

ACTUARIAL STUDIES

No. 5

DISABILITY BENEFITS
IN
LIFE INSURANCE POLICIES

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THE ACTUARIAL SOCIETY OF AMERICA



DISABILITY BENEFITS
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PREFACE

The first edition of this Actuarial Study was published in April 1920. Since that time there has been a great development in permanent total disability benefits which has necessitated a complete revision of the original study. The present text book is several times the size of the original. At the time the work was begun early in 1931, it was questioned whether the study should be published, as radical limitations were expected in the scope of the disability coverage in 1932, to which reference has been made under the appropriate headings. The student, however, cannot obtain a proper background without a review of the history, conditions and methods of preparing premiums and reserves. Hence, it was decided to proceed with the publication of a new edition. All of those listed as associate contributors have borne their share, in varying degrees, in the preparation of the material on this complicated subject, and in addition, many others, although not on the committee, have assisted materially in the preparation of this book. The thanks of the Actuarial Society of America and of actuarial students are due to them as they have given freely of their time and labor with the sole purpose of helping others.

A. H.

TABLE OF CONTENTS

Part I

	PAGE
CHAPTER I—HISTORY.....	1
Contracts	2
Premiums	6
Standard Disability Provisions.....	8
Total and Permanent Disability Benefits in Group Life Insurance Policies	11
CHAPTER II—THE PRO-RATING CLAUSE.....	13
Advantages	13
Disadvantages	14
CHAPTER III—INCONTESTABILITY.....	16
CHAPTER IV—CAUSES OF DISABILITY.....	19
CHAPTER V—TERMINATIONS AMONG DISABLED LIVES.....	22
CHAPTER VI—SELECTION OF RISKS.....	24
Occupation	24
Medical Impairments	25
Moral Hazard	29
Maximum Amount of Coverage on Individual Lives.....	30
Women	31
CHAPTER VII—SURRENDER VALUES	35
CHAPTER VIII—DIVIDENDS	37
CHAPTER IX—ADDITION OF DISABILITY BENEFITS TO POLICIES PREVIOUSLY ISSUED, CHANGES IN DISABILITY BENEFITS, RE-INSTATEMENT	40
Reinstatement	42
CHAPTER X—TREATMENT OF CLAIMS.....	44
CHAPTER XI—LEGAL DECISIONS	48
Total Disability	50
Permanent Disability	51
Proof of Disability—Effective Date of Payments.....	52
Discontinuance of Disability Benefits upon Recovery of Insured	52
Default in Premium Payment While Disabled.....	53
Claims Delayed Until After Recovery.....	53
Claims for Disability Benefits After Death.....	53
Insanity	54
Actions for Breach of Contract.....	54
Incontestability Clause	54
Overpayment of Benefits	55
General Remarks	55
Digest of Typical Decisions.....	56

	PAGE
CHAPTER XII—METHODS OF ACCOUNTING.....	70
Premium Income—Waiver of Premiums.....	70
Income and Instalment Payments.....	72
Income payments	72
Instalment payments reducing face.....	73
Income payments reducing face.....	75
Maturity instalment benefit.....	75
Assets and Liabilities.....	76
Gain and Loss Accounting	79
(a) Analysis by Means of a Running Account.....	79
(b) The Gain and Loss Exhibit.....	81
Subdivision into active and disabled—Recoveries	81
Active life reserve on recoveries.....	83
Reserves for unreported and pending claims...	83
Canadian Annual Statement	85

Part II

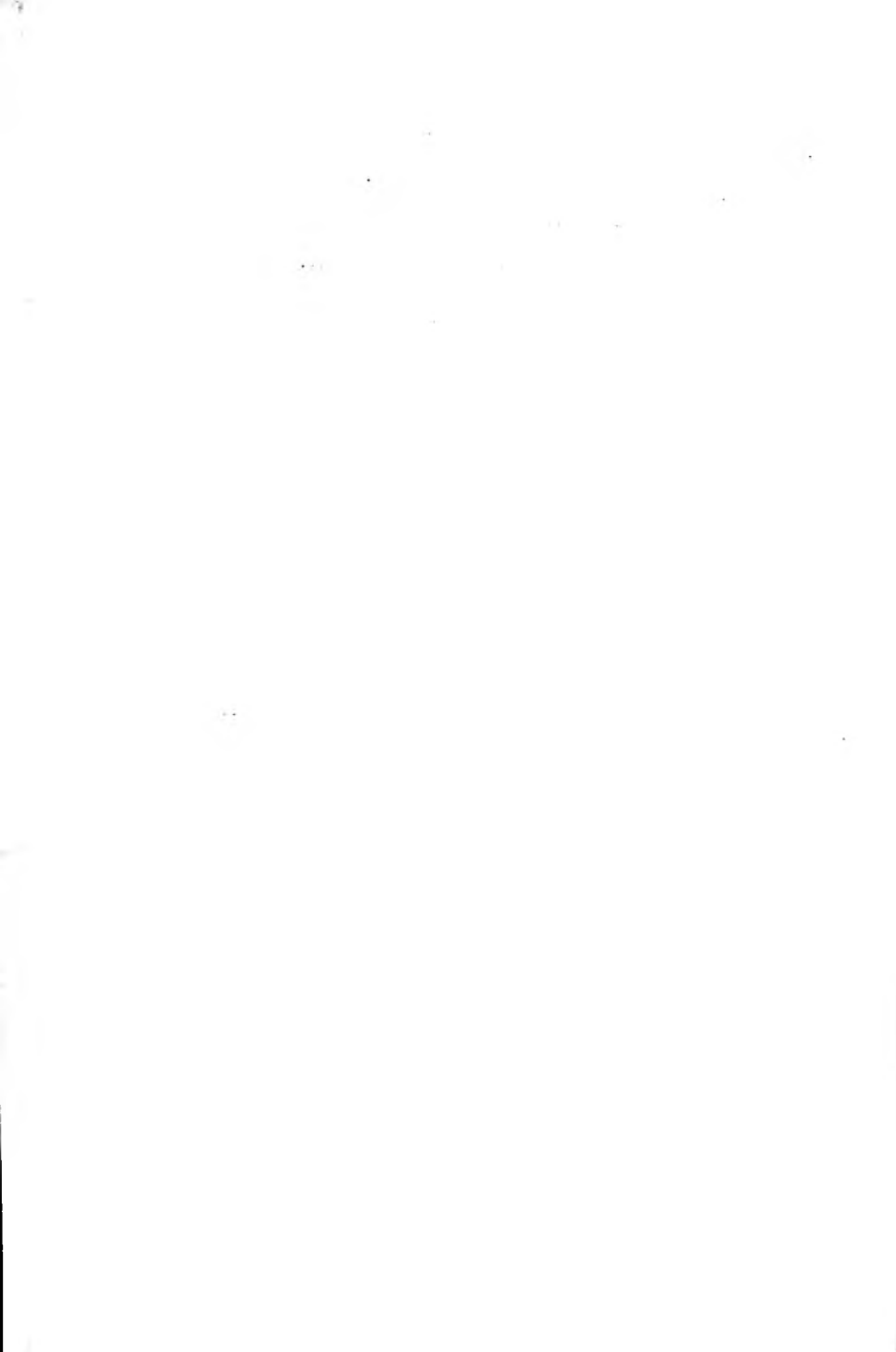
CHAPTER I—INTRODUCTION—COMBINED MORTALITY AND DISABILITY TABLE.....	87
CHAPTER II—RATE OF DISABILITY.....	96
CHAPTER III—DEFINITION OF DATE OF DISABILITY.....	103
Effect on Statistical Analysis of Claims.....	105
Present Day Practice.....	107
CHAPTER IV—EXISTING TABLES OF RATES OF DISABILITY AND RATES OF TERMINATION AMONG DISABLED LIVES.....	109
Tables based on fraternal experience.....	109
Inter-Company investigation	110
Mutual Life's experience.....	112
New York Life's experience.....	112
Effect of medical selection.....	114
Rates of Termination among Disabled Lives.....	115
Disability Experience among Women.....	119
CHAPTER V—GRADUATION	121
CHAPTER VI—CONSTRUCTION OF COMBINED MORTALITY AND DISABILITY TABLE	126
(a) When r_x is given.....	126
(b) When r'_x is given.....	127
(c) When p_x^{ad} is given.....	128
Approximate Methods Used in Recent Investigations	131

	PAGE
CHAPTER VII—COMMUTATION COLUMNS FOR THE HAMZA	
METHOD OF COMPUTATION—ANNUITIES.....	132
Annuities	133
Calculation of Disabled Life Annuity Values.....	136
CHAPTER VIII—CALCULATION OF PREMIUMS BY THE HAMZA	
METHOD	138
Waiver of Premium Benefit.....	138
Ordinary Life policy.....	139
Limited payment life and endowment policies.....	139
Disability Annuity Benefit.....	140
Payment of Face Amount in Instalments during Disability,	
Balance, if any, Payable at Death or Maturity.....	142
Ordinary Life policy.....	143
Limited payment life policy.....	144
Endowment policy	144
Maturity Instalment Benefit	147
Ordinary Life policy.....	147
Limited payment life policy.....	148
Endowment policy	148
Total Premium for Disability Benefits. Period during which	
Payable	148
CHAPTER IX—CALCULATION OF PREMIUMS BY PENSION FUND	
METHODS	150
A. Pension Fund Method Applicable to Early Types of	
Benefits	150
Instalment Benefits	154
Life policy	154
Endowment policy	155
Maturity Instalment Benefit.....	156
Ordinary Life policy.....	157
Endowment policy	157
B. Pension Fund Method Applicable to Present Day Benefits	157
CHAPTER X—APPROXIMATIONS IN CALCULATING PREMIUMS FOR	
MODERN BENEFITS	163
CHAPTER XI—USE OF MIXED LIFE TABLE INSTEAD OF ACTIVE	
LIFE TABLE	165
CHAPTER XII—ADJUSTMENTS FOR VARIATION IN THE WAITING	
PERIOD	168
First Method: Adjustment in Rate of Disability and Claim	
Annuity Values	168
Second Method: Adjustment in Premium Formula.....	170
(1) Non-retroactive benefit, m months' longer waiting	
period	171
(2) Income and waiver benefits retroactive to commence-	
ment of disability.....	172
Alternative Treatment	173
Valuation of Claims and Comparison of Actual and Expected	
Claims	180

	PAGE
CHAPTER XIII—RESERVES—GENERAL DISCUSSION—RESERVES ON ACTIVE LIVES	181
General Discussion	181
Reserves on Active Lives.....	182
Formulas according to Hamza Method Notation.....	183
Waiver of Premium benefit.....	183
Annuity during disability.....	184
Payment of face amount in instalments, balance, if any, at death or maturity	185
Maturity instalment benefit.....	186
Formulas According to Pension Fund Notation.....	187
Pension fund notation applicable to older types of benefits	187
Pension fund notation applicable to modern benefits	189
Accumulation formulas for active life reserves.....	191
Modern active life reserve practice.....	192
Approximate methods of valuation.....	195
Mean reserves	195
CHAPTER XIV—RESERVES ON DISABLED LIVES.....	197
Waiver of Premium.....	197
Annuity during Disability.....	197
Annual annuity	197
Monthly annuity	198
Payment of Face Amount in Instalments, Balance, if any, Payable at Death or Maturity.....	198
Ordinary Life policy.....	198
Limited payment life policy.....	201
Endowment policy	201
Payment of Face Amount in Instalments for a Term Certain, Commuted Value of Unpaid Instalments Payable at Death or Maturity	202
Mean Reserves	202
Average Reserves	203
Select Disabled Life Annuities.....	204

APPENDICES

	PAGE
APPENDIX A—Specimen Disability Clauses.....	207
APPENDIX B—Specimen Disability Claim Forms.....	223
APPENDIX C—Standard Provisions	227
APPENDIX D—Definition of Symbols.....	236
APPENDIX E—Bibliography	244



CHAPTER I.

HISTORY

While commenting on A. G. Finlaison's Report on the Sickness Experience (1846-1850) of the Friendly Societies of England, J. A. Higham (J. I. A., VII, 115) made the following statement:

"Insurance Companies will confer another boon on men who, in common with their families, depend on their professional exertions, when, on sufficient data, they can afford the means of providing against permanent sickness—the only ill, legitimately within their province, against which they do not afford protection. The man who has insured his life, or contracted for an endowment or annuity yet deferred, and who is disqualified by sickness from continuing his premiums and even from maintaining himself, is in sad case, and to this case we can at present apply no remedy."

It was nearly sixty years after the foregoing was written before life insurance companies added to their policies an income disability coverage on a liberal basis. In Britain this form of insurance has not been largely developed but in America and Canada it has become a popular addition to life and endowment policies.

In Great Britain sickness insurance is principally done by the Friendly Societies which were recognized by an act of Parliament as early as 1793. In America, sickness protection, usually associated with accident insurance in the same agreement, is offered by local societies, private employers, municipalities, trade unions, fraternal organizations and casualty companies.

For a number of years the only form of income protection similar to that in life insurance policies, was that known as non-cancellable disability insurance issued by the casualty companies. Owing to the losses incurred, this form is not freely written now—in fact is granted by a few companies only. While it would be interesting to trace the development of sickness and disability coverage in this country, this study must be limited to a review of total and permanent disability benefits issued in connection with life insurance policies. As its name implies, the original

intention of the companies was to provide indemnity in event of total disability of a permanent nature only. Various types of disability occur, of which we might consider incurable insanity as total and permanent disability, typhoid fever as total disability of a temporary nature, the loss of a foot as partial disability of a permanent nature and a curable injury to a hand as partial disability of a temporary nature.

Contracts

The first life insurance policies containing this form of disability insurance appear to have been issued in Germany in 1876 and by American fraternal orders in the succeeding year. The disability clauses of the fraternal organizations usually provided that in event of total and permanent disability one-half of the face of the policy would then become payable, the balance being due at death. In 1894 Arthur Hunter prepared premiums for a disability policy which would provide for the waiver of premiums under the life insurance policies carried by the applicant in all companies, but on account of financial difficulties, the company which was to grant this coverage was not launched. Soon thereafter, the Fidelity Mutual Life Insurance Company, of which L. G. Fouse was president, issued a policy which provided that in event of the insured becoming totally and permanently disabled he would have the option of making his policy paid-up, or of purchasing an annuity in place thereof, based on a disabled life table. The premiums were prepared by Mr. Hunter and were based on the experience of fraternal orders.

The next company to adopt any form of disability benefit was the Travelers Insurance Company, which, in 1906, commenced to issue the waiver of premium benefit on total and permanent disability. The New York Life Insurance Company followed in 1910 with the same benefit which was then adopted by many companies. After the waiver of premium benefit had been in general use for only two or three years, several companies commenced to issue a benefit consisting of a payment of 10% of the face of the policy annually, such payments to be deducted from the face of the insurance at death. The payments usually ceased when they equalled the amount of insurance and are frequently referred to as "instalment payments". A modification of this type of benefit was subsequently introduced by some companies,

the disability payments being continued during disability, although the total might be greater than the face amount of insurance. These forms may be termed "income payments reducing sum insured". In a variation of the instalment benefit mentioned above, the policy was matured on disability for a twenty year annuity certain of one-twentieth of the face amount per annum. Under this form the company paid at death the present value of unpaid instalments. In case of death before all the instalments had been paid the cost of this benefit would be less than that under the other instalment form. Under either of these instalment benefits the present value, at disability, of future payments was less than the face amount of insurance. Another form of instalment benefit has been issued called the "maturity" type wherein the policy was matured on disability for the full face amount, the proceeds being paid in the form of an annuity certain. This last type was issued by at least one company until a year or two ago under ordinary insurance. This form is still used under group insurance contracts. In event of disablement and subsequent recovery, various courses were followed: one company based its premiums thereafter on the reduced face amount, i.e., the sum originally insured, less the instalments paid. The cash, loan and paid-up values were correspondingly reduced. Another company treated the instalments as a loan against the policy, without interest, the premium remaining the same as at the inception of the claim.

Several companies provided that, after age 60, the premium would be waived on total and permanent disability, but that such amount would be deducted from the face of the insurance at death. This was equivalent to granting a loan without interest equal to the amount of the premium waived. Usually, however, the cash, loan and paid-up values and also the premiums were reduced in proportion to the reduction of the sum insured.

In 1915 a policy was introduced providing for an income benefit of the same amount (10% of the face annually), but without any deduction of the disability payments from the sum payable at death or maturity. The premium was not waived until the anniversary following proof of total and permanent disability, and the first payment of 10% was not made until a year thereafter, so that the first income payment was made one and a half years on the average after disability occurred.

One of the most prominent companies for a number of years provided for the payment of the face amount of the insurance on proof of total and permanent disability, but the form was not generally followed.

The next development (1920) was the payment of a monthly income of 1% of the face of the policy, the payments to commence one month after proof of total and permanent disability.

A further important development, which has had a great influence on the disability experience, took place in 1921 when, in the policies of several companies, continuous total disability of three months (known as a "waiting period" or a "qualification period") was presumed to be permanent. In some companies the income payments began on the first of the month following proof, but the practice was soon changed to provide that payments should begin upon presentation of proof of disability.

In 1926 a number of companies decided to date the monthly disability income from the beginning of disability after there had been three months of total disability, the income being payable for each completed month of disability. This form rapidly came into general use so that at the end of 1929 the majority of the larger companies used it.

Mention should be made of the plan begun in 1922 of increasing the monthly payments with the duration of disability. Two prominent companies provided that the monthly disability income would be increased by 50% after five years of continuous total disability and by 100% after 10 years. Other methods were adopted by a few companies, such as a yearly increase of 10% in the income for each of the first five years of disability, such income remaining constant thereafter during the continuance of total disability.

Beginning in April, 1929 one life company which had previously not engaged in disability insurance in any form issued a disability contract with a definition of disability different from that which had theretofore been generally used, namely that disability should be regarded as total when, by reason of accident or sickness, the insured's average monthly earned income for a period of four months had not exceeded one-fourth of his former earned income. Earned income is defined as wages, salaries and other amounts received as compensation for personal services actually rendered, not including amounts received

as a pension or retirement allowance. The contract was issued as an entirely separate supplementary policy in that the life insurance policy in connection with which it was issued contained no reference whatever to the disability feature. Other special features of the policy are: (1) it does not become incontestable at any time, (2) premiums and benefits are subject to equitable adjustment in the event of material change in the insured's occupation or duties, (3) it terminates when the co-existing life insurance policy terminates unless disability then exists, (4) it is a participating contract, (5) it provides for cash surrender values equal to the net Hunter 3% reserve, and (6) the company is protected against "over-insurance" by a clause which renders the supplementary policy void beyond certain limits depending upon the relation between earned income and total income insurance. (For specimen clauses see Appendix A.)

In 1930 practically all the principal companies modified their disability clauses for new policies in order to conform to the so-called standard provisions. These provisions, which will be discussed later, require, among other matters, a waiting period of not less than four months, with no income payable for the first three months.

In practically all companies a clause is inserted to the effect that the loss of both eyes, or of both hands, or of both feet, or of one hand and one foot shall be considered as total disability although such loss may not seriously hinder a man from carrying on certain occupations. Many companies have liberalized this clause to include the loss of "the use of" both hands, eyes, etc.

Provision is usually made that the companies shall have the right to call upon the insured for proof of the continuation of the disability, but not oftener than once a year after two years of disability. The first part of such a clause was desirable for the protection of the company against continuation of payment of disability benefits after the insured had recovered and the second part for the protection of the insured against having to furnish proof of continued disability too frequently after he had apparently become permanently disabled.

In the original forms the insured was required to be totally and permanently disabled before he could present a claim. There were, however, so few diseases or injuries in connection with which the resulting disability could be called "total and

permanent" that the phrase was broadened to cover "*presumably* permanent" disability. The force of competition then led to a liberal interpretation of this phrase so that many companies considered that if a person had been disabled for six months he was presumably permanently disabled, although it was likely that he would eventually be cured. This liberalization was still further extended so that, for example, a person who had incipient tuberculosis and had ceased to work was considered permanently disabled, although there was a very good prospect of his recovery. It was not only the force of competition but the decisions of the courts which made companies more liberal in their attitude, especially with regard to the insured's inability to carry on *any* occupation for remuneration or profit. The adoption of the provision that three months' total disability should be considered permanent disability was intended to meet the opinions of a number of courts that the rights of the insured should be more clearly defined and to make it easier on the part of the companies to determine the conditions under which total disability should be considered permanent disability.

Premiums

The only available material in the early days for the construction of net premiums was the experience of fraternal orders. The first legal standard for reserves was Hunter's Disability Table and naturally the premiums were based thereon. For several years these tables, based on the experience of three fraternal orders, were universally used, as none other except the Manchester Unity experience which was not considered suitable could be obtained until the companies had developed data of their own. When waiver of premium only was granted or the instalment income, deductible from the face amount of insurance, the net premiums on Hunter's Table were generally sufficient, partly because the old definition of total and permanent disability was used. The experience under the income benefit, however, under which the payments were not deducted from the face of the insurance, indicated that the premiums might be inadequate, especially in companies with a very liberal treatment of claims. This became more apparent when the monthly income was allowed under a liberal interpretation of the policy.

An investigation was made in 1926 of an Inter-Company disability experience by a committee of the Actuarial Society. The experience for the latest forms of contracts was very limited in duration, but approximate premiums were calculated for three types with a monthly income benefit—Class (1) payable on total and permanent disability, and Classes (2) and (3) payable after three months of total disability. Classes (2) and (3) covered the same type of benefit, the only difference being in the companies' attitude towards the disability coverage. Different methods of selection of risks and of administration of claims undoubtedly have a great influence on the resulting claim experience. As a result of the approximate premiums obtained from this investigation an addition of about \$1.00 or more per thousand of insurance was made to the disability premiums when the payment of the income benefit was made retroactive to the beginning of disability. This, however, proved inadequate, and the basis for calculating premiums was made more severe in 1930, and a further increase was made in 1932. The increase in the disability loss ratio has been ascribed to various causes—the education of the public with regard to the advantage of disability benefits, the presentation of more claims than formerly on account of retroactive payments, the effect of more liberal benefits, a greater selection against the company on the part of the applicants, an increasing sickness rate among the population with a decreasing death rate, the legitimate increase in nervous disorders and heart troubles during years of depression, and the illegitimate increase due to unemployment. With regard to the natural increase in disability under certain diseases, the example of tuberculosis may be given. A generation ago one-third of the cases came to the sanitariums in the incipient stage, while now-a-days more than two-thirds are in that condition. The same applies to several other diseases where the insured ceases work and adopts the most advanced hospital treatment which may decrease the death rate but increases the disability claim rate.

When it was evident in 1929 that the basis for premiums should be more conservative, a number of consultations were held by the actuaries of several companies to exchange experiences and to determine what would be a reasonable basis on which to calculate new premiums. The experience of the Mutual

Life (T. A. S. A., XXX, 172) and that of the New York Life (T. A. S. A., XXX, 373) showed that a basis different from the experience of fraternal orders should be sought. A comparison of the Inter-Company Class (3) net premiums, prepared in 1926, with the net premiums based on the experiences of the Mutual Life and the New York Life, showed that there was not a great deal of difference between them. It was then decided to employ Class (3) as the basis of premiums with a suitable loading, although such premiums were founded on a limited experience as to duration. All the large companies and many others throughout the country adopted that basis or something approaching thereto. In 1931 it again became apparent that the basis of premiums should be made more conservative and that benefits should be restricted. Accordingly, in 1932, companies which continued to issue the disability income benefit, generally based the premiums on the rates of disability in Class (3) increased by 65%, with an increase of 3% additional for each age above 45, using such increased rates of disability in conjunction with the Class (3) rates of termination of claims.

