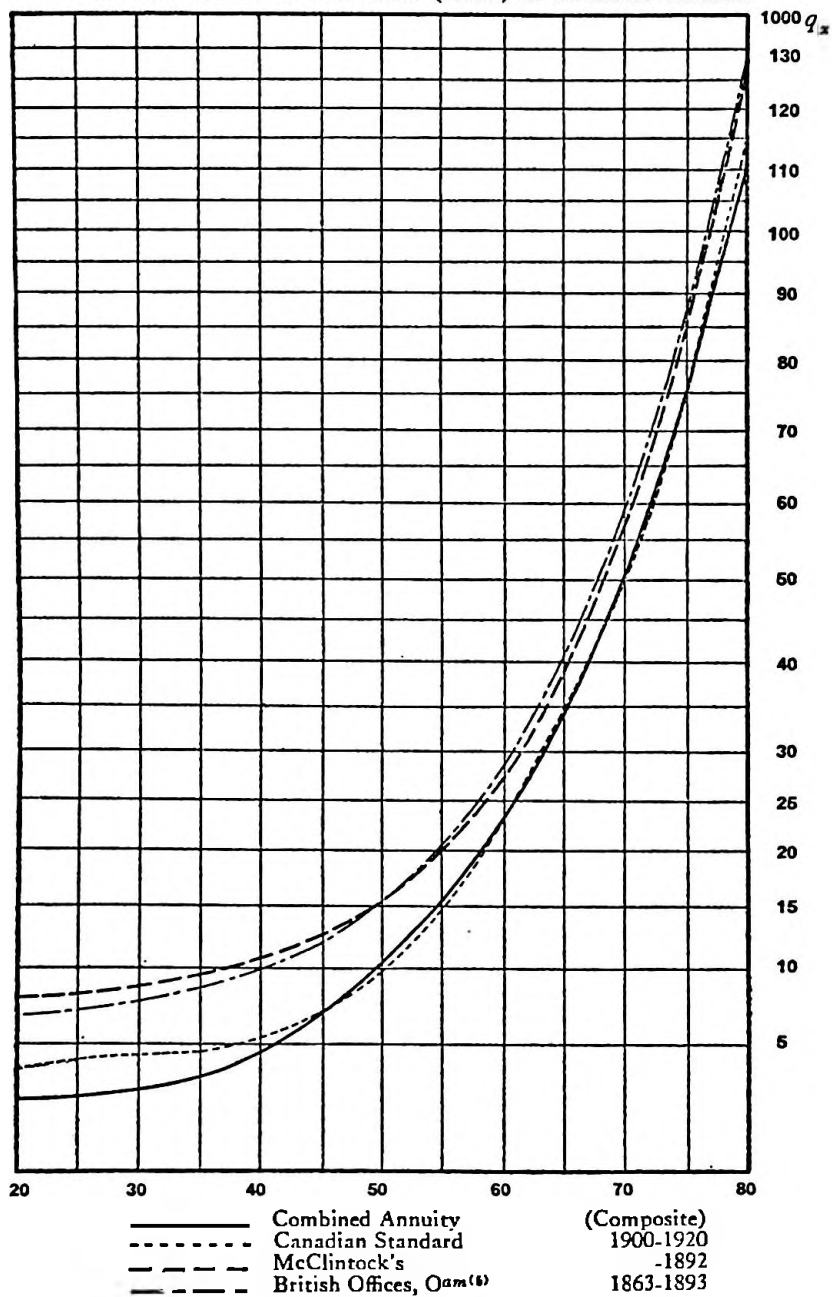
POPULATION, INSURANCE, AND ANNUITY EXPERIENCE IN ENGLAND