A conference of actuaries suggested in 1931 that the monthly income be restricted to \$5 instead of \$10 per thousand dollars of insurance, that the presumptive period of permanency be six months instead of four, that income payments commence at the end of six months and not run beyond the maturity or expiry of the policy, that the disability occur before age 55, and that a much stricter underwriting be adopted, including a further restriction in the number of occupational classes to be covered. Many companies adopted these suggestions in the early part of 1932 while others went still further by entirely eliminating the income benefit. When the waiver of premium benefit is issued alone the disability coverage is generally extended to age 60 and the premium based on the rate of disability in Class (3) increased by 50% up to age 45 with 2½% additional for each age thereafter.

Standard Disability Provisions

Prior to the introduction of standard disability provisions in 1930, there were many features of the disability coverage which

differed in various companies. In 1928, E. B. Morris (P. C. A. S., XV, 9) found that 44 of the 50 largest companies provided disability coverage to age 60 and 6 to age 65, while 10 companies granted a modified coverage above these ages. In 34 companies total disability was presumed to be permanent if it lasted three months, and in 3, if it lasted six months. Proof of permanence of disability was required by 12 companies. In 31 companies the disability income was continued after the maturity of an endowment.

In 1928 the Superintendent of the Insurance Department of the State of New York appointed a committee to consider the whole question of disability benefits and in particular the drafting of standard disability provisions. In his letter appointing the committee he stated that suggestions had been made to him of statutory standard provisions for disability benefits, that some of the prevailing features were undesirable, many of them having been fostered by competition, that "those features should be prohibited which are likely to be misleading to the policyholders, and that there is a real need for a standard definition . . . which will clarify the real meaning of the disability clause for the insuring public".

In December of 1928 the Acting Insurance Commissioner of the State of Massachusetts presented a resolution to the National Convention of Insurance Commissioners, which called attention to the large and persistent losses being experienced by the life companies in the conduct of their disability business and suggested, among other matters, that no disability benefits should be paid unless after six months of total disability, and that disability payments should not be dated back. The resolution was not actually adopted but the Convention appointed a Committee of Department Actuaries to confer on the subject with the actuaries of companies appointed by the Superintendent of New York State. The recommendations of these committees were approved by the National Convention of Insurance Commissioners at Toronto, Canada, in September, 1929. The National Convention added the recommendation that reserves be put on an adequate basis. The recommendations were later put into effect by nineteen states, while fourteen others expressed themselves favorable to the changes but had no authority to put them into effect. The main features were:

1st. Disability benefits should be allowed, whether disability were undoubtedly permanent or not, when total disability had continued for a definite period—not less than four months and not more than one year.

2nd. Monthly income payments should not begin until the end of the fourth month of disability and should not exceed 1% of the face of the policy.

3rd. The waiver of premium benefit could be allowed as of the beginning of disability but at least four months of disability must have existed.

4th. The benefits should not be granted if disability occurred after age 60 nearest birthday, except in the case of endowments and deferred annuities maturing at an age not greater than 5, but in such cases the disability income must cease at maturity.

5th. Disability benefits providing for an income in excess of 1% per month, or for an income increasing with the duration of the disability should not be permitted, nor the so-called Professional Men's disability clause providing for the payment of disability benefits so long as the insured was unable to continue his occupation.

6th. If disability were established as of a date not later than the last day of grace, the disability benefits should be allowed the same as if default had not occurred, provided application were made within a certain period (not less than six months) after default of premium. Under such circumstances, the insured should be liable for the premium in default, with interest.

7th. Dividends should be the same as though disability had not occurred.

There were several other matters which will be considered in their proper place. The Joint Report of the Actuarial Committees is reproduced in full in Appendix C.

The principles outlined by the Joint Committees of Actuaries and adopted by the National Convention were very generally accepted by the insurance companies, and policies in accordance with those principles were put into use as of July 1, 1930.

*Total and Permanent Disability Benefits in
Group Life Insurance Policies*

Total and permanent disability benefits were first incorporated in group life contracts about the year 1915. Although waiver of premium only was provided in a few of the early policies, the usual benefit has been the payment of the face amount of the policy (or its equivalent in instalments) in case of total and permanent disability before age 60. This is known as the "maturity" type of disability benefit. The group policy with such a disability benefit provides, in effect, for the payment of the face amount in event of either death or total and permanent disability, the disability benefit merely anticipating the payment of a death claim. Some companies, under the above provision, have paid the face amount in one sum, while others have paid it in a limited number of annual, semi-annual or monthly instalments for a period not exceeding five years. In case of death before all instalments had been paid, the commuted value of future instalments, usually at $3\frac{1}{2}\%$, would be paid in one sum to the beneficiary. If the insured recovered and returned to work with his former employer, provision was generally made that insurance under the group contract for the amount of the commuted value of any unpaid instalments might be reinstated.

Group contracts have generally provided disability benefits only in event of total and permanent disability. The principal companies have never included in their contracts provision that total disability continuing for a specified waiting period would be presumed to be permanent. Proof of permanency of disability, therefore, is still required by group life contracts.

Certain companies have made the first payment on account of total and permanent disability as of the date of receipt of proof, while others have delayed the first payment for as much as six months after receipt of proof.

The disability experience of a number of the leading companies writing group insurance was found to have become less favorable in recent years. In 1922 the ratio, by number, of the actual to expected claims based on Hunter's table was 54%, while in 1930 the ratio had increased to 158%. Because of the unsatisfactory experience indicated for recent years, a number of the larger companies in 1931 adopted several restrictions in their clauses. Three of the more important changes were:

- (1) Payments are not to begin until at least six months after the commencement of continuous total and permanent disability, and not until at least three months after receipt of proof of the existence *and permanency* of total disability.
- (2) New employees entering after the effective date of the policy are not eligible for disability benefits unless total and permanent disability commences after they have been continuously insured under the group policy for one year.
- (3) Written notice of claim must be submitted within one year after the cessation of premium payments on account of the disability of an employee.

Other companies contemplate restricting the coverage to a waiver of premium benefit only.

See Appendix A, Section III for clause issued at present time by one of the large companies.

CHAPTER II.

THE PRO-RATING CLAUSE

A clause providing that the total income from all policies with disability benefits on a specified policyholder's life shall not exceed a specified proportion of that policyholder's income is known as a pro-rating clause. For example, it might be provided that the total monthly disability income from all companies and societies must not be greater than three-quarters of the insured's average monthly earned income during the last two years and if so the disability income shall be reduced proportionately. In California a bill was introduced in the legislature during 1929 to compel companies to use such a clause. It was intended to protect companies from the unfavorable consequences of a larger amount of disability coverage on a life than was warranted. An accepted principle of disability underwriting is that indemnity in all companies for loss of earned income in event of disability should not exceed a certain proportion of the applicant's net earnings during a specified period prior to disability. This proportion may run from 25% to 75%, depending on the amount of income, the age and other factors. Net earnings through personal efforts and not those arising from invested capital are generally considered. The advantages and disadvantages of a pro-rating clause may be briefly stated as follows:

Advantages

1st. It would protect the companies in event of insurance for an unduly large proportion of the applicant's earned income.

2nd. The coverage should decrease with decreasing income so that, if a man had originally taken an income benefit of \$500 per month and his earned income had decreased through economic misfortune to \$200 per month, the latter figure should properly determine the amount of the coverage.

3rd. The life insurance companies cannot cancel the disability benefits when conditions change with regard to the insured's

finances, although the casualty companies issuing health insurance can generally do so, and accordingly the former are entitled to the protection of a pro-rating clause.

4th. After the insured passes middle life, his earned income in certain occupations is likely to decrease, while the moral hazard due to over-insurance is increased.

Disadvantages

1st. The difficulty of determining the earned income would be considerable in occupations where there are large fluctuations from year to year and where no books are kept as, for example, in the case of many farmers.

2nd. A discrimination would exist against the company which does not have a pro-rating clause in favor of the company using the clause, the former company having to pay its full share in event of over-insurance, while the latter would have its liability decreased or eliminated.

3rd. The difficulty would arise of obtaining the records of all companies in which disability benefits are carried, including casualty companies.

4th. An insured who was properly covered at the time the policy was issued but who could collect only a small income benefit on account of his earned income having shrunk materially during the years immediately preceding his disability would consider the operation of the clause unfair.

5th. Where the insured, in consequence of a cessation in his earned income, has not been covered, the company would be expected to return the cost of the coverage for the unprotected period or make some similar equitable adjustment.

6th. The policyholder should be notified each year before paying premiums that his coverage is only proportional to his earnings—an expensive procedure.

7th. Difficulties would arise in the treatment of claims with different forms of disability in several companies.

The standard provisions adopted by the Insurance Commissioners permitted the use of a pro-rating clause, but only one prominent company took advantage of it. The section of the permissive standard provisions applicable thereto is as follows:

“A proportionate reduction of income payments, accompanied by return of premiums paid on the amount of such reduction, may be made in case the aggregate monthly amount payable to the Insured on account of disability exceeds the percentage specified in the provision (not to exceed 100 per cent) of monthly earned income at date of disability, or alternately at date of application.”

A paper on the subject of pro-rating by J. M. Laird (T. A. S. A., XXX, 40), with the discussion thereon, will give the student the ideas prevailing in 1929. The views of many actuaries may be changed as the result of the limited disability coverage adopted in 1932.

CHAPTER III.

INCONTESTABILITY

Under the laws of many states, a life insurance policy must contain a provision that it shall be incontestable after a certain period. This period is either one or two years from a fixed time which may be the date of the policy, or the date of issue. A number of companies require that the policy be in force either one or two years "during the lifetime of the insured" before becoming incontestable. Unless an exception is stated, this incontestability would apply to the entire policy. There are, however, exceptions allowed by most statutes which have been adopted by a large number of companies, so that many of the incontestability clauses read substantially that the policy shall be incontestable after the period stated, "except for non-payment of premiums and except as to provisions and conditions relating to benefits in the event of total and permanent disability."

There has been, however, some question as to the interpretation of the phrase just quoted. The chief question of doubt is whether the foregoing language would be interpreted by the courts to imply (1) that only the restrictions in the disability clause would apply after the contestable period had expired, such as a restriction that benefits shall not be granted if disability results from military service, or (2) that not only would the restrictions in the clause be permanent but that, as to any claim for disability made by the insured any time after the contestable period had expired, the company could still contest such claim on the ground that the insured had made misrepresentations in his application for the insurance. A recent decision of the New York Courts (*Connecticut General Life Insurance Co. vs. Jos. S. Brandstein*—1931) has aided materially in establishing the latter interpretation. The policy in that case read as follows:

"INCONTESTABILITY. This policy shall be incontestable after it shall have been in force during the lifetime of the Insured for two years from its date of issue except for nonpayment of premiums and except as to provisions

and conditions relating to benefits in the event of total and permanent disability and those granting additional insurance specifically against death by accident; but if the age of the Insured be misstated, the liability of the Company shall be the amount of insurance which the premium would have purchased at the correct age."

The company sought to have the disability provisions stricken from the policy long after the contestable period, on the ground that fraud and misrepresentation had induced the company to issue the disability coverage. The plaintiff contended that, since the contestable period had long since expired, the company could not contest on the ground of misrepresentation. Upon appeal to the Appellate Division, the Court sustained the holding of the Lower Court that the exceptions stated in the incontestability clause clearly intended that the company could always defend a claim for disability if misrepresentations were made in the application for the policy.

This one decision does not entirely settle the question and, in view of the remaining doubt, at least one company has attempted to avoid the difficulty by granting the disability coverage in an entirely separate contract to run concurrently with its life contract issued to the insured at the same time. Although no court decisions have been rendered in such a case, it seems that the matter could be handled in this manner, so as to avoid the incontestability clause in so far as the disability coverage is concerned.

When a policy has a one year contestable period with regard to the life insurance provisions, claims are occasionally made for disability benefits in the early part of the second year which indicate that the policy was obtained by misrepresentation. On account of the great incentive to fraud under the present liberal forms of disability benefits, several companies have recently lengthened their contestable periods from one to two years, thereby giving them a longer time within which to rescind policies fraudulently obtained. The basis for such cancelation is sometimes the information given by the policyholder in connection with claims for disability benefits made during the second policy year. Instances have occurred where, through conspiracy, persons with tuberculosis had been insured and had been advised not to make claims until after two years had expired, but there

is less incentive in such cases to apply for a policy in a company with a two year incontestability clause than in a company with a one year clause. As a further protection one company, for example, stipulates in its disability clause that benefits are payable only if disability did not arise from bodily injury or disease occurring before the insurance under the policy took effect and known to the insured but not disclosed in the application for the insurance.

CHAPTER IV.

CAUSES OF DISABILITY

It is evident from the published experiences of different companies that the causes of disability in relation to the total disability claims depend upon the nature of the disability provisions, the liberality of their interpretation, and the number of years during which the company has been granting the benefit. A company which granted the waiver of premium on proof of total and permanent disability would obviously have a different distribution of causes of disability than one which granted a 1% monthly income benefit, such payments commencing at the end of three months of total disability. For example, in the first mentioned company the percentage of claims from tuberculosis might be 45% of the total, while in the second company it might be less than 15%. The claims from accident might be 4% in the company with the waiver of premium only, and 20% in the case of the company allowing the benefits after three months of total disability. To illustrate this point a table is given of the experience of a company which had granted the waiver of premium only upon proof of total and permanent disability, the six principal causes covering 82% of the claims.

WAIVER OF PREMIUM ON TOTAL AND PERMANENT DISABILITY

Causes of Disability	Percentage to Total
Tuberculosis of lungs.....	44%
Insanity, melancholia, paresis.....	23
Accidents	5
Rheumatism, arthritis, neuritis.....	4
Heart disease	3
Cancer and other malignant tumors.....	3

For comparison a table showing each cause accounting for 3% or more of the total, based on 6,000 claims, is now given from the experience during 1930 of a company under policies with the three months' waiting period with retroactive payments issued during the years 1926 to 1929 inclusive.

THREE MONTHS' WAITING PERIOD WITH RETROACTIVE PAYMENTS

Causes of Disability	Percentage to Total
Accident	19%
Tuberculosis of lungs.....	12
Rheumatism, arthritis, neuritis.....	7
Appendicitis	6
Pneumonia, influenza, pleurisy, asthma, bronchitis	6
Diseases of nervous system.....	5
Cancer and other malignant tumors.....	5
Heart disease	3
Ulcers of stomach and duodenum.....	3
Insanity, melancholia, paresis	3

Policies with a three months' waiting period but without retroactive payments had only a slightly different distribution by cause of disability, as will be seen from the following table:

THREE MONTHS' WAITING PERIOD WITHOUT RETROACTIVE PAYMENTS

Causes of Disability	Percentage to Total
Accident	16%
Tuberculosis of lungs.....	11
Rheumatism, arthritis, neuritis.....	6
Cancer and other malignant tumors.....	6
Diseases of nervous system.....	5
Heart disease	5
Insanity, melancholia, paresis	4
Pneumonia, influenza, pleurisy, asthma, bronchitis	4
Appendicitis	3
Ulcers of stomach and duodenum.....	3

Different distribution of causes of disability would also appear in the case of two companies with identically the same contract but with different treatment of claims. Under a policy providing for approval of claims on proof of the permanency of disability the conservative company might assume that an insured was not permanently disabled until it was quite evident that he would not recover, while the other might assume all cases permanently disabled when total disability had existed for six months. In the case of tuberculosis there would be a wide difference of treatment if one company did not grant disability benefits until there was little chance of the insured's recovery, and if another allowed such benefits on the insured giving up work or going to a sanatorium, irrespective of the prognosis. It hardly seems

necessary to give further tables showing the difference in the distribution of causes of disability as it would vary widely with the rules in practice in the various companies.

There are other matters which would enter into the different distribution of causes of disability, several of which are now given:

1st. If one company had been granting disability benefits for twenty years and another had recently commenced to issue them, there would be a larger proportion of the diseases of middle life in the former and of those peculiar to the younger ages in the latter.

2nd. The age after which the disability coverage ceases must also be considered. If one company granted the disability coverage under the monthly income plan until age 65, there would be a larger percentage of degenerative diseases than in another with coverage ceasing at age 60, assuming that both companies had been granting the disability benefits for a sufficient length of time.

3rd. The practice of a few companies of extending the waiver of premium automatically to all policyholders under a specified age would affect the distribution by cause as compared with companies which granted the waiver of premium to new entrants only.

4th. The underwriting rules of the companies, whether granting disability benefits to substandard risks at an extra premium, or limiting them to first-class risks from the disability standpoint, would affect the distribution by cause.

5th. The practice of the companies with regard to paying disability benefits after death, although no claim had been made during lifetime, or the practice of allowing such benefits after recovery or of dating back claims to the beginning of disability would have an influence on the distribution by causes.

6th. Where the premium was not waived until the ensuing anniversary or the income benefit was not obtainable until a specified date, there would be a tendency to delay the presentation of claims until immediately before benefits could be received. Hence a number of cases would not be presented which would appear if the disability were payable at the end of three months of total disability, for instance. This would reduce the causes with short duration of disability such as accident.

CHAPTER V.

TERMINATIONS AMONG DISABLED LIVES

While this subject will be dealt with in detail in connection with the statistical basis for tables and the method of calculating premiums and reserves, a general statement may properly be made here.

The rate of termination (recovery and death) largely depends upon the same factors as the rate of disability, such as the type of the disability benefit, the interpretation of the clauses, and the liberality in the treatment of claims. Under a clause with a three months' waiting period there would be a larger percentage of recovery in the early months of disability than under a policy providing for total and permanent disability. As an example of this, we may give a brief table showing the experience of the companies in the Actuarial Society's Inter-Company investigation of 1926, comparing Class (1), which did not have the 90 day clause, with Class (3), which covered cases with the 90 day clause under a liberal treatment of claims. The table shows the ungraded rates of termination among disabled lives per 1,000 exposed, for all ages combined.

RATE OF TERMINATION PER 1,000 AMONG DISABLED LIVES		
Period after Date of Disability*	Class (1)	Class (3)
1st quarter year.....	74	267
2nd " "	80	220
2nd half "	148	291
3rd " "	133	150
4th " "	126	112

As a further illustration of the termination rate we may take the experience of the New York Life (T. A. S. A., XXX, 373) under two recent forms of disability coverage. Under the issues of 1921 to 1925 there was a waiting period of 90 days, the first payment to commence at the end of that time, while under the issues of 1926 to 1928, with the same waiting period and the same definition of disability, the disability payments were made retroactive, the first payment at the end of three months being

* See Inter-Company definition of "Date of Disability" on page 103.

\$30 per thousand. The figures are given for each of the first six months after the first 90 days of disability, the differences being most marked during that period.

CRUDE MONTHLY TERMINATION RATES PER 1,000
Based on Amounts Insured for Both Sexes, All Ages Combined

Period after Date of Disability	Issues 1921-1925	Issues 1926-1928
1st month	93	191
2nd "	95	142
3rd "	93	136
4th "	94	114
5th "	91	110
6th "	73	92

As an illustration of the experience of different companies under the same benefit, we may compare the Class (2) and Class (3) termination rates of the Inter-Company investigation, 1926. These two classes covered the experience under policies with similar disability clauses but in Class (3) the companies were more liberal than those in Class (2) in the treatment of claims.

RATE OF TERMINATION PER 1,000 AMONG DISABLED LIVES

Period after Date of Disability	Class (2)	Class (3)
1st quarter year	180	267
2nd "	216	220
2nd half "	305	291

The differences in the rates of termination under different conditions clearly indicate the desirability of each company preparing its own disability experience and analyzing it with a view of determining not only the sufficiency of the premiums but also the practice of those selecting the risks and passing upon claims.

There are of course instances of recurrence of disability after recovery. In one company 8% of the total recoveries became disabled again (from the same cause) after recovery, the insured in all cases having recovered a sufficient time before the recurrence to have paid a premium. The rate of recurrence would depend upon the factors which have been already mentioned in connection with the rate of termination.

The differences between the termination rates for men and women are discussed in Chapter VI.

CHAPTER VI.

SELECTION OF RISKS

The selection of risks under disability benefits has become almost as important as that under purely life insurance policies. An applicant may be a standard risk for life insurance but a substandard one for disability benefits. For example, two attacks of rheumatism might not result in an applicant being charged an extra premium or declined for life insurance, but they might result either in a refusal to grant the disability benefits or in the charge of a substantial extra premium. We shall first deal with the question of occupation.

Occupation

The principal hazard in the majority of occupations is accident. With this in mind accident and health underwriters have prepared classification pamphlets with ratings for the various occupations based on the probability of non-fatal accidents and on the probable duration of the disability. Over 3,000 occupations have been grouped under ten general classifications, but the selection of risks is practically limited to the four or five most favorable occupation classes. Examples of the better class of occupations would be bank officials, lawyers, office clerks (100%); commercial travelers, clergymen, illustrators (120%); and dentists and physicians (140%). The higher classifications would be represented by barbers, bakers, embalmers, building contractors (170%); chauffeurs, butchers, policemen (200%); and railroad engineers (300%).

In determining the health hazard a classification pamphlet, such as has been referred to, must not be given too much weight as the underwriter must be governed by his knowledge of the general conditions prevailing in the various occupations. For example, doctors are exposed to septicaemia, stone cutters to dust, blast furnace employees to extreme heat, cold storage employees to sudden changes in temperature, pottery workers to

dampness, workers in certain chemical industries to fumes, and manufacturers of paints to lead poisoning, all conditions detrimental to health.

For many years life insurance companies have issued occupation pamphlets which have stated whether disability benefits would be granted or not. It has been the practice to decline to grant such benefits to those who have a higher liability to sickness or non-fatal accidents than the normal. Those who are subject to extremes of heat and cold, to dust and fumes, are excluded from the coverage, and also those in the "industrial" group, such as laborers. It may be mentioned, as examples, that the following are generally excluded from disability coverage: drivers of newspaper delivery wagons and public trucks, workmen in cement mills, cotton-gin employees, dynamo tenders, employees of city fire departments, furniture movers, furriers, etchers in glass, granite cutters, grinders of metal (dry process), underground miners and workmen in asbestos mills. The experience of the companies has indicated that further restrictions than formerly should be made. One of the principal occupations on which there has been a change of sentiment is that of farming. A number of companies will not now grant the income disability benefit to farmers who do not employ help, and in fact, a few will not grant even the waiver of premium to any farmer unless he acts in a supervisory capacity only.

In certain occupations such as farming, teaching, and nursing, there is a seasonal hazard. In other occupations there is a hazard due to irregularity of employment as disability is most likely to occur during periods of enforced idleness. There is also an extra disability hazard in the case of persons who have markedly fluctuating incomes, such as vocalists, violinists, lecturers and the like. Actors, especially those connected with moving pictures, who have made very rapid successes, are poor disability risks.

Medical Impairments

There are a number of impairments which do not result in the applicant being treated as a substandard risk from the standpoint of life insurance, but which would be substandard from that of disability. In the following list appear the principal of

these impairments, the best of which are usually accepted for insurance at regular rates:

- Albuminuria, intermittent
- Albuminuria, history of, constant, not now found
- Asthma, history of, moderate or severe
- Family history of two deaths among parents, brothers or sisters from cancer, apoplexy, insanity or tuberculosis
- Gastritis, history of
- Hernia in manual workers
- Insomnia, history of, severe
- Lumbago, history of, recent
- Malarial fever, history of, more than one recent attack
- Neurasthenia, history of
- Neuritis, history of
- Otitis Media
- Rheumatism, history of
- Underweight at young ages
- Varicose veins in manual workers

The foregoing is not a complete list but covers the main types which occur in practice. It would not be feasible to suggest a treatment for several of them, as such treatment would depend upon the severity of, and the number of years elapsed since the attack. For example, a history of rheumatism of a severe nature within the last year would call for entirely different treatment from an attack of a mild nature five years ago. Where such cases are accepted at the regular rates of premium for life insurance the practice is to increase the disability premium by a percentage of itself, to charge an extra premium per thousand of the face of the policy, or to limit to the waiver of premium, with or without an extra charge. In recent years the tendency is not to grant income benefits in cases of an extra hazard from the disability standpoint.

Where the insured is a substandard risk for life insurance and an extra premium is charged or a lien imposed, it is the general practice to decline the applicant for disability benefits under serious impairments in which there is likely to be a long continued illness. There are, however, impairments on account of which the insured is not likely to be disabled for a long period, under which the disability benefits may be accepted at the regular rates or at an increase thereon. It was the practice of many companies to include certain types of heart murmur in that classification, but a paper of Arthur Hunter's (T. A. S. A.,

XXXII, 31) has modified this action. He showed that where disability claims had been approved the duration of the disability was slightly greater than the average for all causes of disability. Impairments which may be considered with a suitable extra for income disability benefits or which should be limited to waiver of premium, are:

- Albuminuria, constant, small amount
- Alcoholic habits, best types
- Cystitis, history of
- Gallstones, history of attack, not recent
- Gastric and duodenal ulcers, history of, several years ago
- Glycosuria, small amount of sugar
- Goitre, except severe and except exophthalmic
- High blood pressure, moderate degree
- Lung trouble, old history of, cured
- Overweight, moderate

In the case of an attack of, or operation for, gastric ulcers, the most favorable treatment calls for a rating as high as three times the normal after five years have elapsed. A more general treatment is to refrain from granting these benefits until at least six to ten years have passed since the attack or the operation. In the case of duodenal ulcers a more favorable attitude is taken, provided the diagnosis is confirmed by operation, X-ray or other satisfactory evidence. The most liberal treatment is to grant disability benefits after two years have elapsed since the operation and the least liberal to require a waiting period of six years with twice the usual rate for the following four years. With regard to other cases in the foregoing list, there are many disability claims occurring among persons who gave a history of high blood pressure, and accordingly companies are disposed to decline to grant disability benefits except where there has been only a moderate degree of it. The disability benefits are not usually granted with a history of lung trouble or tuberculosis unless a number of years have elapsed since the attack, and the applicant is above the average weight.

There are of course a number of types in which disability benefits are not granted, such as applicants who use alcoholic beverages too freely, those with a fragile appearance, and those with a history of syphilis.

A number of policies provide that the loss of the use of both feet or of both hands, or of a leg and an arm constitute total

disability, thus covering the case of a man who has become paralyzed and may be worse off than if he had lost both legs. This applies whether or not the insured is able to engage in any work. Some companies are experimenting with charging an extra premium for disability to persons who have lost a hand, or a leg, or an eye. The accident hazard of the applicant must be considered in connection with granting such coverage. His occupation should not throw any strain on the remaining member or increase the chance of losing it. In the case of the loss of one eye it is not usual to grant disability if the loss of the eye was due to disease. An extra charge of \$1.00 or more per thousand of insurance is usually made for the income benefit, a smaller charge applying to the waiver benefit only. A few other companies insert a clause in the policy providing that if one member is lost, the loss of the other member or the loss of its use does not constitute total and permanent disability.

The question of build, i.e., the relation of weight to height, may now be considered. Among lightweights at the younger ages there is a marked tendency towards tuberculosis and anaemia, which results in the disability benefits being frequently declined or an extra premium charged. Overweights at the older ages are more liable to heart disease, cancer and Bright's disease, which may result in a long period of disability. Some of the other principal causes of disability, such as accident, rheumatism and arthritis do not seem to be dependent to a marked extent on build. With the exception of the young underweights and the elderly overweights, the general practice of the companies in connection with build is to treat the disability benefits in the same general way as the mortality hazard, i.e., if standard insurance is granted then disability benefits are usually issued, and if an advance in age is charged for overweight, the same advance is made for the disability benefits or a suitable extra premium charged.

The general tendency at present is toward greater strictness in granting the disability coverage because of the continued unsatisfactory disability experience of many companies. This results in a smaller percentage of policies being issued with the income disability benefits.

In connection with the selection of risks for disability benefits, the rates of recovery and death after approval of the disability

claim should be considered. The paper by Mr. Hunter just referred to (T. A. S. A., XXXII, 31) gives a comparison of these rates of termination for the principal impairments, with the average rate for all impairments under claims with a 90 day qualification period. The actual cost, under disability claims depends, however, not only on the value of the termination rate, but also on its incidence. Comparing termination rates alone does not give, therefore, a complete comparison. For our purpose it is necessary that we compare the values of disabled life annuities. In Mr. Hunter's paper cost indices, based principally on select temporary disabled life annuities, are assigned to the more important causes of disability. Temporary annuities were used as the data, when subdivided by sex, age and cause, were not sufficiently extensive to determine life annuities. It appears that the cost was distinctly lower than the average from accident, cancer, pleurisy and pneumonia, gastric ulcer, duodenal ulcer, gall bladder troubles, nephritis, cardio-vascular-renal disease, and the group of cystitis, pyelitis and prostatitis. The cost was higher than the average from tuberculosis, insanity, asthma and blindness. From rheumatism the cost was slightly below the average and from neurasthenia, practically the average. The cost from heart disease was low at the younger and high at the older ages, where most of the claims were incurred. The low cost was sometimes due largely to a high proportion of recoveries, such as under accident claims, or to a high proportion of deaths, such as under cancer claims.

Moral Hazard

The factor of moral hazard must be considered with the utmost care in connection with the selection of risks. Applicants with the reputation of being dishonest or untrustworthy would obviously be poor risks for disability benefits, particularly with the income feature. Applicants who have retired from business or who have practically given up work and those with incomes principally from investments should not generally be granted the income benefits. Environment and family relations may also be a factor in selection.

It may be necessary in the case of those actively at work to consider the pensions to which they are entitled, in determining

the amount of disability income to be granted. Let us consider a city employee with a salary of \$150 per month who has worked for the city for fifteen years and who could retire after twenty years service with a pension of from 50% to 75% of his wages. The temptation to stop work five years hence and claim disability benefits should be taken into account, especially as in certain branches of municipal service serious obstacles are not put in the way of retirement on the ground of ill health.

We have already dealt with the question of moral hazard in connection with certain occupations, in which there is seasonal work and irregular employment.

Maximum Amount of Coverage on Individual Lives

The maximum amount of insurance issued at their own risk by the large companies until the year 1931 under policies with both waiver of premium and income disability benefits was not more than \$50,000 on first class risks. Such companies would generally grant in addition thereto the waiver of premium benefit on another \$50,000. Where a company had, for full coverage, a maximum of \$50,000, the waiver of premium only would usually be issued up to \$250,000. The rule seems to prevail of considering insurance with waiver and income benefits as equivalent to three to four times as much insurance with waiver alone. The maximum coverage is not usually granted at the younger and older ages. On account of the general unsatisfactory experience in the last few years there is a marked tendency towards reduction of the coverage. Several large companies, for example, which granted \$50,000 with income disability and waiver, or \$200,000 with waiver alone, have reduced these amounts to one-half; one company has reduced the coverage with income benefits but retained its maximum for waiver of premium only, while another is granting income benefits of \$125 per month as a maximum.

Each company has its own rule for determining the maximum amount of disability benefits, in relation to the applicant's income, which should be granted in individual cases. For moderate amounts of insurance many companies do not wish the disability coverage on men under all policies to be more than 50% of the average *earned* income or more than \$750 per month; other companies are willing to grant up to 60% or 70% in special cases.

On the other hand, there is a strong feeling that the disability coverage should be greatly restricted in policies for large amounts. In the case of women the maximum is much smaller than on men. Some companies do not grant disability benefits to women for a larger amount than 25% of their average earnings. On impaired risks the maximum amount is much less than on standard lives, depending on the nature of the impairment and the age of the applicant. Obviously, the amount granted would be greater under an impairment likely to result in a short than in a prolonged illness.

Women

The practice of the companies in the early days of disability benefits was not to grant any form of such coverage to women. It was not long, however, until a number of companies gave to self-supporting spinsters the waiver of premium benefit only and later the instalment income deductible from the face of the policy at death. When the 1% monthly income policy was generally adopted a wide difference arose in the treatment of applicants. Some companies declined to grant disability benefits to any woman and have continued that attitude. At the other extreme were companies which treated women for moderate amounts of insurance exactly the same as men, whether they were self-supporting or dependent, single or married. There were many intermediate practices, such as those adopted by the companies which allowed the disability feature to self-supporting women, whether single or married, and those which accepted self-supporting single women or widows, with a clause providing for the cessation of the coverage on marriage or remarriage.

In 1929 the experience of the New York Life Insurance Company on women was published (T. A. S. A., XXX, 373). It appeared that, based on 4,265 disability claims on women, the net premiums required were approximately twice as large as the corresponding premiums for men. In the discussion of the paper W. A. Hutcheson stated that "the Mutual Life's disability rates for women as a class were about twice those of men, while the termination rates were slightly less for women than for men". A. T. Maclean stated that the Massachusetts Mutual's claim rate was approximately 50% more than on male lives, and

that the rate of recovery had been less than on men. As a result of these unfavorable experiences, which had been confirmed by those of other companies, a marked change came in 1930 with regard to the treatment of women. Many companies began to charge women twice the disability premium for men. They also restricted the classes to which disability would be granted. If they had been giving disability benefits to factory workers, employees in stores, and the like, it was discontinued. A more severe selection was made with regard to certain other occupations, such as nursing, the practice ranging from restriction to the waiver of premium at two or three times the premium for men to declination of any form of disability. Women who owned, or whose husbands owned, small stores and who worked in the stores either occasionally or regularly, were found to be poor risks from the disability standpoint.

Some companies decided to give the disability benefits to women at the same premium rate as to men, but to restrict the monthly income benefit to \$5 instead of \$10 per \$1,000 of insurance, and to terminate the coverage at age 55 instead of at age 60. A few companies continued to issue policies for moderate amounts to single or married women, self-supporting or dependent, but the majority limited the coverage to single self-supporting women with a provision that the coverage would cease at marriage. Towards the close of 1931 most companies decided to limit women to the waiver of premium only.

In the early papers and discussions in connection with disability benefits on women it was generally admitted that the rate of disability would be higher than among men, but it was thought that the rate of recovery would be greater, that is, that the period of sickness would be less. From recent statistics this does not appear to have been the case, the rate of recovery having been practically the same as among men, although the rate of disability has been from 50% to 100% higher. Several companies, which allowed disability benefits to women on the same terms as to men, were prepared for a higher cost on disability as they expected to have a greater saving on mortality and also greater profit on the accident cost under the double indemnity provision. The higher cost, however, for disability benefits has evidently been greater than the savings from the other two sources.

In the selection of risks reference has already been made to certain occupations in which there has been an unfavorable disability cost. Unfortunately, there has been published very little on the subject of the rate of disability among women according to occupation. Most of the disability investigations made by insurance companies under the different occupations have been by approximate methods on account of the great labor of making investigations on exact lines. Comparisons have generally been made of the proportion of business issued to women in the principal occupations with the proportion of female disability claims to total claims in the same classes. This had to be supplemented by investigations to determine whether the ages were the same in the different classes compared, and whether the rates of termination (recovery and death) were approximately the same. From these investigations it appeared that certain classes, such as factory workers, domestic servants, waitresses, employees in stores, and women proprietors of small stores, had a high cost of disability. The experience of several companies showed that nurses had a high rate of disability, and this was confirmed through investigation of a large beneficial society for nurses. In general, the actuary can determine from a study of the details of the occupation, the social and sanitary surroundings and the class usually employed, whether or not the disability rate is likely to be above the average for women. Although it had been anticipated that teachers would have a higher rate of disability, the unpublished experience of a large company showed that while the cost of disability benefits on account of nervous troubles was high, it was practically offset by the low cost of some other causes, notably tuberculosis.

With regard to the selection of risks among women on account of medical impairments, a separate technique has not been developed from that among men. Greater consideration, however, is given to a history of nervous prostration, rheumatism, and other similar diseases of which there may be but few, if any, physical signs. There is not the same incentive to attend to office work as in the case of men, and where married women are insured it is difficult to determine whether or not they are able to carry on their usual household duties. Histories, therefore, of diseases which might largely though not totally disable, if they recur, are more significant in the case of women than of men.

While there has been no experience published with regard to women who were married at the date of application, or who were in a gainful occupation at the date of application for insurance and who subsequently married, the unpublished experience of one company does not indicate any marked difference between the cost of the disability on these two groups and that of unmarried wage earners.

CHAPTER VII.

SURRENDER VALUES

The almost universal practice of life insurance companies is to grant the same surrender values for policies with disability benefits as are given for policies without disability benefits—a fact which cannot be considered independently of the history of the disability business. It has been stated that the earliest form of disability benefits provided only for waiver of premium in event of total and permanent disability, and was issued at a very small premium. The reserves for this benefit on individual policies were negative under certain plans and at many durations, and when positive were never sufficiently appreciable to warrant any adjustment in the ordinary surrender values. In the event of disability the policy provided for the regular increase in the non-forfeiture values, and in the event of subsequent recovery the insured was in the same position as if the premiums had been paid. Even when disability provisions were changed to include the payment of income the reserves for the older forms were not large, and the established practice of not adjusting surrender values was continued except for one form of policy. Where the disability income benefit consisted of the payment of the face amount in instalments, it was usually provided that the non-forfeiture values were subject to reduction in the proportion which the amount paid under the disability benefit bore to the face amount of the policy. This avoided the condition which might otherwise have existed of the cash value at some time exceeding the remainder of the face amount.

Commencing with the introduction of the 90 day clause, the disability provisions of most companies underwent rapid and substantial changes. Experience with the innovations was lacking, so that appropriate premiums and reserves could not be based upon any existing table. At this stage of development there were, therefore, two reasons for not granting any surrender values. First, the expense of providing even the simplest form of surrender value was disproportionate to the amount involved. Second, the business was in an experimental stage, which made it

desirable to maintain the conservative practice of making no adjustment in the regular surrender values. The subsequent experience of most companies adopting the liberal disability features was not favorable, so that the question of allowing surrender values was never an active one. It was not until 1929 that an American life insurance company for the first time provided a cash value in respect to its disability benefits. The benefits were provided in a separate supplementary contract which also contained other features differing from those in general use.

Theoretically there is no reason for not allowing some form of surrender value. Under the present disability coverage the reserves are no longer trivial so that the practical objection to the expense of dealing with them does not apply with the same force as formerly. While a working basis for the calculation of reserves has become available by the publication in 1929 of monetary values based on the Class (3) experience contained in the 1926 Report of the Committee on Disability Experience there still remains the generally unfavorable experience of recent years and the lack of knowledge of the future which hinders the introduction of surrender values at this time. The standard provisions for disability benefits which were adopted by many states in 1930 do not provide for surrender values.

CHAPTER VIII.

DIVIDENDS

Under the earliest forms of disability benefits, it was the common practice for companies to charge a disability premium which was sufficient to waive only the net premium for insurance. During disability some companies hold that there should be no dividend from loading under these forms, as the loading is not assumed to be paid, or from mortality savings as the insured is an impaired risk, and that accordingly the only source of profit is excess interest earnings. Such earnings are generally paid on the regular life insurance reserve held by the company and not on the total reserve, including that carried for future disability payments. If the disabled policyholder is allowed excess interest on the claim reserve, there might properly be charged against the dividend an expense charge for administering claims. Another method is to allow the excess interest earnings on the regular life insurance reserve plus the same mortality savings as under premium paying policies, on the ground that the company determines its dividends from the mortality experience among the total group of policyholders and not among active lives alone.

One of the standard disability provisions prescribed in 1930 by the insurance departments of various states is that the same dividends shall be payable during disability as those which would have been payable if disability had not occurred. An exception is permitted when, in lieu of disability income payments, there shall be payable an annuity certain for a period of not more than ten years, the present value of which shall be equal to the amount of insurance. In determining the premium, therefore, for the waiver of premium benefit, the actuary must provide for the waiver of the gross and not the net premium. In the event of disability a bookkeeping transaction transfers the amount of the life insurance gross premium from the disability reserve funds to the premium income account, and the company makes the same distribution of surplus in respect of the life insurance as if the insured had not been disabled.

Commencing with the very small premiums charged for the early forms of waiver of premium, additional dividends had seldom been paid to active lives on account of the disability benefits. As new disability clauses were evolved, the premium rates were increased, but these have apparently not been sufficient in many companies to cover the cost of the more liberal benefits. On account of the loss on disability benefits the question of the treatment of dividends under policies with these features was brought to the attention of insurance companies authorized to do business in New York State by the Superintendent of Insurance, on the 11th of October, 1930. After quoting from the Gain and Loss Exhibit to show the losses from disability benefits during 1928 and 1929, he makes the following statement:

"In the case of a mutual company, this means that holders of participating contracts which do not contain a disability benefit, or which do not contain a disability benefit of substantial form, are being assessed with a large share of such annual deficiencies. Further, as the gross disability premiums charged for the business issued prior to July 1, 1930 are undoubtedly inadequate in many cases, such losses will in all probability continue, in fact may increase. I would therefore request that each company give this matter careful consideration and advise me what action, if any, is contemplated to correct this inequity through an adjustment in dividends."

One prominent insurance company had already been charging for the preceding three years part or all of its losses on disability benefits against the policies with these features. Another company adopted the suggestion of the Superintendent of Insurance of adjusting the dividends under policies with disability benefits for the year 1931. At this time the losses for 1931 appear to be as great as for the year 1930, and a number of mutual companies will be faced with the question of readjusting their dividend scales, unless they decide to charge such losses against the general surplus. One method is to charge the losses against all policyholders, old and new, whether with or without disability benefits. Another is to take into account in the formula for the distribution of surplus under policies with disability benefits, the loss from these features. If the second plan is adopted the question of the method requires very serious consideration. Should the adjustment be made by age at issue and

by plan of insurance? If so, that requires a very thorough analysis of the experience of the company and, when subdivided into the different types of disability benefits, may not be a practical basis for the adjustment of dividends. It seems inadvisable to discuss this subject further until the matter has been fully developed by the companies themselves.

CHAPTER IX.

ADDITION OF DISABILITY BENEFITS TO POLICIES
PREVIOUSLY ISSUED, CHANGES IN DISABILITY
BENEFITS, REINSTATEMENT

The companies are frequently asked to add disability benefits to policies which have been issued without them and also to grant the modern benefits to those with the earlier clauses. The former includes cases where disability benefits were originally refused on account of some defect which has now been removed, or the insured had not known of the disability benefits and had therefore not applied for them.

Where the insured could have had the disability benefits if he had applied for them, the practice of a number of companies is to refuse to add the benefits to these policies. The principal reason for this action is that the expense of making the change absorbs a good part of the first year's premium for the disability benefits, as it involves rewriting the policy or adding a rider, changing the records and paying commission to the agent. It may also be the opinion of the officers of these companies that the disability risk is greater on those who ask to have the disability benefit added to their policies than among those who applied for it originally. Where the benefit is granted it is customary to require the applicant to pay the fee for the medical examination in connection with the change, unless he is taking additional insurance. Many companies freely add the benefits, advising their agents to endeavor to obtain new insurance through this introduction, while others discourage the practice by paying no commission to agents on the disability premium. This applies both to adding the present form of disability benefits and to changing a less to a more liberal form.

When changes are made three methods may be adopted, partly depending upon the age of the policy and the plan,—(a) payment of the back premiums with interest, (b) payment of the reserve, the premium being taken at the original age at issue, (c) payment of the premium at the attained age. Under (c) the insured would be required, for example, on a Twenty Payment

Life policy issued four years ago at age 35, to pay the disability premium for a Sixteen Payment Life issued at age 39. As method (a) involves an overcharge and as it is sometimes difficult to convince applicants of the propriety of (b), the third method is usually favored.

Method (b) may be termed "original age" method and (c) the "attained age" method. The original age method has an advantage to the company in that the total premium charged for a policy can be ascertained at all times by reference to readily available publications such as the agents' rate books. Knowledge of the date of change and special tables of premium rates are necessary when the change has been made at attained age. The original age method causes comparatively little extra work in the valuation of the reserve liability, but an exact calculation of the liability under disability benefits which have been added or changed by the attained age method is laborious and requires special treatment. As indicated by J. S. Thompson in his paper "Changes in Policy Contracts" (T. A. S. A., XXIII, 204), the attained age method was largely avoided until comparatively recently for this reason, and also for the reason that the amount required by the original age method was not very large. In the past decade, however, the premiums for disability benefits have been increased substantially and the cost of adding benefits by the original age method is now considered unsuitable by many companies. There is a small overcharge in the attained age method. For instance, in the example above the insured pays the disability premium for a Sixteen Payment Life policy and obtains, in event of disability, the waiver of only a Twenty Payment Life premium. This overcharge, however, is usually quite small and applies only to the waiver portion of the benefit. If desired, a slight adjustment in the waiver portion of the premium may be made to allow for the amount of the premium which will actually be waived if disability occurs. This would require, however, a separate calculation for each case.

Where a form of benefit is changed to a more liberal one, the general procedure is the same as already outlined except that credit is sometimes given for the reserve, if any, on the existing coverage. If the policy were issued with waiver of premium only or with instalment benefits deductible from the face of the policy at death or maturity, requests for change to the

monthly income benefit are becoming less frequent, especially as the companies have increased the bases of their rates under the recent forms.

For changes in plan of insurance with a continuation of the same form of disability benefits, the adjustment is usually made as of the original date of issue on the basis of the difference in disability reserves. If, however, the company's basis for changing insurance to a higher premium plan as of the original date is to charge the differences in premiums with interest, the adjustment may also be carried out on the "difference in premiums" basis. By either method an adjustment for negative reserves would be disregarded.

In connection with requests for addition to or change of benefits the companies generally require a statement of the amount of disability in force under all forms of insurance, of present occupation, and occasionally of participation in airplane flights. Information regarding the insured's financial position is usually obtained by an inspection report, unless the change desired is not material. The medical requirements generally depend upon the increase in the company's risk, and vary from a complete medical examination to a simple statement that the insured is in good health at the time of change.