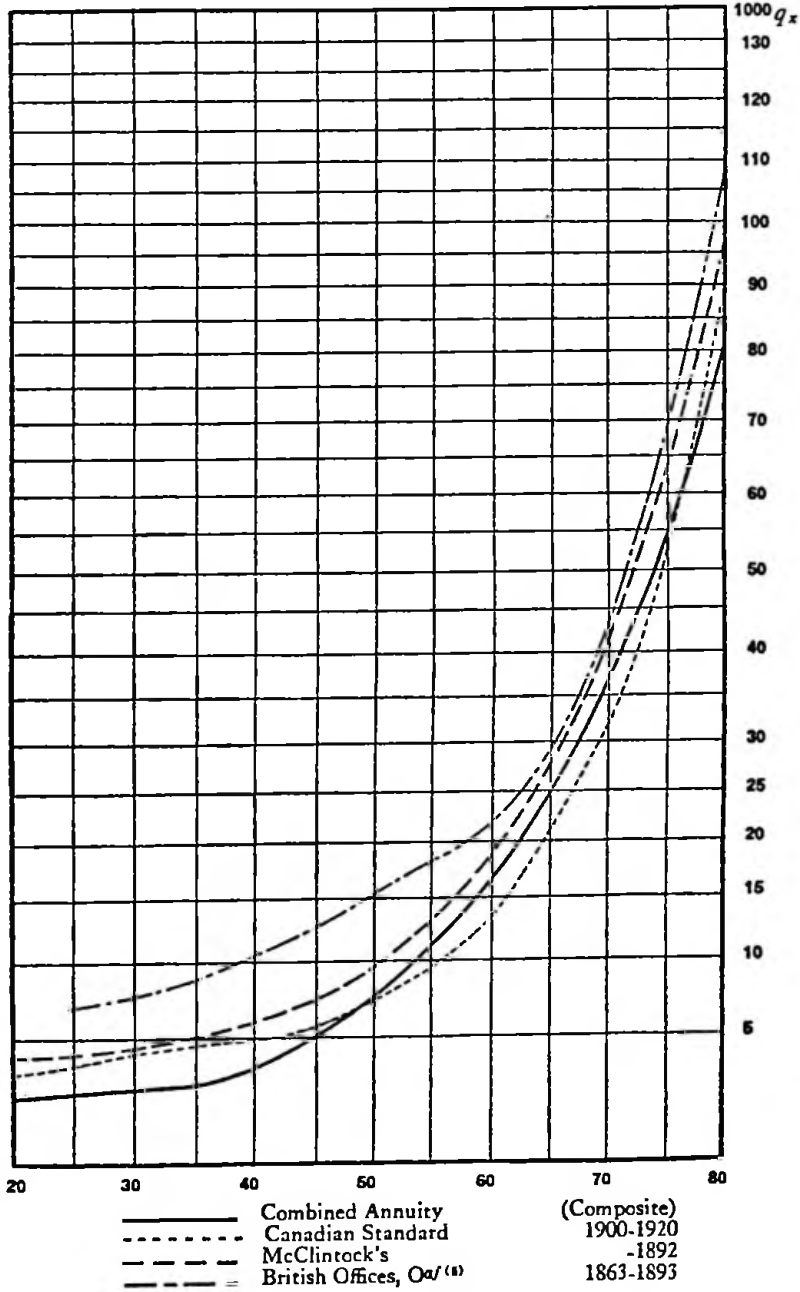
POPULATION, INSURANCE, AND ANNUITY EXPERIENCE IN THE UNITED STATES



ANNUITY VALUATION STANDARDS (MALE) IN U. S. AND CANADA

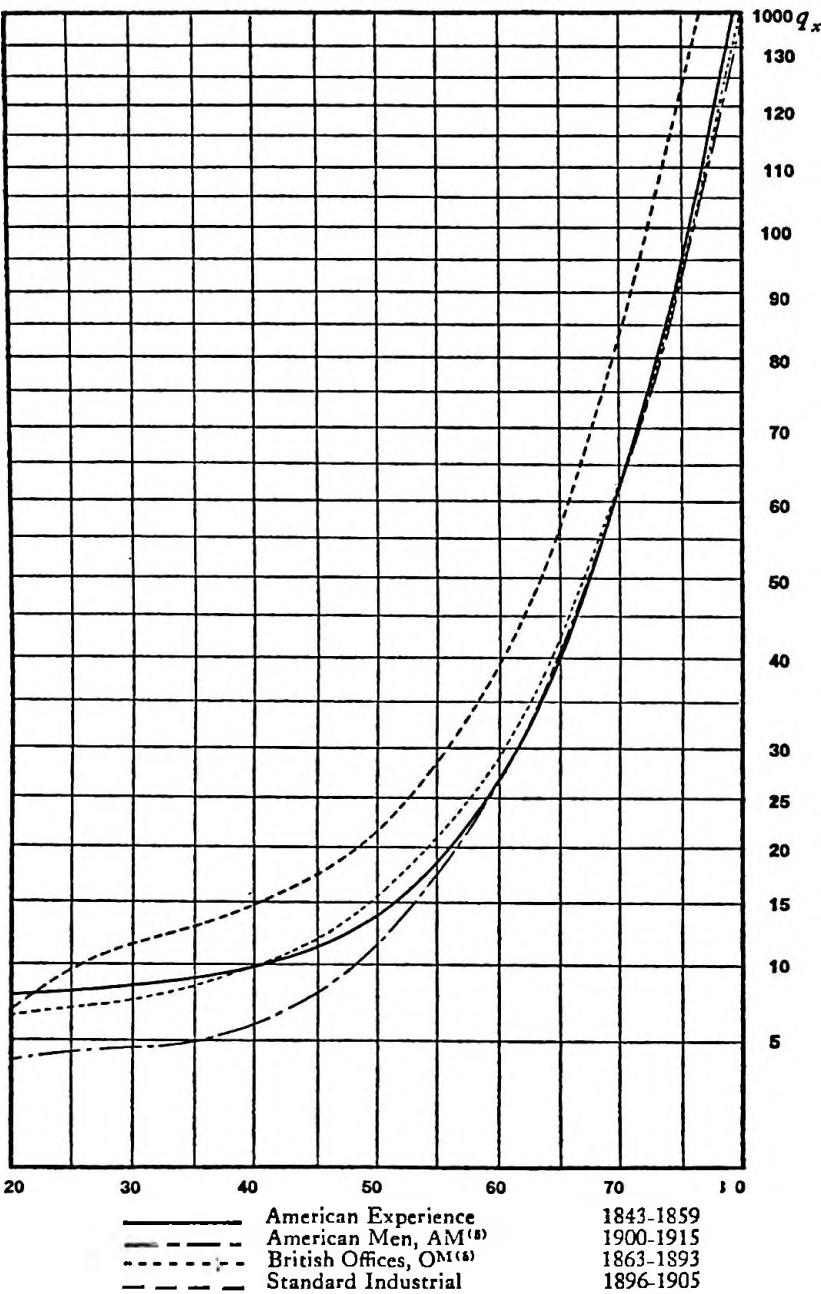


ANNUITY VALUATION STANDARDS (FEMALE) IN U. S. AND CANADA



MORTALITY TABLES

INSURANCE VALUATION STANDARDS IN U. S. AND CANADA



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