Reinstatement

The question of the reinstatement of disability benefits generally occurs in connection with the reinstatement of insurance. A policyholder who requested the cancelation of the disability benefits alone very seldom applies for their reinstatement. The practical questions to be dealt with, therefore, concern the effect of the disability provision on the reinstatement of policies of insurance.

As already stated, one of the standard disability provisions prescribed by various states for policies issued on and after July 1, 1930 is that if total and permanent disability is shown to have begun during the grace period for payment of a premium in default, the policy must be reinstated, provided notice of claim is given within a specified time. Payment by the insured of the premium in default with interest is provided for.

The reinstatement of a policy containing disability benefits

may necessitate medical information which would not be necessary if disability coverage were not included. An example is the case of the reinstatement of a policy which is being continued for a long period of extended term insurance. Such a policy, if without disability benefits, would probably be reinstated irrespective of the health of the insured, but if it contains disability benefits the company may offer to reinstate the policy on condition that these benefits be eliminated, should the insured's health be impaired. No general rule can be adopted, as the question of reinstatement must depend on the degree of impairment of the policyholder and on the nature of the disability coverage, ranging from waiver of premium on total and permanent disability to the retroactive monthly income after three months total disability. The underwriting rules applicable to reinstatement should generally be those in use at the date of issue of the policy and not any less liberal ones which may now be in force with regard to new applicants provided that there has been no material change in the nature of the risk since the date of issue.

In determining the cost of reinstatement to the policyholder the practice is to be follow the same method for the benefits as is employed for the insurance. If the company charges the unpaid premiums with interest, such premiums should include the extra premiums for disability and if the practice is to base the charge on reserves, such reserves should include the reserve for the disability feature.

CHAPTER X.

TREATMENT OF CLAIMS

On receipt of notice of a claim the practice of the insurance companies is to ask the policyholder to fill out a statement as to the cause of disability, its duration, the date on which he ceased to work and the names of the physicians who attended him. He is then requested to obtain at his own expense a report on a prescribed form from his attending physician, stating the disease or injury for which he has been attended, the date of the beginning of the disease or the date of injury, its cause and its probable duration. In most cases the insured is then requested to appear before the company's medical referee, who makes an examination of the claimant, reporting directly to the home office. In the meantime the company obtains an inspection report, which seeks to confirm the date on which the insured ceased to work, and to obtain the details of his disability, particularly of his activities since the date of the accident or the beginning of the illness. The inspector is also expected to report on any previous illnesses of which he can obtain information. While the foregoing generally covers the routine, additional information may be sought from employers or from hospitals, sanatoriums or laboratories. (Specimen claim forms appear in Appendix B.)

All companies receive complaints from their agents that there is delay in approving disability claims. An analysis of the causes for delay generally indicates that the fault lies principally with the policyholder: he has not forwarded the required report from his own physician, or has omitted to give exact information as to the beginning of his illness. Where the claim is made during the contestable period of the policy and there is an indication that all the facts were not submitted to the company at the date of application for insurance, there may be delay in order to make investigations which would satisfy the company that good faith had been observed by the policyholder originally.

In the majority of cases there are no serious difficulties in

disposing of claims promptly, but there are of course more points to be considered than in the case of death losses. Consideration must be given to the bias of the attending physician in favor of his patient, and to the natural feeling on his part that the company can afford to pay a sum of money which would meet the expenses of his patient's illness. In spite of this bias, it is sometimes necessary to rely entirely upon the attending physician's opinion where the cause is not immediately apparent or the symptoms are purely subjective. The justice of claims from persons in certain occupations is occasionally hard to determine. For example, the work on a farm is very heavy during the summer and the farmer may attend to most of his duties although handicapped, but in the winter season he may be tempted to consider himself totally disabled. If he has several men under him he may continue to supervise the work although unable to plow or do similar hard work. Teachers, especially women, have a high rate of disability from neurasthenia, and in a few cases it may be difficult to determine the justice of the claim when the disability commences at the end of the school session and continues during the summer months. The occupation must frequently be considered, as a mechanic with a broken arm may be entirely incapable of carrying on his work for several months, while a man in a clerical position with the same injury might be able to attend to his duties within a month.

Where married women are granted disability protection or where they receive, while they are spinsters, a policy providing for disability benefits, and later marry, it is occasionally difficult to ascertain the extent of their disability as they may be able to perform part or all of their household duties although suffering from a serious condition.

With regard to those mentally unbalanced, the family may decide not to have the insured declared mentally incompetent, and accordingly the certificate of an institution is not obtainable. He may be entirely incapable of carrying on any work, but "safe to be at large". In these cases the degree of the impairment and the date from which the total disability began are occasionally difficult to determine.

Several companies, prior to the adoption of the standard provisions as of July 1, 1930, gave the insured the right to reinstate his policy in the event of total disability occurring before the

date of default in payment of the premium or within the grace period. These companies provided that if the insured were totally disabled at the time the premium became due, or became so during the days of grace, and were still disabled at the time he applied for reinstatement, the policy would be reinstated without evidence of insurability provided application were made within, say, six months after the due date of the premium. Under the standard provisions the policy must be reinstated, irrespective of the state of health of the policyholder, provided application is made within a specified period (not less than six months after date of default). If the insured became totally disabled before the due date of the premium, the premium is waived; but if disability began during the days of grace, the insured is required to pay the overdue premium with interest.

The date from which the claim should be allowed may occasionally raise difficulties. In many of the older policies it is provided that the disability benefits are not allowed for any period prior to the date of receipt of proof. The practices of the companies differ widely regarding the dating of the claim prior to such proof. Some companies consider that if the insured has been so seriously ill that he was incapable of thinking of his affairs, or is insane, then the disability benefit should be allowed from the date of total disability. Others date back the benefit for six months or a year, and still others do not date the benefit prior to receipt of proof, thereby standing by the terms of the contract.

Questions also arise with regard to allowing the benefit after the insured has recovered. Some companies make payment after recovery, provided they are entirely satisfied as to the validity of the claim, while others consider that recovery proves that the policyholder was not permanently disabled. The cause for the delay in filing the claim should be carefully investigated. It may have been due simply to lack of knowledge of the conditions of the policy, or to a doubt in the insured's mind as to the genuineness of his claim. Cases occur in practice, however, where the sole reason for delaying the notice was to wait until the contestable period had expired, as the policy had been obtained through misrepresentation.

Some companies admit claims for disability benefits after the death of the insured although the policies may have provided

that due proof must be received during lifetime and during continuance of disability. Where such a claim is paid after death, the policy having lapsed, these companies generally apply the same rules regarding retroactivity and reinstatement as they apply to a delayed claim presented during the insured's lifetime. Many companies will not allow claims made after death unless there has been at least three months' continuous disability prior to the date of death. However, if the coverage is of the total and permanent form they generally exclude cases where death was due to an acute illness as such an illness would not as a rule be covered by the policy.

Under the older forms of policy, where the insured was not entitled to any benefit unless he was permanently disabled, many companies have a rule under which, in doubtful cases, they presume disability permanent during continuous total disability which has existed for six, nine or twelve months. Under policies with the three months' waiting period, there may arise the question of permanency but, where the evidence does not show that the disability will be permanent, the policy settles the question of presumably permanent disability in that it provides for presumption of permanency after continuous total disability for three months.

In respect to the question of what is total disability the "rule of reason" is generally applied. This matter will be dealt with more in detail under the heading of "Legal Decisions".

CHAPTER XI

LEGAL DECISIONS

The questions which would be put before a law court for decision would depend largely upon the type of the disability clause. If the clause provided for granting the benefits in event of total and permanent disability, the questions for adjudication would be distinctly different from those under a policy providing for the assumption of permanent disability after four months of total disability. Again, a policy which provided for paying the disability benefits after receipt of proof would have more questions raised than one under which the benefits were payable from the beginning of disability. Under the former, claim would be made that the company should in equity date back the benefits as the insured had been too ill to make the claim or was unaware of the provision in his policy until many months after the disability had begun. The cases for judicial decision would also involve questions regarding benefits not claimed until after recovery because, according to the terms of policies in general, all claims for disability benefits must be made while the insured is totally disabled. An insured might hope for success in having the disability benefits paid, despite the terms of the contract, if a sympathetic jury were to consider the case.

The student should bear in mind that questions of law are decided by a judge, while questions of fact are usually settled by a jury; also that it is customary to take the interpretation which is least favorable to the party which drew up the contract if it proved to be ambiguous. Furthermore, policies are subject to interpretation by forty-eight separate and independent jurisdictions in the United States, as the courts of one state are not bound by the decisions of courts in any other state. Even the decisions in one state do not necessarily create precedents in that state for a particular case, due to lack of uniformity in the language of the policies, not only between the contracts of different companies, but between those of the same company.

It is evident that there would be more difference of opinion with regard to disability than to death, and that therefore there

would be proportionately more claims for disability submitted to judicial tribunals than those for life insurance. In the experience of a large company which approved claims for disability under about 9,000 policies covering about 6,500 lives in the year 1929 there were 50 disability suits started during that year. These were all cases in which the claim was based on the merits of the disability, and not those involving rescission of the policy due to fraudulent statements. Of the 50 cases 35 have since been settled without any court action. In most of these cases additional information had been furnished which enabled the claimants and the company to arrive at a satisfactory settlement. Of the remaining 15 cases, 3 were pending at the end of 1931, 7 had been won by the company and 5 had been lost. This makes a total of 12 claims which had been actually brought before the courts for decision, which is 1 in 500 cases, approximately. In the cases which had been lost, the sympathy of the jury rather than the merits of the claim seem to have prevailed.

The great majority of the cases of disability which have appeared before the courts, have been in connection with health and accident policies, or with disability policies of casualty companies. In the early days of disability benefits the principal question was whether or not the insured was permanently disabled, as these policies provided for benefits only in the event of total and permanent disability. With the change to the clause providing for the presumption of permanent disability after three or four months of total disability, the principal question to be considered is whether or not the disability is total. To what extent, for example, is a farmer with a broken hip incapable of carrying on his work? If his usual occupation is to carry on all the work of his own farm, is he totally disabled when he has so far recovered as to be able to do a few of the duties, such as milking the cows, or feeding the chickens? If he works largely in a supervisory capacity and is able to drive around the farm in a buggy, giving instructions, is he totally disabled? Again, where a spinster has taken a policy when she was engaged in teaching, and on marriage engages in housework, to what extent may she do her household duties, in the case of a nervous breakdown, and still be considered totally disabled?

Before taking up any of the decisions, it may be well to state

that the courts frequently apply the "rule of reason". It has been held that the insured is disabled within the intent of the policy if he is so disabled as to be prevented from doing the important duties of his occupation or is unable to do practically all the material acts necessary. If he should be so disabled that he cannot carry on his own occupation, he is not debarred from applying for these benefits although capable of engaging in another occupation for which he has neither training nor aptitude. If common care and prudence indicate that the insured should stop work, as to continue would be to the detriment of his health, he should be considered totally disabled. He must not be expected to do his duties with "great pain or inconvenience". The "rule of reason" which the courts have applied in determining what is total disability, is the one which is generally in force in deciding claims in the offices of the life insurance companies. Accordingly, where there is a difference of opinion and the companies decide to submit the matter to the courts, it is due to the conviction that the insured is not equitably entitled to the benefits claimed.

Equally strong decisions in favor of, or against, the insurance companies' point of view may be found upon practically every question involved in the settlement of claims for disability benefits. The following paragraphs briefly discuss the principal causes of differences of opinion connected with disability claims, aside from those based upon fraud or misrepresentation in securing the policy or a reinstatement thereof.

Total Disability

Whether or not an insured is totally disabled so as to be entitled to benefits under the disability provision contained in a life insurance policy is a question which the jury is occasionally called upon to decide. The law as laid down by the judges in instructing the juries has usually been based upon precedents involving health and accident policies.

Reference to typical decisions will now be made, as they appear at the end of this chapter. The courts construe the policy provisions in the insured's favor whenever possible, and have generally held that, in order to be entitled to benefits, the insured need not be unable to do any work, but that, if he is

prevented from performing a substantial part of his duties, the benefits should be allowed (Case 1). A few courts have ruled that the insured need only be so disabled as to be prevented from following the occupation in which he was engaged at the time disability began (Case 2); others have ruled that the word "occupation", in the expression "prevent him from engaging in any occupation", means one which the insured might reasonably be expected to fill considering his education, training and general adaptability (Case 3). The courts of certain states have interpreted the clauses strictly and have insisted that the insured must be so disabled as to be unable to follow any occupation, in order to receive the benefits (Case 4). Some cases may be noted where the courts have held that where the disability is such that ordinary care and prudence would require cessation of business for a time sufficiently long to effect a cure, the insured should be considered totally disabled (Case 5).

Permanent Disability

With the extension of clauses providing for "presumably permanent" disability after total disability has existed for a stipulated period of time, the question as to whether or not a total disability may prove permanent becomes less important. The great expansion of disability coverage has taken place since the introduction of such "presumptive" clauses, so that in practice it becomes necessary in the large majority of cases to determine the duration of total disability.

Under the early policies providing benefits in event of total and *permanent* disability only, cases sometimes arise where the disability claimed is obviously of a temporary nature, and the company accordingly contests the claim. The decisions in the courts of the different states are at variance. The New York courts, for example, have held that the disability in such cases must be a permanent one (Case 6), while the courts in some other states have required benefits to be paid, even though it was evident that the disability was not of a permanent nature (Case 7). The courts explain their interpretation of the contract by laying particular emphasis upon the provisions for termination of benefits in event of recovery, which fact, in the opinion of the courts, proves that the word "permanent" did

not necessarily apply to a disability which would last for the remainder of life.

Proof of Disability—Effective Date of Payments

Although policies issued in many companies during the past few years have provided for retroactive payments, the clauses generally in use prior to 1926 specified that the benefits would begin "upon receipt of proof . . . before default in payment of premium . . . etc." An examination of the cases submitted to courts as to the effective date of payments under the latter type of policies indicates two well defined lines of decisions. The courts of some states have held that the benefits under such a clause did not begin to accrue until after proof of disability had been furnished (Case 8). The courts of other states have held exactly the opposite view, and have decided that benefits accrue from the beginning of total disability (Case 9). In some of the cases decided, the latter view appeared to be so directly contradictory to the clearly stated policy conditions that it is difficult to understand the conclusions reached in favor of the insured, except on the assumption that the decision was based upon precedents established after consideration of other forms of policies, which involved conditions differing from those found in the usual total and permanent disability clauses of life insurance policies.

Discontinuance of Disability Benefits upon Recovery of Insured

The disability clauses of life insurance policies provide for the payment of disability benefits "during the continuance of total disability" and specifically state that the benefits will cease when the insured is no longer disabled. Where a company, after having admitted that the insured was disabled and after paying benefits, learns that the insured is no longer disabled and accordingly discontinues payments, the company has been held to bear the burden of proving that the insured is no longer entitled to the benefits, if it has not made demand upon the insured for presentation of proof of continued disability (Case 10). On the other hand, if the company has demanded proof and the evidence discloses the termination of the total disability, the burden

of proof has been held to be on the insured to show that such evidence is incorrect (Case 11).

Default in Premium Payment While Disabled

Several cases may be noted where the insured, while disabled, allowed the policy to lapse. After the death of the insured, the beneficiaries sought to obtain the face of the life insurance and were upheld in their contentions, the courts ruling that the disability excused failure to file the necessary proof of claim, and consequently the policies were not to be considered as having lapsed (Case 12). Other states have ruled to the contrary on the same question, and have not held the company liable in such a case (Case 13).

Claims Delayed Until After Recovery

Occasionally it happens that a person who has been totally disabled for a period sufficiently long to entitle him to disability benefits, delays presenting his claim to the company until after total disability has ceased to exist. The company may then deny his right to the presumption of permanent disability, as he is no longer disabled, and suits have arisen for benefits in such cases. The weight of opinion appears to be to the effect that the insured is not entitled to such benefits under the usual total and permanent or presumably permanent disability clause of a life insurance contract (Case 14), and further that ignorance under such conditions does not excuse failure to comply with the terms of the policy.

Claims for Disability Benefits After Death

In drafting the provisions for total and permanent disability the companies did not intend to pay such benefits where an insured died following an acute illness which would not normally have resulted in permanent disability. However, in one or two cases submitted for court decision the question arose regarding the waiver of a premium which happened to fall due during such short illness, and a court ruled, in effect, that death was proof of permanent disability (Case 12), but other courts have required a strict compliance with the terms of the disability clause

calling for proof of disability during the insured's lifetime (Case 15).

Insanity

The expression, "totally disabled by bodily injuries or disease", was decided by a court to include insanity (Case 16), but this decision seemed wholly unnecessary, as the main point was the rescission of the policy.

The question of whether or not insanity excuses the filing of notice and proof is one in which there is a conflict of authority, the courts in some states having ruled that it does furnish a valid excuse (Case 17), and in other states that it does not (Case 13). The general practice of the companies is that, within a limited period of time, insanity is a valid reason for failure to file claim promptly.

Actions for Breach of Contract

A few cases have been brought before the courts where an insured, whose claim to benefits has been rejected because the company did not deem him to be totally and permanently disabled within the meaning of the policy, has sought to obtain judgment for an amount equal to the sum of all incurred income payments and the present value of future payments for life, claiming a breach of contract. In one of these cases, involving an accident policy, the recovery sought was allowed (Case 18). Cases may, however, be cited where it was held that the denial of benefits was based upon the policy contract, and did not constitute a repudiation of it (Case 19).

Incontestability Clause

The law in many states provides that policies of life insurance shall contain an "incontestability clause", under the terms of which the company is prevented from contesting payment for any reason whatsoever after the insurance has been in force, during the lifetime of the insured, for one or two years. Many states expressly exempt the disability clause from the operation of this limit. On the other hand, at least one state (Indiana) provides that the incontestability feature shall apply to the disability clause.

Where a company adds disability benefits to existing policies by rider, some companies deem it desirable to have a separate incontestability clause in the rider, bearing its date of issue, and not that of the policy to which it is attached. In one case it was held that the incontestability feature became effective at the expiration of the contestable period of the parent policy (Case 20).

Overpayment of Benefits

In practice, cases arise in which, through error or on account of fraud, an insured is paid disability benefits for a period of time during which, it subsequently develops, he was not entitled to these benefits. In such cases the courts have held that the company is entitled to recover the amount overpaid (Cases 21-22).

General Remarks

The differences of opinion by the various courts should not be taken as proof that they constitute great difficulties in handling disability claims. The rules of the companies are generally based on the most liberal interpretations of the various court decisions unless they constitute an injustice to the insurance companies. Some of the inconsistent or apparently unfair decisions were rendered by courts of minor importance, and if the cases had been appealed, the appeal would probably have resulted in a reversal of the verdict of the Lower Court. The decision of the courts in the state in which the insured resides must be considered in dealing with claims, but, if less liberal than the rules adopted by the company, may be ignored. Where they are more liberal than the company's general rule, then the company has the option of allowing the claim in that state, or of refusing to pay in the hope that the previous decision will be overruled. In most cases the insured, or his lawyer, is willing to accept a fair decision by the company irrespective of what the Lower Courts in his state may have decided.

The standard provisions which became effective on the 1st of July, 1930 in many states, will result in new questions being submitted for judicial decisions. As these provisions were prepared with a knowledge of the various court decisions and with a view to lessening differences of opinion between the claimant

and the company, it is hoped that the small percentage of cases submitted heretofore for court decision will be still less in the future. The student will find it instructive to read the standard provisions (Appendix C), bearing in mind the various points discussed in this chapter.

Digest of Typical Decisions

Case 1. *Fitzgerald vs. Globe Indemnity Co.*, District Court of Appeals, 3rd District, California, 258 Pac. 458 (1927), where the Court said:

"The weight of authority supports the view that provisions in accident policies for indemnity in the event the insured is totally or wholly disabled do not require that the accident shall render the insured absolutely helpless, but such provisions are construed as meaning such a disability as renders him unable to perform the substantial and material acts of his business or occupation in the usual and customary way."

Case 2. *Fogleson vs. Modern Brotherhood of America*, Kansas City (Mo.) Court of Appeals, 97 S. W. 240 (1906), in which the Court said:

"The language of the policy, 'permanent and total disability of said member, which renders him unable to carry on or conduct any vocation or calling,' the vocation of the insured not being designated should be construed as meaning, if anything, the vocation or calling in which he might be following at the time he became disabled, and not any vocation whatever which he might be able to follow after he had been disabled."

Case 3. *Carson vs. New York Life Insurance Company*, Supreme Court of Minnesota, 203 N. W. 209 (1925), in which it was stated:

"The intention was to provide an income as a substitute, in a measure, for the earnings in an occupation he was capable of filling, but which the disability prevents him from engaging in. Any other occupation for this insured, an ordinary farmer and telephone repair man, could not reasonably be said to include that of a highly specialized scientist, the work of which the physical disability may not prevent his doing, but in which he can never hope to engage because of lack of mental qualification and training. It must mean any occupation similar to that in which he had ordinarily been engaged, or for which he may be capable of fitting himself within a reasonable time. If the dis-

ability prevents the insured from performing the essential parts of such an occupation with substantial continuity, it should entitle to the income payment promised."

Case 4. *Starling vs. Supreme Council Royal Templars of Temperance*, Supreme Court of Michigan, 66 N. W. 340 (1896), from which we quote:

"The fact that a man may carry a bucket of coal, or may carry a stick of wood, or perhaps may run a lawn mower over a lawn, will not, in itself, necessarily show that he is competent to follow an avocation. The fact that a man may work for a few moments, even though, perhaps, he may work for a few months, will not, necessarily—it is not conclusive evidence that he can follow some avocation. But, if you find that he can perform some kind of employment,—if you find, as suggested by the counsel in this case, that he could keep a newspaper stand, or a peanut stand, or could do any work, or follow any line of employment, why, then, gentlemen of the jury, under those circumstances, he would not be entitled to recover."

And also:

Lee vs. New York Life Insurance Company, Supreme Court of North Carolina, 125 S. E. 186 (1924), from which we quote the instructions to the jury in the Lower Court, which were approved by the Supreme Court:

"Now, you will want to know what is meant by the language in the contract 'wholly incapacitated and thereby permanently and continuously prevented from engaging in any avocation whatsoever for remuneration or profit.' It does not mean merely that this disability may incapacitate him from pursuing his usual avocation, from working on his farm with his hands, but that it must incapacitate him from engaging in any avocation for remuneration or profit.

"Our courts hold that the act shall be in force as it reads and that the insured cannot recover because totally disabled for his own trade or business, if he retains health, strength, and physical ability sufficient for the pursuance of other avocations by which he might engage for profit or remuneration.

"It would not be necessary that the business in which he engages should actually be profitable or remunerative and he would have to take the risk that all men take in things of that sort, and the test would not be whether he could have actually made a profit, if it was such a business that he was able to engage in it in the expectation of profit or remuneration."

Case 5. *Brown vs. Missouri State Life*, Supreme Court of South Carolina, 134 S. E. 224 (1926), in which the Court said:

"The rule prevailing in most jurisdictions is that the 'total disability' contemplated by an accident insurance policy does not mean, as its literal construction would require, a state of absolute helplessness which can result only from loss of reason, since, as long as one is in full possession of his mental faculties, he is capable of transacting some part of his business, whatever it may be, although he is incapable of physical action. On the contrary, these Courts, giving consideration to the object of the contract, hold that the 'total disability' contemplated by the agreement is inability to do substantially all of the material acts necessary to the prosecution of the insured's business or occupation, in substantially his customary and usual manner.

"If the prosecution of the business required insured to do several acts and perform several kinds of labor, and he is able to do and perform only one, he is as effectually disabled from performing his business as if he were unable to do anything required to be done, and while remaining in that condition, he suffers loss of time in the business of his occupation. Nor does the provision contemplate absolute physical disability to transact any kind of business pertaining to one's occupation, but it is sufficient if his injuries are such that common care and prudence require him to desist from transacting any such business in order to effect a cure."

Case 6. *Ginell vs. The Prudential Insurance Company of America*, Court of Appeals of N. Y., 237 N. Y. 554 (1923). The provision in the policy used the expressions "permanent disability" and "permanently disabled" without defining the word "permanent". The insured was disabled from January 6, 1921 to May 10, 1922, from pleurisy followed by tuberculosis. On February 1, 1921, proof of disability was offered and claim was rejected. The case came before the Court after the recovery of the insured. In the Trial Court and the Appellate Division it was held that this seventeen months' disability was permanent disability but there was a strong dissenting opinion in the Appellate Division. This dissenting opinion (205 N. Y. App. Div. 494) is in part as follows:

"The payment is not dependent upon actual total permanent disability but on proof of such; and it is no uncommon experience that that is established by proof to be a fact which in truth was never a fact. • • • If the word 'permanent' in the policy has

any natural meaning at all there was no proof furnished of a total permanent disability. Each doctor said plaintiff would recover, and one of the physicians named the time about one year with accuracy; the other named a longer period."

Elsewhere this opinion also stated that the construction approved by the majority of the Court gives no natural meaning to the word "permanent".

The Court of Appeals reversed the Appellate Division, and adopted as its opinion the dissenting opinion from which quotation was made above.

Case 7. *Wenstrom vs. Aetna Life Ins. Co.*, Supreme Court of North Dakota, 215 N. W. 93 (1927). The insured had been totally disabled for some time due to a series of accidents, in the course of which three ribs and both legs were broken. The Court said:

"* * * The policy continues:

" 'And if such disability was sustained before the insured attained the age of 60 years, the company will pay to the life beneficiary the sum of ten dollars for each thousand of the sum insured and will pay the same sum on the same day of every month thereafter during the lifetime and during such disability of the insured.'

"What is the meaning of this phrase, 'during the lifetime, and during such disability of the insured'? If the disability must be incurable and continue during the life of the insured, it would be sufficient to say that the same sum would be paid on the same day of every month during the lifetime of the insured. Is not this provision in the policy the same as if the contract said, will pay the same sum on the same day of every month during the lifetime of the insured, or as long as he is disabled? We must assume that the phrase, 'and during such disability of insured,' means something, and if it means anything it means that the amount will be paid during such disability, whether it be for life, for years, or for months, and it would seem that it is placed there to cut off the indemnity in case the insured recovers. It is settled law that in construing insurance policies the language of the entire policy must be considered, and when capable of two constructions the most favorable to the insured must be given. Under this rule we are of the opinion that the words, 'and during such disability of the insured,' qualifies the preceding language in that paragraph so as to permit a recovery when the disability is curable, but the indemnity ceases if the insured recovers. From this construction it follows that the insured is

entitled to the indemnity during the entire period of his disability."

Case 8. *Courson vs. New York Life Insurance Company*, Supreme Court of Pennsylvania, 145 Atl. 530 (1929), in which the Court held that where insured had furnished no proof of claim, he was not entitled to recover premiums paid over a course of years, during which period he was in fact totally and permanently disabled.

See also *Jones vs. New York Life*, Supreme Court of Washington, 290 Pac. 333 (1930), and *Peoria Life Insurance Company vs. Bergholm et al.*, Circuit Court of Appeals, 5th Circuit, 50 Fed. (2nd) 67 (1931).

Case 9. *Missouri State Life vs. Le Fevre*, Court of Civil Appeals of Texas, 10 S. W. (2nd) 267 (1928), as summarized in Legal Bulletin (American Life Convention):

"Company contended that policy had lapsed by reason of non-payment of premium due March 8, 1927 prior to insured's death on June 6, 1927. Company further contended that insured had not been in sound health when the policy was delivered to him and that he had misrepresented his previous condition of health. The Trial Court in response to special issues found that insured was in sound health when the policy was delivered; that insured was totally incapacitated from bodily infirmities from February 20, 1927 to the date of his death; that he had made no false representations as to material matters in his application. The evidence disclosed that insured from February 20, 1927 was totally disabled by an attack of typhoid fever; that he thereafter grew worse and died on June 6, 1927. The policy contained a total and permanent disability provision, which provided for waiver of premiums, same to be effective 'only upon receipt at the company's home office while no premium is in default of due proof of existing total and permanent disability.' No notice of disability was given the company prior to insured's death. Beneficiary contended that under the facts of the case, the making of such proof was impossible, and that all that was required was to furnish same within a reasonable time after it could be made.

"Held: (1) The effect of insurer's obligation to the insured upon a sufficient consideration, was, if he became totally and permanently disabled, to allow him a sick benefit to the amount of the premiums thereafter accruing, and in lieu of paying the same to insured, to apply same to the premiums accruing on his

life insurance. The evident purpose of the disability provision was to preserve the insurance in the event the insured, on account of disability, became unable to pay the premiums, and said provision should be construed so as to effectuate this intention. (2) The stipulation as to the time within which notice or proof of disability should be given is not necessarily to be literally complied with. (3) Where failure to give prompt notice of such disability has been prevented or rendered impossible by the nature of the situation, this would furnish a sufficient legal excuse for delay in giving the stipulated notice. (4) The time of making the proof of disability was not of the essence of the contract. (5) No duty rested upon anyone to make the proof until a reasonable time after same could be made, and beneficiary's offer to make such proof on June 17, 1927 after insured's death was within a reasonable time after same could be made. (6) Since beneficiary pleaded the policy provision which stated that disability benefits would be effective only upon receipt of due proof of existing total and permanent disability while no premium was in default, but failed to allege that such proof of disability was furnished and failed to allege any excuse for this failure to furnish due proof as required, the petition was fatally defective. The Trial Court's action in overruling insurer's special exception to this question was in error and requires a reversal of the case. (7) Where the evidence disclosed that insured had been suffering from a swelling of the neck, and insured had stated in his application that he had undergone a tonsillectomy operation, his failure to mention the fact that X-ray treatments had been taken for his neck swelling did not constitute a false statement. Judgment of Trial Court in favor of beneficiary reversed. *Dissenting opinion by Barcus*, Associate Justice, who considers that the Court should not make a contract for the parties; that the only construction of the contract is that insured is relieved of payment of premiums only when he has strictly complied with the conditions precedent, and that proof of total disability must be furnished the insurer before the premium actually becomes due. In his judgment the cause should be reversed and judgment rendered for insurer."

Case 10. *Beasley vs. Pacific Mutual*, Supreme Court of Tennessee, 13 S. W. (2nd) 330 (1929), as summarized in Legal Bulletin (American Life Convention):

"Insured had previously sued company and secured judgment for total and permanent disability benefits. After a course of time company discontinued such payments, and this suit is for back payments alleged to be due. Policy provided that insured might be required at least once each year to provide satisfactory proof of continuance of disability.

"*Held*: (1) The burden was upon the insurance company to allege and prove that it had made request for satisfactory proof of disability and that the insured had not complied therewith. (2) The former judgment established the total and permanent disability of insured, and it was not incumbent upon him in suit for subsequent payments due to prove that his disability still existed. Decree overruling demurrer affirmed and case remanded."

Case 11. *Fields vs. Equitable Life Assurance Society*, Supreme Court of North Carolina, 141 S. E. 743 (1928), as summarized in Legal Bulletin (American Life Convention):

"Policy provided that if insured became physically incapacitated to such extent that he would be wholly and presumably permanently unable to engage in any occupation or perform any work for compensation of financial value, company would waive payment of further premiums and pay a monthly disability benefit. In March, 1922, insured became sick and filed claim for disability benefits payable under the policy. Insurer allowed such claim and made payments beginning December 6, 1922, and terminating May 9, 1923. In April, 1923, a blank form, designated as proof of continuance of disability, was filled out by insured's attending physician, in which the doctor indicated that insured was able to perform light work on farm and that he was not permanently disabled. Company thereupon declined to make further payment and restored policy to a premium paying basis. Insured insisted that the doctor's statement that he was not permanently disabled was not a correct statement of the true facts, and at the trial, insured's attending physician testified that when he used the term 'not permanently disabled,' he meant that insured was not confined to his bed.

"*Held*: (1) The proofs of loss, though not conclusive, are prima facie true as against him. The burden was upon the plaintiff to show that the statement made in the proofs of loss was erroneous in fact. The plaintiff having filed them, has vouched for their truth. He must show mistake. (2) Company's contention that the proof of loss itself contained evidence avoiding liability cannot be sustained, although same could be available as a defense to an action on the policy, since the question of permanent disability was for the jury. Judgment of non-suit against insured reversed."

Case 12. *Minnesota Mutual Life vs. Marshall*, Circuit Court of Appeals, 8th Circuit, 29 Fed. (2nd) 977 (1928), as summarized in Legal Bulletin (American Life Convention):

"Policy dated October 14, 1925, provided for a grace period of one month. The second premium, due October 14, 1926, was not paid. Insured died November 29, 1926. Evidence disclosed that on the morning of November 16th, insured came to father's house from his home in the country in a sick condition and that afternoon was operated on for appendicitis. The operating surgeon testified that insured's appendix had bursted and that he was suffering from peritonitis from which he died. His opinion was that insured had been suffering from appendicitis for several days prior to the date of his operation; that on account thereof he had been disabled and prevented from performing his ordinary duties. Policy provided that if insured became totally and permanently disabled and 'shall furnish satisfactory proof thereof, the company will waive the payment of premiums thereafter becoming due.' The question which is raised is whether the provision of the policy required waiver of payment of premium which became due October 14, 1926, merely because insured became totally and permanently disabled during the grace period. No notice of disability was given the company and it contended that the evidence was insufficient to sustain the jury finding that insured was disabled before the expiration of the grace period.

"*Held*: (1) The decisions are conflicting in the construction of same or similar language as that here used. These decisions establish doubt as to the construction and meaning of the provisions which this Court is called upon to interpret. Where contracts are prepared by insurer and there is doubt as to the meaning of their provisions, they will be construed most favorably to the insured. (2) The Court says: 'The right of insured to have his premiums discontinued during disability is one that he paid for. To make its operation depend upon the proof of disability and not upon the time of disability itself, which was the real thing that he was protecting himself against, renders the provision of the policy under construction inoperative and the right of no value.' (3) If insured had died during the grace period no question would be raised as to the right of his beneficiary to recover. Insured's policy should not be allowed to forfeit where his disability occurred during the grace period, and continued until his death. So long as insured was in good standing and he became disabled, under the provisions of the policy he had a right to protection. (4) A construction making the disability benefits begin when proof of disability was made is satisfactory where insured is living, but this provision, where disability occurs near the due date of the premium and continues until death, is made worthless by holding that proof of disability and not the disability itself makes it operative. Such construction is harsh and unreasonable and should not be

adopted if the language used is susceptible of one more favorable to insured. (5) The term 'permanently' is not used in its ordinary meaning since the policy reserved the right to again collect premiums should disability be removed. Length of time is not the measure by which the permanent disability was to be determined according to the provisions of the policy. A complete disability, which extended over the date for the premium payment or until the disability resulted in death, is such a disability as would relieve from further payment of premiums. Judgment for beneficiary affirmed."

Case 13. *New England Mutual Life Insurance Co. vs. Reynolds*, Supreme Court of Alabama, 116 Sou. 151 (1928). The insured in this case beyond any controversy became insane prior to the due date of the premium that was not paid and for which the policy lapsed. The contention was made that, notwithstanding the wording of the policy, his total incapacity to act in the matter excused the failure to file proofs of disability prior to the default in the payment of the premium. In relieving the company of liability the Court said:

"We are of opinion that furnishing proof of disability to the insurer is made a condition precedent to the waiver of premium payments under the supplemental agreement set out in the special plea above. This agreement declares:

"If the insured * * * shall furnish due proof to the company at its home office * * * that he has become wholly disabled by bodily injury or disease * * * the company will waive payment of each premium as it thereafter becomes due during the continuance of such disability.' * * *

"Dealing with the argument that the insured could not be expected to make proof while mentally incapable of so doing, let us look at this policy without the waiver clause. In that event the actual payment of the premium when due could alone prevent a lapse of the policy. No sickness, insanity, or disability of any kind would excuse payment as stipulated. The law does not class such event as one rendering performance impossible, nor its requirement unreasonable. The insured, by his contract, assumes the risk of any disability rendering it impossible for him to make payment in person, and is thereby warned to place the beneficiary or some next friend in position to take care of his contract in such event. * * *

"Now, by the terms of the waiver agreement before us, the insured may pay the premium or cause it to be paid, or may avoid so doing by furnishing the proof of disability as stipu-

lated. In case of insanity, the required proof could be furnished by the beneficiary or next friend, just as the premium could be paid."

Case 14. *Gottlieb vs. New York Life*, Supreme Court of New York, Appellate Term, First Department, 240 N. Y. S. 568 (1930), as summarized in Legal Bulletin (American Life Convention):

"Policy provided for disability compensation and waiver of premiums upon proof of total and permanent disability, or where such proof establishes 'that the insured is, and for a period of not less than three consecutive months immediately preceding receipt of proof has been, totally disabled.' Insured offered proof of disability one year after the disability ceased. Insured claimed that he was ignorant of the provision entitling him to compensation.

"*Held*: (1) Insured's offer of proof of disability one year after the disability had ceased was not a compliance with the terms of the policy. (2) Insured's ignorance of the provision entitling him to compensation is no excuse for his failure to comply with the terms of the contract. Judgment for insured reversed, with costs, and complaint dismissed on merits."

See also *Equitable Life Assurance Society of U. S. vs. De Johnson*, Supreme Court of Arizona, 286 Pac. 817 (1930).

Case 15. *Yohalem vs. Columbian National Life*, City Court of New York, Trial Term, 240 N. Y. S. 666 (1930).

"The action is brought under a total permanent disability clause contained in a policy of life insurance for \$10,000. The policy was paid after the death of the insured. This action is brought by the administratrix for an additional sum of \$1,333.93 for premiums and benefits on the total and permanent disability clause, for which a separate premium was paid. It is agreed by the stipulation that the insured became ill on October 1, 1926, and was unable to engage in any occupation for remuneration or profit up to the date of his death on March 13, 1928. After the death of the insured the plaintiff offered proof of total and permanent disability, which defendant refused to receive because it was not notified of the disability prior to the date of the death of the insured. No claim was made because of such disability until after the death of the insured. The policy provides total and permanent disability benefits as follows: (A) waiver of premiums; (B) annuity payments. These benefits are predicated on due proof 'that he is and thereby will be'

permanently disabled and that such disability has existed for sixty days prior to the furnishing of proof. The waiver of premiums is to be by endorsement on the policy. The annuity payments are to be made in monthly instalments, the first monthly instalment is to be made six months after due proof of total disability and subsequent payments monthly thereafter during the continuance of total disability. Plaintiff contends that the policy provision requiring notice and proof of disability fixes the time of payment by defendant, so that no liability accrued until such proof was made or waived; that late proof is to be distinguished from failure of proof and that the courts are disinclined to deprive the insured of the benefits of insurance unless his default works a forfeiture by the definite language of the policy. * * *

"The language of the insurance contract 'that he is and will be' permanently disabled has reference to a condition present during the lifetime of the insured. The provision in subdivision (B) of the policy that the first instalment in monthly payments shall be made six months after due proof of such total disability accompanied by the policy for endorsement and subsequent payments monthly during the continuance of total disability clearly negatives the claim that proof of disability supplied after death entitles the claimant to recover. * * * Notice of disability and endorsement of the waiver of premiums were substantial provisions of the contract and became conditions precedent to recovery. * * *

"Nor can the two premiums of \$251.30 be recovered in this action under the waiver of premium clause. * * * The entire structure of the contract negatives the idea of a self-operating waiver in the event of total disability. Endorsement on the policy was a condition precedent to such waiver, and the insured was liable for the premium which became due before the insurer was notified of total permanent disability. Judgment for defendant, dismissing complaint."

Case 16. Old Colony Life vs. Julian, Supreme Court of Arkansas, 299 S. W. 366 (1927).

"Insurance ⇌ *524—In insured's cross-complaint to recover against life insurance company under disability provisions of policy, insanity held to be disability due to bodily injury or disease.*

"Where, in life insurance company's suit to cancel policy, the insured, by counter-claim, sought payment of the disability compensation because of his insanity, *held*, that insanity came within the disability provision of the policy which provided that

the total and permanent disability of the insured must be due to bodily injuries or disease in order for him to recover compensation therefor."

Case 17. *Levan vs. Metropolitan Life Insurance Company*, Supreme Court of South Carolina, 136 S. E. 304 (1927), where the insured was insane and lived more than a year after default in payment of the premium, the Court said, in holding that notice after the death of the insured was sufficient:

"The evident purpose of such a provision in the policy was to preserve the insurance in the event the insured, on account of disability, became unable to make the money to pay the premiums; and the clause should be construed so as to effectuate this intention. Accordingly, where the insured was rendered incapable of furnishing proofs of his disability by reason of disability, then it must be presumed that the parties did not intend by the language used to deprive the insured of the benefits he was to receive."

Case 18. *Federal Life Insurance Company vs. Rascoe*, Circuit Court of Appeals, 6th Circuit, 12 Fed. (2nd) 693 (1926). The insured took out an accident policy by the terms of which the company agreed to pay the sum of \$25.00 per week so long as she suffered total disability from accidental injuries sustained while a passenger on a common carrier. The insured had an accident while riding on a common carrier and submitted proofs to the company. The company made a number of payments to the insured under the policy and then refused to make any further payments. Insured then brought this suit for the monthly instalments which were then due and for those to become due, during the remainder of her life. The insured pleaded that the company repudiated the entire contract, but the form that the repudiation took does not appear in the opinion. The policy provided that if the insured was disabled for more than thirty days that she should furnish the company each thirty days with a report in writing of her attending physician or surgeon fully stating her condition and the probable duration of the disability. The Court permitted recovery of the past due instalments and also the present value of the instalments which would become due in the future throughout insured's life expectancy. A dissenting judge wrote:

"In my judgment this case illustrates the evils of laxity in permitting a premature recovery. In such a case as this (as facts often are, though possibly not in this case) plaintiff in a year or two may recover entirely or (if the case is as bad as here claimed) is very likely not to live long. In either case the true liability is for a short term; but the recovery has been upon the basis of the full expectation of life of a healthy person,—though no error in this respect was duly saved."

Case 19. *Indiana Life Endowment Company vs. Reed*, Appellate Court of Indiana, 103 N. E. 77 (1913). This case involved a life policy with a total disability provision payable in instalments.

"Insured lost his left arm and claimed total disability. The company replied that this didn't totally disable the insured within the terms of the contract and refused to pay the instalment claimed to be due. Suit at once was brought to recover on the theory that the contract had been repudiated. The Court held that the denial of liability was based on the contract and was not a repudiation of it. There was a clear dictum that if there had been an unequivocal repudiation of the policy that future damages would have been recoverable."

See also *Howard vs. Benefit Assn. of Railway Employees*, Court of Appeals of Kentucky, 39 S. W. (2nd) 657 (1931).

Case 20. *Wamboldt vs. Reserve Loan Life Insurance Company*, Supreme Court of North Carolina, 131 S. E. 395 (1926).

"Insurance ⇐ *400—Supplemental contracts attached to policies held subject to incontestable clause in policies, and defenses not available in action on policies not available on supplemental contracts.*

"Where supplemental contracts attached to policies recited that they were component parts of such policies and bore same dates as policies, such supplemental contracts were subject to incontestable clause contained in policies, and supplemental contracts became effective from dates appearing in them and not from date on which they were attached to policies, and no defense was available to insured on the supplemental contracts which was not available in action on policies."

Case 21. *Couper vs. Metropolitan Life*, Supreme Court of Michigan, 230 N. W. 929 (1930). Policy provided that insurer's liability could not extend more than six months prior to date of proof. By a mistake on the part of an employee, insurer

made payment for disability extending from twenty months before date of proof, and, on learning error, sought to recover overpayment. According to Legal Bulletin (American Life Convention), the Court held that:

"Payment, though voluntary, may, if made under mistake of material fact, be recovered, though mistake is due to lack of investigation. 'Waiver' is the intentional relinquishment of known right, and implies an election to forego some known advantage, inducing belief of intention to waive. The policy provision limiting recovery for disability benefits to six months before proof was not waived by the mistaken payment of an excessive amount, where facts negative any intention to waive such provisions, especially where such payments included payments for a period prior to commencement of insured's disability. Judgment for insured reversed."

Case 22. *Peoria Life vs. Smith*, Federal District Court, Eastern District of Michigan, 47 Fed. (2nd) 279 (1931). The policy provided that disability must be due "to disease occurring while the policy is in full force". The period of contestability had elapsed, but the insurer, after having paid disability benefits, learned that the insured's disability was due to tuberculosis which commenced before the policy was issued. The Court decreed that insurer should recover the disability benefits paid and the premium waived.

CHAPTER XII.

METHODS OF ACCOUNTING *

Disability benefits affect a life insurance company's accounting in a number of ways. Bookkeeping entries must be made to cover the disability premium income, the premiums waived on disability, and the disbursements on claims. A system of accounting is also necessary in connection with some of the reserve items and in the ascertainment of the profit or loss as required by the Gain and Loss Exhibit.

Premium Income—Waiver of Premiums

In the majority of companies the practice is to state on the first page of the policy the total premium for life insurance, disability and double indemnity. There also appear in the policy, or in the riders attached thereto, the premiums for each of these forms of coverage. The premium notice sent to the insured shows the total premium for the policy and the cashier at the branch office or agency records such premium in his books. The company is required, however, to show separately in its annual statement the premium income for each type of protection and such information is also necessary in determining its gain or loss. The company's total premium income may be separated into its component parts by treating each individual case, which may be done either at the branch offices by the cashiers or at the home office by means of perforated cards. On account of the labor thus involved, some companies determine at the end of the year the disability premium income by methods of approximation which give reliable results.

The company's premium income is also affected by premiums waived. If any premiums have been waived during the year under disability claims, the life insurance premiums thus waived should be included in the premium income and charged to Premiums Waived. Accordingly, a record should be kept of all premiums waived during the year, whether net or gross. Where

* The student may find it easier to understand the discussion of disability accounting methods if he first reads the demonstration given on page 181.

the net premium is waived, the gross premiums are sometimes used in such a record and the corresponding net premiums determined by applying a ratio to the total for the year.

The fundamental principle in the accounting is to charge to the account pertaining to a certain form of coverage all the benefits granted for which that form of coverage is responsible. It has been found desirable in practice, however, to make some calculations and adjustments by approximate methods at the expense, occasionally, of theoretical accuracy. It should be noted, however, that approximations used in determining some of the items involved in the disability premium income, the waiver of premiums and the double indemnity premium income do not affect the total net income of the company. The total premium income received is a definitely ascertained quantity and it is only in the analysis of it for special groups that approximations are sometimes made.

The foregoing references to the waiver of premiums apply to the usual type in which the face amount of insurance is not reduced by the amount of premiums waived. The modified type of waiver issued under older forms and usually granted under disabilities occurring after age 60 or 65, requires special treatment. The face amount is, as a rule, reduced by the amount of the premiums waived. Therefore, where the original premium for the contract, less the disability extra, is waived each year, reducing the face by a like amount, the benefit is equivalent to a loan without interest for the amount of premiums waived. As this is analogous to the instalment benefit which reduces the face amount of insurance, the theoretical considerations presented below as applicable to the latter may also be taken as applicable to the former. In practice, this modified waiver is treated the same as other waiver benefits. Under many contracts containing a modified waiver benefit for disabilities occurring after age 60 or 65 provision was made that the amount of premium waived each year after the first would be reduced in proportion to the decrease in the face amount through the waiver of previous premiums. A similar proportionate decrease was made in surrender values, dividends, etc. Where this benefit is granted the insured receives an additional benefit over and above the loan interest on the amount of premiums waived as the face amount

is reduced for premiums waived after the first, not by the amount of the full premium, but by the proportionately reduced premium. Theoretically, different accounting methods may apply to this form of modified waiver of premium benefit but even under this form the usual practice is to use the accounting methods applicable to other waiver benefits.

Income and instalment benefits are always accompanied by a waiver of premium benefit except in one year renewable term contracts, such as issued under group insurance. Since the accounting methods outlined above as applicable to waiver of premium benefits are equally applicable to the waiver of premium portion of benefits which grant income or instalment payments in event of disability, no specific mention of the accounting methods pertaining to such waiver portion will be necessary, as a rule, in the presentation under the next heading.

Income and Instalment Payments

Income payments. Income payments which do not reduce the face amount present no difficulty. They appear as disability income disbursements in the annual statement. In bookkeeping phraseology these amounts are debited to Disability Income Payments and credited to Cash. Some companies, instead of debiting the premiums waived and the income payments made as they fall due, prefer to make a single entry at disability, debiting the present value of premiums and income payments on a disabled life basis to Premiums Waived and Disability Income Payments respectively, the corresponding credits being to Renewal Premiums (occasionally part to First Year Premiums and part to Renewal Premiums) and Supplementary Contracts involving Life Contingencies. The argument in favor of this treatment is that the entire disability claim loss is charged to the Premiums Waived and to the Disability Income Payments accounts in the year in which the claim occurs. The principal objection urged against this method is that it does not charge Disability Claims with the true cost of the benefit. If the amount transferred at disability overstates the actual value of the benefits, the Supplementary Contracts account will show a profit and if there is an understatement, it will show a loss; this profit or loss should be merged with the total profit or loss in the disability account.

Incidentally this method may call for reversal entries when the insured recovers. A further objection to this method is that the company's income and disbursements are inflated through duplication of entries.

Instalment payments reducing face. We shall now consider the instalment type under which the amount of insurance at death is reduced on account of the disability instalment payments. The problem is to separate the various transactions into their disability and life insurance parts. Obviously it would not be correct to charge each instalment payment to disability claims, then the balance of the face at death or maturity, if any, as a death claim or a matured endowment. An understanding of the fundamentals involved in the accounting under such types of disability benefits may be best obtained by a consideration of the methods used in the calculation of the premium (Part II, Chapter VIII). It will be noticed that the disability benefit provided under the instalment type is, in addition to the waiver of premiums on the original face amount of the policy, merely the interest on the instalments received. The accounting should show how much is to be assessed against disability claims and how much against the life insurance account. To accomplish this in the instalment type of benefit two new ledger accounts may be introduced—Disability Loans and Disability Loan Interest. As already mentioned, the payment of an instalment is equivalent to granting a loan without interest. The instalment payment accordingly represents only a change in the form of the company's assets, in this case, from Cash to Disability Loans, so that it would not enter into the income and disbursements of the annual statement. The interest on this loan which the company foregoes is, however, a debit each year to Disability Claims and a credit to Disability Loan Interest. At death or maturity the Death Claims or Matured Endowments would be debited by the full amount of insurance, Disability Loans credited with the amount previously transferred to that account and Cash credited with the balance of the face amount, if any. In the case of the modified premium waiver benefit which reduces the sum insured by a constant amount each year, the amount of premiums waived would be credited to

Renewal Premiums and the assets of the company increased by the corresponding debit to Disability Loans. The interest on these loans would be debited each year to Disability Claims and credited to Disability Loan Interest. At death or maturity Death Claims or Matured Endowments would be debited with the face amount, Disability Loans credited with the amount of loans and Cash credited with the balance. In event of the insured recovering from disability before all the instalments have been paid, the policy is usually continued in force for the reduced amount of insurance. Disability loan interest on the instalments paid and the waiver of premiums on the reduction in the face amount should continue to be an annual charge to the disability account until the maturity of the policy, and the instalments paid should be treated as a disability loan until such maturity. However, in order to close the disability entries, we might credit the total reduction in the face amount, due to instalments paid, to Disability Loans and debit to Surrender Values the insurance reserve on such reduction in face amount and make the balancing debit, if any, to Disability Claims. Similar entries might also be made on the elimination of the face amount during the insured's lifetime because of disability benefits granted not only under this type of benefit, but also under any type wherein the payments made or premiums waived are considered as a policy loan which is to be deducted from the face amount in any settlement thereunder. The student will find a discussion of these accounting methods by J. S. Elston in T. A. S. A., XXI, 134.

The instalment type of benefit has not been issued generally in America for some time and has ceased to be an important form. Since it usually is contained in a relatively small proportion of the total amount of insurance with disability benefits, many companies do not attempt an exact separation of the instalment payments into their insurance and disability parts. One method is to charge to Disability Claims the instalment payment less the life insurance reserve. The reserve thus released would be debited to Surrender Values and the amount of the instalment credited to Cash. The amount of insurance reported in the annual statement would be reduced by the amount of each instalment paid. At death or maturity only the balance, if any, of

the face amount would be charged to Death Claims or Matured Endowments.

If a company used the Disability Loans and Disability Loan Interest accounts the interest foregone by the company would appear as an interest income item in the annual statement and a question might arise with regard to its treatment in the United States Federal Income Tax returns, which are based on the total amount of interest and rents received. As interest is not actually received, the exclusion of it in determining the tax might be permitted by the federal authorities. Such interest might also be properly excluded in arriving at the company's earned interest rate.

Income payments reducing face. Under the "income reducing face" type of benefit, payments made before the face amount has been entirely reduced would be treated in the same way as instalment payments which reduce the face, and those made after the face amount has been entirely reduced, in the same way as disability income payments which do not reduce the face amount.

Maturity instalment benefit. There is another type of instalment benefit which should be referred to, namely, the form under which the commuted value of the instalment payments at date of disability is equal to the face amount of insurance, hence known as the "maturity" instalment benefit. On a $3\frac{1}{2}\%$ interest basis the annual instalment payable for 20 years certain is \$67.98 per \$1,000 which, in practice, is usually taken as \$68. The amount payable at death on a disabled life under this type would be equal to the commuted value of any unpaid instalments. Since this is equivalent to paying the face amount of insurance at date of disability, the value to the insured, exclusive of the waiver of premiums, is equal to interest on the \$1,000 instead of interest on the instalments only as under the instalment type previously discussed. The amount properly chargeable to Disability Claims would, therefore, be the interest in advance on the face amount, which at $3\frac{1}{2}\%$ on \$1,000 is \$33.82. Of the first instalment payment (\$68.00) \$33.82 would be charged to Disability Claims and \$34.18 to Disability Loans. In the second year the charge to Disability

Claims would again be \$33.82, but Disability Loans would be debited not only with \$34.18, but also with one year's interest on the previous loan of \$34.18, under which the company is foregoing interest. In each succeeding year the Disability Loans account would be similarly debited with \$34.18, plus interest on previous debits to that account. Disability Loan Interest would be credited with these amounts of interest on previous debits to the Disability Loan account. Each year Cash would be credited with \$68.00. In event of death, Death Claims would be charged with \$1,000. The credit entries—the commuted value of future instalments to Cash and the part of instalments previously charged to Disability Loans, together with the interest thereon, now a credit entry to that account—should amount to \$1,000. The amounts will differ slightly, due to instalments being taken at \$68.00 instead of \$67.98. Numerical illustrations of this accounting will be found in the paper by Mr. Elston, already mentioned.

The maturity instalment type of disability benefit is issued at the present time under group insurance contracts. Since group insurance is usually on the one year renewable term plan, the above accounting methods should be suitably modified. As indicated in Chapter I, the disability benefit under such contracts is equivalent to paying the face amount at disability. The practice of one of the largest companies issuing group insurance is to cancel the face amount of insurance in event of disability. The mode of cancelation in the Policy Exhibit is by "Disability". Each year the instalment payments are charged to Disability Claims. In event of death before all instalments are paid, the account of Death Claims is charged with the commuted value of unpaid instalments; insurance to the same amount is revived in the Policy Exhibit, then canceled by "Death". No charge is made to Disability Claims on account of premiums waived, as the insurance is assumed to be canceled in event of disability.

Assets and Liabilities

Disability benefits affect not only the company's income and disbursements but also various items of assets and liabilities.

Among the assets shown by the companies are "Gross pre-

miums due and unreported" and "Gross deferred premiums" less loading. The gross premiums required for these items are the full premiums including any for disability benefits. Although the company is not expected to show separately the amount of disability premiums due and unreported or deferred, such amounts must be determined since they are used in the preparation of the disability section of the Gain and Loss Exhibit.

Disability liability items are several in number. A company is required to hold disability reserves on active lives and on disabled lives. The active life reserve appears in the annual statement under the item "Extra reserve for total and permanent disability benefits . . ." and the disabled life or claim reserve under "Present value of amounts incurred but not yet due for total and permanent disability benefits . . .". These reserves are definitely ascertainable amounts according to the reserve basis adopted and comprise the bulk of the disability reserve held by a company. There are, however, other items of importance. Reserves are required to be held for claims which have occurred but have not yet been reported to the company. This item is usually known as the reserve for unreported claims and appears in the annual statement under "Estimated Net Losses Incurred but not yet Reported". A reserve must also be held for claims reported but not yet adjudicated, known as the reserve for pending claims or for claims in course of settlement, which now appears under "Incomplete Proofs, Under Adjustment, or Adjusted but not Due". A reserve is also required to be held for claims filed with the company but on which the company for one reason or another is resisting payment, the reserve appearing under "Resisted".

The amount of the active and the disabled life reserves is obtained by means of the company's valuation schedules. The other reserves are readily ascertainable from the records of the disability claim department, with the exception of claims incurred but not reported. The reserve for these claims varies in companies of the same size according to the promptness with which claims are reported. In some companies, for example, where disability benefits have been extended to all outstanding policies, it frequently happens that a considerable length of

time will elapse between the date of disablement and the date of filing claim. A common practice of the companies is to keep a record of delayed claims received during the year where the claim could have been presented prior to the current year. Such a record is the basis for the incurred but unreported liability item at the end of the year, due allowance being made for the normal increase in the total yearly claims.

Various smaller liability items are also held, either under the above item of "Incomplete Proofs . . ." or under "Due but Unpaid . . .". Income payments approved for payment but unpaid at the end of the year would appear under the latter item. These may occur, for example, if the insured has changed his residence so that he could not be reached, or if he has been committed to an asylum for the insane but no guardian appointed. To avoid delays due to the latter, many policies now provide for payment to the beneficiary when disability is caused by insanity, but even this provision does not always make it possible to dispose of the income payments promptly. If the beneficiary is a minor the company would ordinarily prefer to make payments to the legal guardian of the beneficiary. Again, under the instalment type of benefit which reduces the face amount, a company may make a practice under delayed claims of paying instalments for a period prior to receipt of proof of disability, although the policy may provide for payments beginning only from receipt of proof. Since such instalments are not according to the provisions of the policy and affect the amount payable to the beneficiary, some companies deem it advisable to obtain the beneficiary's consent before the instalments are paid. A reserve would be held for such instalment payments approved but awaiting consent of beneficiary. There might also be a reserve to cover current benefits on claims previously admitted, which would normally fall due prior to the end of the year of account, the approval of which is delayed until receipt of evidence of continuance of disability.

If disability premiums payable fractionally are true fractional premiums instead of instalment premiums, the reserve item "Reserve to cover the non-deduction of deferred fractional premiums at the death of the insured" may include a reserve in respect to the disability portion of the premium.

This reserve may be obtained by treating as additional insurance the amount of deferred premiums which would not be received in event of death. In other words, the benefit would be valued by using under Ordinary Life policies the tabular insurance reserve factors and under endowment and limited payment life policies the reserve factors for a term policy covering the premium paying period. The present convention annual statement blank does not permit the valuation of this benefit by means of yearly renewable term factors. This item is a partial offset to the deferred premium asset item. As already stated, it represents a reserve for those deferred premiums which will not be received on account of the death of the insured. The question may be asked: Should not a reserve also be held for deferred premiums which will not be received on account of the insured becoming disabled? The student will see in the calculation of premiums for present day benefits (Part II, Chapter IX) that a continuous disabled life annuity is generally used for the evaluation of the waiver benefit. In such cases the premium calculation provides for the waiving of premiums, if any, for the balance of the policy year in which disability occurs, and the reserve to cover the non-deduction of deferred fractional premiums at the disablement of the insured would be covered by the disability active life reserves.

Gain and Loss Accounting

(a) Analysis by Means of a Running Account

The foregoing accounting methods enable the company to analyze its disability experience. The various items of income and disbursements, together with the reserves carried, determine the total profit or loss but, in interpreting the results, one must of course consider whether the reserves are inadequate, excessive or approximately correct. Analyses of the rates of disability and the rates of claim termination enable the actuary, after making provision for unpaid and unreported losses, to determine the premiums based on experience according to age and plan. In order, however, to obtain the current aggregate profit or loss, or the accumulated aggregate profit or loss to date, either for the total company or for homogeneous groups, it is the practice to keep a running account. Sometimes it is desir-

able to analyze the experience, for example, by type of benefit, by sex, or by year of issue. For this purpose it is necessary to subdivide all items of income, disbursement and reserve according to the desired groups. In a typical account on a calendar year basis the gross premiums received less expenses would be credited, and the charges to the disability account, such as premiums waived, annuity payments, disability loan interest, etc., would be debited. These items would be assumed to enter the account on the average at the middle of the year and accordingly a half year's interest would be added at the effective rate. The net amount, plus the funds at the beginning of the year with one year's interest, would give the funds at the end of the year. A comparison of the funds at the end of the year with the total of the various reserves required to be then held would show the total accumulated profit or loss to date. The accumulated profit or loss so obtained may be illustrated by the following symbolic expression:

$$F_1 - V_1 = \text{accumulated profit or loss,}$$

where

$$F_1 = F_0 (1 + i_1) + (I_1 - E_1 - D_1 \pm \Delta_1) (1 + i_1)^n;$$

and F_0 = fund at beginning of year,

F_1 = fund at end of year,

i_1 = effective interest rate for year,

I_1 = disability premium income for year,

E_1 = disability expenses for year,

D_1 = disability claim disbursements (including premiums waived) for year,

Δ_1 = any dividend adjustments on account of disability profits or losses,

V_1 = reserve at end of year.

Another method of crediting interest would be to follow that outlined in the instructions for the determination of the disability items in the Gain and Loss Exhibit. Under those instructions interest is added to the account by applying the effective interest rate to the mean funds or reserves for the year.

The profit or loss for the current year may be obtained by either of the following methods: (1) by comparing the sum of the accumulation with interest for one-half year of the year's

net income (premium income less expense, less disbursements, etc.) and one year's interest on the reserves at the end of the preceding year, with the excess of the reserves at the end of the year over those at the end of the preceding year or (2) by comparing the accumulated profit or loss at the end of the year with the corresponding figure at the beginning of the year, increased by one year's interest. The result should be the same by either method. It is suggested that the student prove by an actual case the equality of the results by these two methods.

In these methods of determining the current year's gain or loss, it is assumed that interest on the reserves at the beginning of the year is credited to the account. If there has been a loss, the above methods imply that the losses to the beginning of the current year have been made up, so that the funds in hand at that time are equivalent to the reserves, and the interest on past losses is not assessed against the current year. On the other hand, this method would not give credit on the current year experience for interest on surplus funds if there had been an accumulated profit. An alternative method would be to determine the current year's profit or loss by crediting interest on the initial fund in place of the reserves. To accomplish this it would be necessary under method (1) to credit one year's interest on the fund at the beginning of the year rather than on the reserves, while under (2) the accumulated profit or loss at the beginning of the year would be used without interest.

(b) *The Gain and Loss Exhibit*

Subdivision into active and disabled—Recoveries. The disability section of the Gain and Loss Exhibit is intended to show the current year's profit or loss, excluding any profit or loss from loading. It brings into the disability section of the account net premiums only as income. The exhibit subdivides the profit or loss into that from active lives and that from disabled lives. The net premium income is credited to the active life section thus establishing the active life funds. When a claim arises the value of the future disability benefits, i.e., the initial claim reserve, is transferred from the active to the disabled life part so as to establish the disabled life funds. If the claim reserve thus transferred understates the payments made,

on the average, under the claim, the disabled life section will in course of time show a loss, and vice versa. The disabled life section receives these claim reserves at date of disability as income. With these sums it is assumed that the payments to the disabled lives will be made and that the claim reserves as of the end of the year will be provided. The question arises, what transfers should be made between the two sections of the disability account on recovery of a disabled person? If the claim annuity is based on a claim termination rate, which includes both death and recovery, then the value of the benefit transferred at disability should theoretically be just sufficient, on the average, to meet all benefits under the claim until the claim terminates either by death or recovery. Accordingly, nothing should be transferred from the disabled to the active life funds when a claim terminates by recovery and the policy re-enters the active life group. If, however, the claim value at disability is based upon an annuity which considers only terminations by death, such as Hunter's disabled life annuities, what entries should be made at recovery? In many cases Hunter's aggregate annuity values are higher for the age at recovery than for the age at disability. In such cases if the value at the age at recovery according to Hunter's table were transferred from the disabled to the active life section, the disabled life section would show a loss. It would have received a certain sum at disability, and, after paying the various benefits under the claim, it would be required to give back to the active life section a sum greater than it received. The Gain and Loss Exhibit in its present form calls for the transfer, from the disabled to the active life section, of the "Present value of claims on contracts held by policyholders recovering during the year" except where claims are valued on the basis of select disabled life annuities. Obviously, such a transfer, based on claim values such as given by Hunter's aggregate tables, makes the results of the subdivisions of gain or loss into active and disabled life groups without significance when dealing with a class which has a high recovery rate.

The Report of the Committee on Disability Experience in 1926, recommended that, on first year disability claims under a 90 day clause, such as in Class (3), the companies hold only 65% of Hunter's reserves. This percentage made allowance for

the lower claim values due to the terminations by recovery as well as by death. If the claim value transferred to the disabled life section at disability is based on some such reduced percentage of Hunter's, so as to allow for recoveries, or is based on a claim annuity which takes recoveries into account, such as the Class (3) annuities, then, on recovery, no amount need be transferred from the disabled life to the active life section.

Active life reserve on recoveries. On the transfer of a recovered claim to the active life section another question arises. Active life reserves are theoretically supposed to be held only on active lives under the total and permanent form. When a claim arises the active life reserve is released and the disabled life reserve is substituted. When the insured recovers, the policy again enters the active life group and an active life reserve is, therefore, again required. Should the disabled life section be charged with this reserve? Or, should the active life section which (apart from the loss sustained through transferring the claim reserves to the disabled life section) gained by the release of the active life reserve when the claim arose, be now required to provide the reserve on the insured's recovery? The latter seems to be the logical treatment, as the active life section released too much reserve at the time the claim was incurred. As a matter of fact, however, many companies apply the active life reserve factors to the total insurance in force with disability benefits, so that active life reserves are held on both active and disabled lives, which, by the way, is a partial offset to the probable inadequacy of active life reserves according to recent experience. Under such a practice the question of providing active life reserves on recovered claims does not arise.

Reserves for unreported and pending claims. There is another question involving reserves which also should be mentioned. As already indicated, besides the regular active life reserve and the claim reserve, reserves are required for unreported and for pending claims. In the subdivision of the disability accounting into active and disabled life sections, which section should be required to hold the reserve for unreported and for pending claims? One treatment is to have the active

life section maintain these reserves. When the claim is approved the pending or the unreported reserve is released and the active life section is charged with the regular claim value, which is transferred to the disabled life section. An alternative treatment is to include in the "Present value of disability claims incurred . . .", of the 1931 gain and loss instruction sheet, the current year's increase in reserve for unreported and pending claims less the amount credited to the disabled life section as the current year's interest earnings on funds for such claims. The active life section would thus be charged each year with the amount included in the "Present value of disability claims incurred . . .". The disabled life section would be credited with such amount and in addition with the interest earned on these reserves, and would in turn maintain the total reserves for unreported and pending claims. By either treatment the active life section would bear the cost of the yearly increase in these reserves in excess of the interest earned, while the disabled life section would show no profit or loss therefrom.

Apart from the foregoing comments regarding the profit or loss on disability in the Gain and Loss Exhibit, one must bear in mind the great changes experienced by the companies from year to year both in the rate at which claims occur and the rate at which they terminate. Such fluctuations render it difficult to obtain standards by which to measure the profit or loss, especially when analyzed into that from active lives and that from disabled lives. One of the functions of the Gain and Loss Exhibit is to facilitate comparisons between the experiences of different companies. When we find, however, in several companies with similar experiences, divergent results in their indicated disability profit or loss, due in part to the considerations above and in part to different accounting methods, it is apparent that the active and disabled life subdivisions are of little value. On the other hand, the total profit or loss for the active and disabled life sections combined is significant and forms a fairly satisfactory basis of comparison. However, as already pointed out, any interpretation of the gain or loss from disability should take into account the reserve basis. The convention form of annual statement, although showing the bases employed in calculating the insurance and annuity reserves,

does not call, at present, for a statement of the basis on which reserves for disability benefits are held.

This discussion of reserves somewhat anticipates what is taken up in Part II, Chapters XIII and XIV, but it seems appropriate to cover some of these points in a discussion of the Gain and Loss Exhibit accounting.

Canadian Annual Statement

The foregoing discussion of annual statement practice has referred to that applicable to the Insurance Commissioners' convention form of statement which is used in the United States. As the accounting methods employed in connection with the Canadian annual statement differ somewhat from those for the convention report, due to certain differences in the respective statements, the difference in accounting prevailing in 1931 will be briefly reviewed.

In the Canadian statement, premiums waived are not included either in the premium income or in the disbursements, but are shown in a separate schedule. Payments on account of disability claims include, in addition to income payments and instalment payments reducing face, the commuted value of future payments under annuity-certain instalment benefits. Such commuted value (in maturity instalment cases, the face amount) likewise enters the income account as "consideration for supplementary contracts, not involving life contingencies."

Differences in treatment also occur in the classification of claim reserves and in the policy exhibit. The disabled life reserve on income types and "instalment reducing the face" types, including the life insurance reserve on the latter, is carried as a disability annuity reserve in the "Annuity Section" of the "Statement of Actuarial Liabilities". On the occurrence of disability, both the instalment reducing face and the instalment annuity-certain types are considered as annuities and are transferred to the annuity exhibit, the former as a disability annuity, and the latter as a supplementary contract not involving life contingencies. Consistent with this treatment the disabled life reserve for instalment annuity-certain types is included under the latter heading in the "Annuity Section" of the "Statement of Actuarial Liabilities." The claim reserve for waiver of premiums,

however, whether in conjunction with other disability benefits or not, is not included in the "Annuity Section" as a disabled life reserve but is carried in the "Assurance Section", presumably with the active life reserves under "Disability Benefits".

The Canadian statement, in addition to the policy and annuity exhibits has an "Exhibit of Net Insurance in Force (Ordinary) Including Disability Benefits". In this exhibit all insurance with disability benefits is shown according to whether the insured is active or disabled, then subdivided by type of disability benefit.

The Canadian statement does not contain a gain and loss exhibit.

PART II.

CHAPTER I.

INTRODUCTION—COMBINED MORTALITY AND
DISABILITY TABLE

The method by which premiums and reserves for disability benefits are calculated depends upon the type of benefit and the definition of what constitutes disability. The earlier benefits were provided only in the event of total and permanent disability so that the termination of the disability claim should supposedly be by death, not by recovery. The present day benefit is granted in event of total and *presumably* permanent disability. Where the *presumption* of permanent disability is based on total disability lasting for a definite "waiting" or "qualification" period such as three months, four months, or one year, many of the disability claims will be terminated by recovery rather than by death, especially in the early disability years. The result is that the values and nature of the claim annuities under the two types of benefits are radically different. The value at date of disablement of an annual benefit of \$1 per annum might be \$8 under the old type but only \$3 or even less under the present type. Of even greater importance is the incidence of the claim termination rate by age and by duration of claim. Under the earlier type the variation was mainly by age so that satisfactory aggregate disabled life annuities could be determined. Under the total and *presumably* permanent type the variation in the annuity value by duration of the claim is much greater than that by age so that in premium and reserve calculations it is necessary to use select disabled life annuities.

There are three principal methods of calculating premiums and reserves for disability benefits. The first will be referred to as the "Hamza" method as it rests on the method devel-

oped by Hamza in his paper presented to the Third International Congress of Actuaries. This method is applicable with greater facility when the type of benefit is such that aggregate disabled life annuities can be used. It is also most readily applicable when the due dates of the disability benefits coincide with policy anniversaries. Typical disability benefits would be a waiver of premium benefit, assuming premiums payable annually, or one of the older forms providing waiver together with an annual income, first payment due one year after the anniversary next succeeding receipt of proof of disability. Such forms of disability coverage involve the rate of disability designated by the symbol " p_z^a " which will be defined later.

The student will also find the designation "Hamza process" used in this chapter. It refers to the method of obtaining the mortality among the "active" and the "disabled" lives. The Hamza process is generally used in this country and applies whether the subsequent calculation of monetary values is by the method which is designated in this book as the Hamza method or by the pension fund method referred to in the next paragraph.

The second method of calculating premiums and reserves for disability benefits is the so-called "pension fund" method. For the student who has not yet become familiar with pension fund calculations it may be stated that, in a pension fund method of calculation, an expression is obtained which represents the total cost of the benefit involved for any policy year. A symbol is then assigned to this expression which is usually a " C_z " with various small letters affixed to some or all of its four corners to indicate the benefit involved. A summation of this expression for the period of coverage gives the numerator of the single premium for the benefit, the summation being represented by an " M_z " corresponding to the C_z already referred to. By its very nature the pension fund method is noted for its facility of application to any and all types of benefits. The difficulty frequently met, however, is the complexity of the symbols, due to the variety of benefits which the " C " and other functions may represent.

The third method is known as the "sickness" method. By this method the individual claim is not considered, nor the cost of each benefit, but all claims are combined and the number of

weeks (or months) of sickness in the year per thousand of the total number of lives is found, including in the latter those who have been sick during the year. The weeks of sickness per annum are usually subdivided according to duration of the sickness so that the resulting tables will show, per one thousand members at age x , that there will be experienced during one year, a certain number of weeks of sickness in the first three months of sickness, a certain number of weeks of sickness in the second three months of sickness and so on. As its name implies, the sickness method may most appropriately be employed where many of the claims terminate after a short illness rather than continue for a long period or for life. Since it does not consider the claims by themselves, it is not suited for the calculation of reserves in America where they are required to be subdivided into those on active lives and those on disabled. One of the important applications of the sickness method was in the investigation of the sickness experience of the Manchester Unity Friendly Society.

The first two methods are the principal ones used in America in connection with calculations for disability benefits included in life insurance policies. Only these two methods, therefore, will be considered.

Whichever method is used it is necessary that we know

- (a) the rate of mortality among the active lives;
- (b) the rate at which the active lives become disabled;
- (c) the rate of death (or termination) among the disabled lives.

(Certain assumptions which have been made in recent years with regard to some of these rates will be discussed later).

Under the earlier type of benefits which were granted only in event of total and permanent disability, no recoveries were anticipated among the disabled lives. The rate under (c) would, therefore, be in such cases the rate of death among disabled lives. With the advent, however, of more liberal forms of disability coverage, benefits being granted in event of *presumably* permanent total disability, recoveries from disability become increasingly more frequent. The function in which we would then be interested for the calculation of premiums and

reserves would be the rate of termination, by either recovery or death, among disabled lives.

Knowing the above three rates for each age required, a "Combined Mortality and Disability Table" may be constructed. Such a table can be prepared whether the termination rate among disabled lives consists entirely of deaths or includes both recoveries and deaths. The inclusion of recoveries, however, considerably complicates the construction of the table. In the methods now used to calculate premiums for modern benefits, under which recoveries are frequent, the combined mortality and disability table is not employed, whereas in the calculation of premiums for the early benefits, under which recoveries were negligible in number, the combined table was used. The combined mortality and disability table developed in this book will, therefore, be based on the assumption that there are no recoveries among the disabled lives. The combined table, however, may be used under modern benefits to test certain assumptions and approximations made in the calculation of the premiums. In such cases the combined table could, with approximate accuracy, be constructed with recoveries taken into account. The student will find in T.A.S.A., XXX, 409, the description of the construction, by an approximate method, of a combined table for the purpose of measuring the effect of using mixed life tables in place of active life tables in the calculation of premiums for modern benefits. This point will be taken up later; for the present, we will consider the form of the combined mortality and disability table on the assumption that there would be no recoveries.

The rate of mortality among the active lives may be obtained in one of the following ways:

1. A table representing the mortality among active lives may be developed by an investigation of these lives.
2. A published table of mortality may be adopted to represent the mortality among active lives.
3. A published table of mortality may be adopted to represent the total mortality among active and disabled lives, the mortality among disabled lives being obtained by investigation. This was the method used by Hamza and is known as the "Hamza process".

Method No. 3 is the one generally used in America. Method No. 2 is only permissible when the table adopted represents,

within reasonable limits, what the mortality among active lives is known or believed to be. The student should note that if a table approximately represents the mortality among the whole body of policyholders, it would not properly represent the mortality among the active lives, because the active lives have a lower mortality than the disabled lives.

Using an ultimate table, such as the American Experience, instead of a select table, to represent the total mortality, results in certain inconsistencies. J. F. Little, in T. A. S. A., XIV, 75, illustrates this as follows:

“For instance, persons insuring at age 30 being all active lives and subject to the American mortality, the rate for active lives aged 30 in the first policy year is equal to the full American q_{30} . The survivors from entrants at age 29, however, will include some disabled lives subject to a high rate of mortality, and the active lives must consequently be subject to a lower mortality than the American, in order that the latter may coincide with the rate for active and invalid lives. Thus lives accepted a year ago, and not absolutely disabled, are treated as better than newly selected lives, and, generally, the longer the period since selection the better the mortality among those not totally disabled. Because of this palpable absurdity, Mr. Mead contends that the American mortality should apply to active lives entering at the youngest age, say 15 or 20, only; and that at all higher ages the rate of mortality among active lives shall be the same as among the survivors of entrants at the youngest age, and consequently lower than that of the American Table. This is theoretically more defensible, as the rate of mortality among active lives is then independent of the age at entry, but it does not comply with the requirement that the American Table shall apply at all ages and durations, as the aggregate mortality of all entrants except those at the youngest age will be less than that of the American Table.

“Mr. Pipe has pointed out that, by neglecting the above mentioned requirement, a life is treated as subject to different rates of mortality for the purposes of life and disability insurance respectively. I think, however, that a mortality table cannot properly be regarded as applying to a single life, but only to a group of lives. It is clear also that an individual may correctly be included in each of two or more groups, although different mortality tables apply to such groups. Thus a clergyman may properly be treated as subject to ‘clergy’ mortality for the purposes of a clergy

pension fund, and, at the same time, as subject to 'insured' mortality for the purposes of life insurance—in the first case as belonging to a relatively small professional group, most of which would be included in the second and larger group. I am by no means certain, however, that practical convenience is not served in some directions by using the American Experience mortality in the manner in question."

The student will find a fuller discussion of this point by referring to Mr. Little's paper. It is interesting to note that in recent investigations the tendency has been to make the assumption that either the American Experience Table or the American Men Ultimate Table can be used for both the active life table and the total mortality table. The error involved in this assumption will be referred to when the various premium formulas are discussed.

The rate of disability adopted is usually ultimate, i.e., it is aggregate with the first few years of insurance excluded.

A section of a combined mortality and disability table (Hunter's) is shown below, the values of l_x and d_x being those according to the American Experience Table (see Chapter VI for changes made in notation used by Mr. Hunter):

COMBINED MORTALITY AND DISABILITY TABLE

Age	(1) T_x	(2) Q_x^l	(3) Q_x^{aa}	(4) l_x^{aa}	(5) d_x^{aa}	(6) i_x	(7) l_x^{ii}	(8) d_x^{ii}	(9) l_x	(10) d_x
15	.000587	.267	.00755	96,285	727	57	0	8	96,285	735
16	.000584	.254	.00747	95,501	713	56	49	19	95,550	732
17	.000581	.241	.00740	94,732	701	55	86	28	94,818	729
18	.000578	.229	.00740	93,976	695	54	113	32	94,089	727
19	.000575	.217	.00740	93,227	690	54	135	35	93,362	725
20	.000574	.205	.00742	92,483	686	53	154	37	92,637	723
21	.000572	.193	.00746	91,744	684	52	170	38	91,914	722
22	.000571	.182	.00750	91,008	683	52	184	38	91,192	721
23	.000570	.171	.00755	90,273	682	51	198	38	90,471	720
24	.000570	.161	.00761	89,540	681	51	211	38	89,751	719

l_x^{aa} = the number of active lives at age "x",

l_x^{ii} = the number of disabled lives at age "x",

d_x^{aa} = the number dying while active between ages "x" and "x + 1",

d_x^{ii} = the number dying, while disabled, between ages "x" and "x + 1",

i_x = the number of active lives becoming disabled between ages "x" and "x + 1",

- r_x = the probability of an active life aged "x" becoming disabled within a year,
 q_x^i = the probability of a disabled life aged "x" dying within a year. (This symbol is also used to represent the probability of a disability claim under a life aged "x" terminating by death or recovery within a year),
 q_x^{aa} = the probability of an active life aged "x" dying within a year while still active,
 l_x = the number living at age "x" according to the total mortality table. This consists of the number of active living and the number of disabled living,
 d_x = the number dying between ages "x" and "x + 1" according to the total mortality table. This consists of active lives dying and disabled lives dying between ages "x" and "x + 1".

The student may more readily master the notation employed if he observes the significance of the small letters used at the upper right hand corner of the main symbol, especially where double letters appear. The letter "a" is used to indicate an active life and "i" an invalid or disabled life. The use of two small letters at the upper right corner has one significance when used with l_x , d_x or any of the commutation symbols (pension fund and disabled life symbols excepted) and something different when used with p_x , q_x or any of the annuity symbols.

When "aa" or "ii" appears at the upper right of l_x or d_x it means that the l_x or d_x is only part of the complete l_x or d_x in a mortality table. That is, l_x^{aa} is only a partial l_x and by itself does not constitute the l_x with which the student is familiar in his study of life contingencies; in other words, $l_x^{aa} - d_x^{aa}$ is not equal to l_{x+1}^{aa} . These statements apply with equal force to l_x^{ii} and d_x^{ii} . The complete l_x is equal to $l_x^{aa} + l_x^{ii}$ and also $d_x = d_x^{aa} + d_x^{ii}$. Later on we will see that $D_x^{aa} = v^x l_x^{aa}$ and $D_x^{ii} = v^x l_x^{ii}$. Therefore, we have $D_x^{aa} + D_x^{ii} = v^x (l_x^{aa} + l_x^{ii}) = v^x l_x = D_x$ and similarly with N_x^{aa} and N_x^{ii} , C_x^{aa} and C_x^{ii} and so on. In brief, when "aa" or "ii" appears at the upper right of any of the foregoing symbols it signifies that two l_x 's, two d_x 's, etc. are required to make the complete l_x or d_x with which we are familiar, one of the partial functions having "aa" and the other "ii" at the upper right. On the other hand, when only the single letter "i" appears at the upper right of l_x , it indicates that the l_x represents the total number living at age x in a certain table; l_x^i is not a partial function but is a complete l_x in a table which consists only of

disabled lives. Such a table would be formed by a group of disabled lives aged x , followed to the death of the last survivor.

When one or two of the letters " a " and " i " appear at the upper right hand corner of a probability or annuity symbol, the first letter indicates the disability status at the present time and the second letter, if any, indicates either the disability status when the event covered by the probability symbol occurs or the disability status which will prevail during the payment of the annuity. For example, p_x^{ai} is the probability that a life aged x now active will be alive and disabled at the end of a year, and a_x^{ai} is the annuity in respect to a life now aged x and active which will be payable when the life becomes disabled. If only one of the letters " a " or " i " is used, it shows the present disability status and also that the disability status in the future is not involved. For example, q_x^a is the probability that an active life aged x will die within a year (whether active or disabled at time of death) and a_x^a is the annuity-due payable for life to a life now active—whether the life subsequently becomes disabled will not affect the future payments. One important exception to this rule should be noted, due to the fact that there are recoveries among disabled lives, even though the combined table may be constructed assuming none, and that is the function a_x^i or a_x^i . This is an annuity paid during the lifetime and continued disablement of a life now invalid and aged x .

The annuity value is equal to $\frac{\sum v^n l_{x+n}^i}{l_x^i}$. As the life is now invalid

and must remain so in order to draw benefit, we might expect the symbol to be a_x^{ii} , if recoveries are taken into account, but such a function, to be consistent with the prevailing notation, should

be equal to $\frac{\sum v^n l_{x+n}^{ii}}{l_x^{ii}}$ which is not the value desired.

It will be observed that the number of persons living at any age according to the total mortality table is divided into active and disabled lives, that the sum of the number of active and the number of disabled lives is the number living according to the total mortality table, and that the sum of the deaths among active lives and of the deaths among disabled lives is the number of deaths according to the total mortality table. The assump-

tion is usually made that at age 15 all lives in the total table are active lives.

The student must keep in mind that in the division of the mixed lives into active lives and disabled lives the combined mortality of these two divisions must remain the same as in the total table.

If the Hamza process of determining the mortality among active lives is adopted the mortality among both active and disabled lives is not required. It is necessary to investigate only the mortality among the disabled lives. The deaths among active lives are obtained by subtracting the deaths among the disabled lives from the total deaths according to the mortality table adopted.

The construction of a combined mortality and disability table will be discussed later (see Chapter VI) after we have considered the determination of some of the rates which must be known before the combined table can be constructed.

CHAPTER II.

RATE OF DISABILITY

In the combined mortality and disability table r_x , the rate of disability, is defined by the relation $r_x = i_x \div l_x^{aa}$. The values of r_x defined in this way are not independent of the rate of mortality among the active lives, because the value of i_x obtained will depend on the average exposure to disability *during* the year while l_x^{aa} represents the number of active lives at the *beginning* of the year. The greater the rate of mortality among the active lives the smaller the number actually exposed throughout the year to disability. Since the l_x^{aa} column is subject to two decrements, i_x and d_x^{aa} , the symbol r_x , as defined above, represents what would be termed in a multiple decrement table the *probability* of becoming disabled (see Life Contingencies, E. F. Spurgeon, Chapter XXI). That function which would be termed a *rate* of disability in a multiple decrement table is called the "absolute" rate of disability in the combined mortality and disability table and is represented by the symbol r'_x .

It is equal to $\frac{i_x}{l_x^{aa} - \frac{1}{2} d_x^{aa}}$ and is, therefore, the rate which is independent of the rate of mortality among active lives.

To investigate the relation between r'_x and r_x , let E''_x be the number active at the beginning of the year exposed to risk of either disability or death in the year from age x to age $x+1$, so that in determining E''_x , active lives passing out of observation through death or disability are taken as exposed for the full year. Let E'_x represent the corresponding number exposed only to risk of disability; then in calculating E'_x the deaths as well as all other terminations, except those due to disability, are considered to have passed out of observation at the nearest, mean or exact duration, so that, on the assumption of a uniform distribution of the deaths among active lives,

$$E'_x = E''_x - \frac{1}{2} d_x^{aa}$$

while

$$(1) \quad q_x^{aa} = \frac{d_x^{aa}}{E''_x}$$

$$(2) \quad r_x = \frac{i_x}{E_x''}$$

$$(3) \quad r'_x = \frac{i_x}{E_x'} = \frac{i_x}{E_x'' - \frac{1}{2}d_x^{aa}} = \frac{i_x}{E_x'' (1 - \frac{1}{2}q_x^{aa})}$$

From (2) and (3) we have

$$(4) \quad r_x = r'_x (1 - \frac{1}{2}q_x^{aa})$$

as the relation sought; while in the combined mortality and disability table we have the analogous relations

$$(5) \quad r_x = \frac{i_x}{l_x^{aa}}$$

$$(6) \quad r'_x = \frac{i_x}{l_x^{aa} - \frac{1}{2}d_x^{aa}}$$

From equation (4) it is evident that the greater the mortality among the active lives the less will be the rate of disability, r_x , although the absolute rate of disability, r'_x , remains unaltered.

The formula $r'_x = \frac{i_x}{l_x^{aa} - \frac{1}{2}d_x^{aa}}$ assumes that the deaths among

active lives will be uniformly distributed throughout the year, which is slightly at variance with the usual assumption that the *total* deaths are uniformly distributed throughout the year. On the basis of the total deaths being uniformly distributed, more active deaths should fall in the first half of the year than in the second half. The greater the value of r'_x the more uneven will be the distribution of the deaths among active lives. The value of r'_x , however, is never large enough to make the practical error important. (For a further discussion of this point the student is referred to T. A. S. A., XIV, 74). To be theoretically correct, r'_x as ascertained from the disability data, should be combined with the rates of mortality adopted for active lives while the other functions, including r_x , should be determined from the relations existing among these functions. The slight error involved in using the values of r_x obtained directly from the disability data is, however, frequently ignored in practice.

In deriving the rate of disability from the experience of an insurance company, the definition of what shall be considered disability determines which deaths shall be taken as deaths

among active lives and which as deaths among disabled lives. Since most deaths are preceded by a period of a few days or a few weeks of total disability the problem is not an easy one, except where a definite period of total disability is considered as presumptive evidence of permanence. Furthermore, the rule that is observed in drawing the line between deaths among active lives and deaths among disabled lives will affect the resulting rates of disability and also the death rates among disabled lives. If, for example, all death claims be examined and all lives that have been disabled 30 days, say, or more, be considered as disabled, and subsequent deaths among them be considered as deaths among disabled lives, the resulting rates of disability and the death rates among disabled lives will be different from those which would be obtained by considering as disabled only those that have submitted proof of disability before death or recovery. It has sometimes been provided in policies containing disability benefits that disability, even if of a permanent nature, must have existed 60 days or more before claim for disability benefits is allowed. On the other hand many contracts issued in recent years have provided that 90 days of total disability are necessary and sufficient to qualify for benefits. (Those issued in accordance with the Insurance Commissioners' standard provisions adopted in 1930 require from four months to one year).

It is moreover imperative that the terms of the policy contract and the practice of the company in allowing claims be carefully considered as to the possible effect upon the number of disability claims. Where the benefit is available on the anniversary next succeeding the receipt of proof of disability many claims will not be filed until shortly before the anniversary and the company may, in order to eliminate useless investigation, advise deferring the filing of proof of disability until shortly before the anniversary—thus causing many deaths among lives that have been disabled less than one year to be regarded as deaths among active lives. If an investigation of the practice of a company shows that the latter is the case, then it is not practicable to obtain directly from the experience either r_x or r'_x ; but it would be possible to ascertain p_x^{ad} , the probability of an active life aged x becoming disabled during the year and surviving to the end of the year.

When the benefit dates from the receipt of proof of disability and becomes available six months or one year thereafter, it is necessary to consider carefully the practice of the company in recording deaths occurring between the time the proofs are filed and the time the benefit becomes available. In this case the policyholder may be depended upon to file proof of disability promptly, so that the company's records may be in the form from which r_x and r'_x may be derived. But, on the other hand, the company may not complete its investigation of the case until shortly before the benefit becomes available and may regard all deaths occurring in the interim as deaths among active lives. In the latter case the records may be used to determine the probability of becoming disabled and surviving six months or one year thereafter, but cannot be employed to determine r_x or r'_x .

If the benefit is entered into immediately upon receipt of proof that the policyholder has been disabled for 90 or 120 days, then r_x or r'_x may be ascertained from the experience. In this case all deaths occurring within a 90 or 120 day period following disability would be recorded as deaths among active lives and only those lives which survived the period of 90 or 120 days would be recorded as disabled. In view of the heavy death rate in the year following the occurrence of disability, especially in the first few months of that year, it is obvious that the resulting rates would be greatly affected by the length of the probationary period. If all deaths occurring after disability has been in existence for 90 days be regarded as deaths among disabled lives, then the rate of disability and also the rate of death among disabled lives should be higher than if the probationary period were 120 days.

In investigating the experience of a company, it is necessary, therefore, to have clearly in mind what type of probability can be derived from the records, so that the resulting probability may be correctly combined with the total mortality table for the purpose of calculating premiums and reserves. From what has been said the records of the company may generally be used to derive one of the following:

- (a) r_x , the probability of an active life aged " x " becoming disabled within a year;

- (b) τ'_x the absolute annual rate of disability among active lives aged " x " at the beginning of the year;
- (c) p_x^d , the probability of an active life aged " x " becoming disabled and surviving to the end of the year.

The investigation should be made in the select form, so that the effect of selection may be measured and so that the experience of the early policy years may be excluded, if necessary, in forming the ultimate table. To be theoretically correct, the policy year method should be used.

The following symbols are used in determining the rate of disability:

- $n_{[x]}$ = new entrants at nearest age " x ".
- $s_{[x]+t}^{aa}$ = active survivors at age " $[x] + t$ ", that is, those who entered at age " x " and who have been " t " years on the books at the anniversary in the year the observation began and who were then active. It should be noted that $s_{[x]+0}^{aa}$ is taken as zero in the following formulas, as such cases are included in $n_{[x]}$ which is separately tabulated.
- $i_{[x]+t}$ = lives who entered at age " x " and who became disabled between the ages " $[x] + t$ " and " $[x] + t + 1$ ".
- $d_{[x]+t}^{aa}$ = deaths occurring between ages " $[x] + t$ " and " $[x] + t + 1$ " among active lives who entered at age " x ".
- $w_{[x]+t}^{aa}$ = withdrawals among active lives who enter at age " x " and with nearest duration of " t " years.
- $e_{[x]+t}^{aa}$ = active lives existing at age " $[x] + t$ " at policy anniversary in the last year of observation.
- $r_{[x]+t}$ = the probability of an active life aged " $[x] + t$ " becoming disabled within a year.
- $\tau'_{[x]+t}$ = the absolute annual rate of disability among active lives in the $(t + 1)$ th year of duration who entered at age " x ".
- $E'_{[x]+t}$ = exposed to risk of disability in the $(t + 1)$ th year of insurance.
- $E''_{[x]+t}$ = active lives exposed to risk of death or disability in the $(t + 1)$ th year of insurance.

(a) To determine $r_{[x]+t}$:

$$(7) \quad E''_{[x]+t} = n_{[x]} + \sum_0^t (s^{aa} - w^{aa} - e^{aa})_{[x]+t} - \sum_0^{t-1} (d^{aa} + i)_{[x]+t}$$

$$(8) \quad r_{[x]+t} = \frac{i_{[x]+t}}{E''_{[x]+t}}$$

(b) To determine $r'_{[x]+t}$ on the assumption that the deaths among active lives are uniformly spread over the year, i.e., that the number of deaths among active lives during the first half of the $(t+1)$ th policy year among lives who entered at age x is $\frac{1}{2} d^{aa}_{[x]+t}$:

$$(9) \quad E'_{[x]+t} = n_{[x]} + \sum_0^t (s^{aa} - w^{aa} - e^{aa})_{[x]+t} - \sum_0^{t-1} (d^{aa} + i)_{[x]+t} - \frac{1}{2} d^{aa}_{[x]+t}$$

$$(10) \quad r'_{[x]+t} = \frac{i_{[x]+t}}{E'_{[x]+t}}$$

(c) To determine $p^{at}_{[x]+t}$:

$$(11) \quad p^{at}_{[x]+t} = \frac{j_{[x]+t}}{E''_{[x]+t}}$$

where $j_{[x]+t}$ represents the lives who entered at age x , and who became disabled during the $(t+1)$ th policy year and survived to the end of the year.

Whichever probability is obtained, $r_{[x]+t}$, $r'_{[x]+t}$, or $p^{at}_{[x]+t}$, the separation of the data into select and ultimate experience should be made in the same manner as when investigating the rate of mortality.

In recent investigations, notably in the Inter-Company disability investigation of 1926, certain approximations have been made in determining the rate of disability, which should be noted here. As the rate of disability is fairly small, it was decided to obtain the exposures to disability from the active life valuation records. The insurance in force at December 31st was taken as the exposure for the policy year then current. The cancelations between the beginning of the policy year and the end of the calendar year consist usually of lapses, deaths among the active lives and those becoming disabled (see comments below). Except for the last item of decrement, the valuation figure corresponds to the exposure designated above by the symbol E'_x . Hence, when we are concerned with a benefit which is payable on receipt of proof of disability the resulting rate of

disability obtained is analogous to r'_x . This rate is referred to as the "unadjusted" rate in the Inter-Company report.

One of two errors is involved in the above method. First, the disability claims occurring in the current policy year up to December 31st should not be excluded from the exposures. As the rate of disability is small, this error is negligible. The second error is due to the practice of some companies in determining the active life valuation. In many cases the amount of insurance entering the active life valuation includes all insurance with disability, i.e., the disabled lives are not excluded, so that instead of the exposed to risk being too small by the first error, the exposures in the second case would be too large, as the first error is not present. This point is covered in the premium formula, as will be seen later.

For simplicity the Inter-Company investigation was confined to the three main plans—Ordinary Life (or Endowment at 85), Twenty Payment Life and Twenty Year Endowment. In certain cases the benefit studied was not introduced at the beginning of a calendar year, so that for the first year of issue the insurance in force at December 31st did not represent the amount in force at the middle of the policy year. The amount at the middle of the policy year was obtained by interpolation from the valuation data.

CHAPTER III.

DEFINITION OF DATE OF DISABILITY

Consideration is given in this chapter to a subject which, although still within the realm of controversy and not of great practical moment due to present methods of approximation, merits thoughtful study if a proper understanding is to be attained of the elements involved in an analysis of the data obtained from an experience under present day contracts containing a so-called waiting or qualification period.

A distinction will be made between the expressions "date of disablement" and "date of disability". The date of disablement is the designation given to the date on which the life actually becomes totally disabled. The date of disability, on the other hand, may be regarded as a technical expression which refers to the date as of which the disabled life is considered to have become disabled for the purpose of a disability investigation. For example, a monthly income benefit may be payable on receipt of due proof that the insured is totally disabled and has been for at least three months. Assume that a life becomes disabled on January 1st and remains so for some time but does not submit proof of such disability until, say, the following November 1st. The *date of disablement* would be January 1st. The *date of disability* would be November 1st, if the company were to adhere strictly to the terms of its policy and if the 1926 Inter-Company disability investigation's definition of the date of disability were to be adopted. (The 1926 investigation defined the date of disability as "the date as of which disability was presumed by the company to be total and permanent"). If the company made a practice of dating back delayed claims six months, it would presume the above case to have qualified for benefits as of May 1st and, following the same definition, the date of disability would be May 1st. With the same company rule prevailing the case would be presumed to have qualified for benefits as of April 1st provided notice of claim were submitted on or before October 1st.

Many policies issued prior to the adoption in 1930 of the

standard provisions provided for benefits either on receipt of proof of total and permanent disability or upon receipt of proof of total disability lasting for three months. Whether or not a disabled life would qualify under the first definition would depend entirely upon the individual company's practice. In one company with such a clause, about 30% of the claims were approved on a "permanent" basis. In another company the percentage was negligible. According to the Inter-Company Committee's definition, claims submitted without delay would have the date at the end of the waiting period as the date of disability if approved on a "total" basis and a date prior thereto, usually the date of disablement, if approved on a "permanent" basis. This of course leads to minor discrepancies in the date of disability as determined by different companies. The effect on the rate of disability, however, would be slight.

Of greater importance is the treatment of delayed claims. It seems logical to expect that, as the public becomes still better informed concerning the advantages of the disability clauses, claims will be reported more promptly, i.e., the so-called "lag" in reporting claims will be decreased. Therefore if no adjustment for the current lag is made in the calculations (i.e., if the date of disability is based upon the date the company receives notice of claim), sufficient provision will not have been made for the future experience. For a safe and conservative basis it is necessary therefore that under delayed claims the date of disability be taken as the date which would have been the date of disability if there had been no delay in presenting the claim. Even this treatment makes no allowance for certain claims of short duration which do not now enter the experience because notice of claim was not given during the period of disability. There is a practical difficulty, however, in the application of this rule. For example, if a claim has been delayed for a considerable length of time the company is not interested in determining the exact date of disablement. It will merely satisfy itself that total disability has lasted for three months or for such maximum period as is covered by its pre-dating rule. However, in each case, there is usually a statement by the insured or his representative as to the date of disablement. This would not usually be later than the true date. Accordingly, the stated date may

be safely taken as the date of disablement and the date of disability based thereon.

Effect on Statistical Analysis of Claims

There is still another important point to be considered in connection with clauses which have waiting or qualification periods to permit a presumption of permanent disability. Let us take a clause which provides that a monthly income benefit will be payable in event of total disability lasting four months. At the end of the four months' waiting period the insured qualifies for benefits. At that time he enters the company's disabled life group and his disability annuity commences. The date at the end of the four months is therefore the date as of which the company presumes the insured to have become totally and permanently disabled.

The data derived from a disability experience under this type of clause may be treated in two ways:

(a) Disability claims may be allotted to ages and policy years with reference to the date of disablement.

If claims are presented and approved exactly at the end of the four months' waiting period in every case, the claims which are approved up to one year and four months from the date of issue are treated as being incurred in the first policy year, those which are approved between one year and four months and two years and four months from date of issue as being incurred in the second policy year and finally, those which are approved between eight months before and four months after the limiting age, as incurred in the final year of coverage. The claims would thus be treated as having been incurred during the period of coverage. (A slight factor of safety involving four months' interest on all payments would be introduced if the average claim value is computed as of the date of beginning of payments, but a suitable correction would be effected by multiplying the claim value by v^4 before introducing it into the various functions as hereinafter explained). Select tables would be evolved which would have the characteristics of select tables in general and, in the construction of aggregate or ultimate tables therefrom, the usual principles would be applicable.

(b) Disability claims may be allotted to ages and policy years with reference to the date of disability.

Under this arrangement the claims approved in the last eight months of the first policy year are all that belong to the first policy year. Those which are approved during the four months immediately following the limiting age belong to the policy year which follows the period of coverage. We thus have a rate of disability for the year beginning with the limiting age. The latter would be either some age such as 60 or the maturity or expiry age, if prior thereto, on endowments or term policies. If select tables were used in the calculation of premiums and reserves, the experience for a given age at entry under a group of contracts having the same period of coverage and treated as outlined under (b), could be used without any adjustment devised to allow for the fact that there is not a full year of exposure in the first policy year and to allow for claims qualifying after the limiting age. This would, of course, produce approximately the same result as the process denoted in (a) if the correction for the four months' deferment of the claim payments, to which attention has been called, were made in the latter. For practical reasons, however, an aggregate rate of disability is desirable and some adjustments in the select data obtained by method (b) are necessary before the ordinary process of compiling an aggregate table is applied. The claims for the first policy year may be increased in the ratio $(1+h)$ to arrive at an assumed number (or amount) of claims which might have been approved if they had been produced by a full year's exposures instead of only eight months' exposures; h may properly be assumed to be .50 although a somewhat different factor might be justified. Similarly, the claims for the year beyond maturity or the limiting age may be increased in the ratio $(1+k)$ to arrive at an assumed number (or amount) of claims which might have been approved if they had been produced by a full year's exposures instead of only four months' exposures; k should evidently be taken as somewhat larger than 2 because, at the upper ages, the rate of disability increases rather rapidly and all claims actually experienced have appeared in the first four months of the year. The values of h and k would normally vary with age. The data are now ready for

consolidation into an aggregate experience in the usual way, the first year being omitted if it is desired to ignore the effect of selection to that extent. When, however, the aggregate table is used for the computation of premiums, the tabular value of the first year's risk should be reduced in the ratio

$\frac{1}{1+h}$ and that of the year beyond maturity or the limiting age

should be reduced in the ratio $\frac{1}{1+k}$, in order to allow for the diminished actual exposures in those years.

The rates of disability exhibited in the 1926 Report of the Actuarial Society's Committee on Disability Experience are based upon the date of disability in cases where the claim has been approved under the provision which regards total disability as presumably permanent if it has continued for at least three months and upon the date of disablement in cases which are approved without any such presumption, that is, where the disability is clearly total and permanent or where one of the "specified disabilities" has been incurred. Since the former group was much more numerous than the latter, the reasoning described in (b) may be regarded as theoretically applicable although no rate of disability is shown for the limiting age 60. On account of the short duration of the policies forming the basis of that experience, there were, in fact, no exposures during the year following the maturity of an endowment nor in the year following the limiting age. In any case, the net effect of the subtractive adjustment for the first year and the additive adjustment for the last year would not be great.

Present Day Practice

As indicated, the process under (b) has been used in actual practice in preparing the statistical data for determining the rate of disability and the claim annuity values. In these investigations the statistics for determining the rate of disability have been compiled in age groups, usually quinary, the data relating to ages at disability 55 to 59 including, as a rule, claims arising at age 55 and over. The rate of disability so obtained for the central age 57 would therefore give effect to the claims qualifying after the limiting age. From such select

data aggregate rates of disability have been determined, eliminating the experience of the first policy year. The calculation of premiums has generally been made by means of such an aggregate table without any adjustment for the first policy year. Premiums have therefore been calculated with allowance for claims qualifying after the limiting age and providing for a full year of exposure in the first policy year. Theoretically the premium is overstated but the actual effect is negligible, especially when it is compared with the much greater variations in the cost of the benefits granted, due to the general tendency in recent years for the claim rate to increase from one calendar year to another. The student will find in the actual calculation of premiums for present day benefits that many methods of approximation are adopted to simplify the calculations. They are generally conservative assumptions which have but minor effects on the resulting premiums.

Many companies, prior to the adoption in 1930 of the standard provisions, issued a disability benefit which provided a monthly disability income for each completed month of disability on receipt of proof of total disability lasting for three months. Under a \$1,000 policy with a 1% monthly income, the policyholder would receive \$30 at the end of the three months and \$10 monthly thereafter during the continuance of disability. Even though some companies date the income as beginning at the end of the first month, the date of disability should be the date at the end of the waiting period, the benefit being viewed as a disability annuity of \$10 per month, first payment at the date of disability, plus a single payment at the date of disability of \$20. In this connection the date of disability might be defined as the date from which exposure to claim termination begins. The foregoing type of disability benefit which provides benefits from the commencement of disability is frequently referred to as a retroactive benefit. This expression, however, is sometimes used in connection with the treatment of delayed claims under other forms.

CHAPTER IV

EXISTING TABLES OF RATES OF DISABILITY AND RATES OF TERMINATION AMONG DISABLED LIVES

One of the earliest reliable disability tables was that prepared by Zimmerman on the employees of German railroads, a synopsis of which appears in J. I. A., XXIX, 306. As that table was not based on lives which were medically examined it was not applicable to insured lives, and as there was no published experience of any American or Canadian company, the experience of fraternal orders was first used as the basis of premiums for disability benefits by American and Canadian companies. The disability benefit in these fraternal orders consisted generally of payment of the sum insured in instalments in event of total and permanent disability, and the members were examined at entry in the same way as the insured in life insurance companies. It was, of course, recognized that fraternal orders would be in a position to exercise more rigid supervision of claims both before and after admission than a life insurance company, and that consequently premiums based upon their rates might prove inadequate for insurance companies. The insurance companies, however, as already stated, had no available experience of their own and consequently the actuarial profession was largely indebted to the fraternal orders which placed their data at its disposal.

Tables based on fraternal experience. Tables based on fraternal experiences were prepared by several actuaries, namely S. H. Pipe, F. B. Mead and Arthur Hunter. The first of these, which appeared in 1909 in a paper prepared by Mr. Pipe (T. A. S. A., XI, 172), consisted of the experience in the Province of Ontario of a large Canadian fraternal society. The experience of that society is given in T. A. S. A., XII, 334. The rates of disability were obtained by excluding the first four years of insurance, and are on an ultimate basis.

The rates of disability appearing in Mr. Mead's papers are based on the experience of an American fraternal order and first appeared in a book by Abb Landis entitled "Analysis of

Fraternal Societies" (page 150). Mr. Mead modified the rates at the higher ages. The experience is an aggregate one.

The rates given by Mr. Hunter (T. A. S. A., XII, 44) were based on the experience of the two orders already mentioned, with the data of a third fraternal society. An ultimate rate of disability (see page 112) was derived from the statistics, the experience of the first five policy years being excluded. Mr. Hunter also produced tables for the calculation of reserves on both active and disabled lives which were adopted by the New York and Massachusetts Insurance Departments. Hunter's tables came into general use by the companies, and proved satisfactory as long as the companies issued only a waiver of premium benefit on a total and permanent disability basis. Hunter's tables were also generally adequate for annual instalment benefits. The nature of the benefits, however, was rapidly liberalized, and in many policies total disability was presumed to be permanent after it had continued for a specified period—usually three months. In order to calculate premiums making allowance for the cost of the temporary benefits, recourse was usually had to the AHJ section of the Manchester Unity experience (1893-97), not because it was considered an entirely satisfactory basis, but because no really suitable table was available (the AHJ section covers agricultural and non-hazardous groups). E. E. Cammack (P. C. A. S., VII, 267) utilized this table for calculating premiums and reserves for non-cancellable accident and health policies. E. H. Hezlett (T. A. S. A., XXIV, 92) employed the same table to determine premiums and reserves for temporary and total disability benefits in life insurance policies.

Inter-Company investigation. In 1926 the first American experience of a broad character was published by a committee appointed by the Actuarial Society of America—twenty-three American and six Canadian companies submitting their material to the committee. The study has become known as the Inter-Company investigation. It revealed not only a great variety of clauses in use, but also considerable diversity of interpretation of the clauses by the companies in the treatment of claims. The committee finally prepared rates on three main types for the experience to the end of the year 1924, as follows:

Class (1)

Policies without the "90 day clause", providing for monthly income during disability.

Class (2)

Policies with the "90 day clause", providing for monthly income during disability.

Class (3)

Policies with the "90 day clause", providing for monthly income during disability.

The clauses in Classes (2) and (3) were practically identical, the only difference being in the companies' attitude towards the disability coverage. The experience was taken to the end of the year 1924, the companies reporting all claims received up to July 1925 where disability had occurred prior to January 1925. In this way the adjustment for unreported claims was materially reduced. The investigation was based on number of policies. Female lives were included. The exposed to risk was, in general, obtained from the companies' valuation sheets, adjustments being made to bring the material to a policy year basis. To further facilitate the work the data for obtaining the rate of disability were confined to the three principal plans of insurance—Ordinary Life (or Endowment at 85), Twenty Payment Life, and Twenty Year Endowment. The total number exposed to risk of disability and the total claims are shown in the following table:

	Exposures	Claims
Class (1)	5,451,221	5,857
Class (2)	598,310	1,293
Class (3)	1,166,396	4,345

The graduated rates of disability for Class (1) were first prepared using only the experience of the sixth policy year, and then, as a second assumption, the combined experience of the fourth, fifth and sixth policy years. In Classes (2) and (3) the experience of the first policy year was excluded and an aggregate table prepared. The following table shows the graduated

112 EXISTING TABLES RATES OF DISABILITY AND CLAIM TERMINATION

rates of disability for decennial ages, and also, for comparison, Hunter's values of p_x^{at} :

RATES OF DISABILITY PER 1,000

Age	Hunter's p_x^{at} (Ultimate)	Class (1) 6th Policy Year	Class (1) Aggregate 4th, 5th & 6th Policy Years	Class (2) Aggregate 2nd, 3rd & 4th Policy Years	Class (3) Aggregate 2nd, 3rd & 4th Policy Years
25	.53	1.69	1.47	2.55	4.12
35	.64	1.80	1.47	2.45	4.21
45	1.15	2.31	1.92	3.82	5.78
55	2.75	3.91	3.63	8.42	10.57

Mutual Life's experience. In 1929 the experience of the Mutual Life Insurance Company on male lives was published by W. M. Strong (T. A. S. A., XXX, 172). The investigation of the rate of disability was made by amount of insurance, but that of the rate of termination among disabled lives by the amount of annual benefit, i.e., income plus premium waived. The exposures on active lives were taken to the policy anniversary in 1926. The volume of the experience is shown in the following table:

MUTUAL LIFE STUDY

Exposures		Claims	
Number	Amount of Insurance	Number	Amount of Insurance
434,072	\$1,321,049,500	1,456	\$5,290,300

The disability rate used in calculating premiums and reserves was based on the total experience excluding the first policy year, the same process as in the Inter-Company Classes (2) and (3).

The crude rates of disability are shown below by number and amount for four age groups:

CRUDE RATES OF DISABILITY PER 1,000
AGGREGATE EXCLUDING 1ST POLICY YEAR

Ages at Issue	Number	Amount
15-24	3.12	3.58
25-34	2.82	3.50
35-44	4.17	5.00
45-55	7.52	8.52

New York Life's experience. In the same year the experience of the New York Life Insurance Company was presented to the

Society in two papers by Arthur Hunter and J. T. Phillips (T. A. S. A., XXX, 373 and 393). The study covered policies issued in the years 1921 to 1925, inclusive, with a 90 day non-retroactive benefit, and policies issued during 1926 to 1928 with a 90 day retroactive benefit. The experience in these two groups was analyzed by sex; also by number of policies and amounts of insurance. The study was carried to December 31st, 1928, all claims approved by the end of April 1929 being included, which made the adjustment for unreported claims very small. The exposures and claims are shown in the following table:

NEW YORK LIFE STUDY

	Exposures		Claims	
	Number	Amount	Number	Amount
Issues 1921-25:				
Males	2,631,032	\$7,765,265,400	11,368	\$38,181,138
Females	398,780	697,202,500	3,214	7,040,166
Issues 1926-28:				
Males	569,516	1,732,177,900	2,578	9,178,631
Females	138,917	227,858,200	1,051	2,175,392

The graduated rates of disability were prepared by amounts on an aggregate basis for all policy years combined, excluding the first policy year.

Two interesting features of the New York Life experience which were preserved, in part, in the graduated rates were:

(1) The male rate of disability decreased from age 15 to about age 23.

(2) In all four tables there was a rapidly increasing rate of disability commencing about age 20, followed by a less rapidly increasing rate for five or ten years. In the case of females for the issues of 1921-25 there was a drop in the rate of disability in the age group 35-39. This hump in the rate of disability curve occurred earlier in the 1926-28 experience than in the 1921-25. An analysis by cause according to age showed that tuberculosis and accident account to a certain extent for the high rate of disability at the younger ages. The New York Life rates are shown below for decennial ages and, for purposes of comparison, Hunter's disability rates and those of the Inter-Company Class (3) are given again:

RATES OF DISABILITY PER 1,000 (GRADUATED)

Age	Hunter's 2 ^d P ^y (Ultimate)	Inter- Company Class (3) (Aggregate)	New York Life—Aggregate			
			Issues 1921-25		Issues 1926-28	
			Males	Females	Males	Females
25	.53	4.12	3.74	7.81	4.66	10.55
35	.64	4.21	4.61	12.17	6.41	13.30
45	1.15	5.78	6.14	15.14	8.68	19.39
55	2.75	10.57	12.66	19.49	13.85	22.45

It should be noted in studying the above that the rates for Class (3) cover the experience for the second to the fourth years inclusive, the rates of the New York Life 1926-28 for the second and third years only, while those of the New York Life 1921-25 show the results for the second to the eighth policy years.

It is common knowledge that, since these tables were constructed, the disability claim rate of many companies has been higher than the above rates.

Effect of medical selection. The effect of medical selection upon the rate of disability wears off very rapidly under the policies containing the modern form of contract with a definite waiting period of short duration. The Inter-Company Report of 1926 quoted very low rates of disability for the first policy year, but in order to compare the effect of selection in the first year with that in subsequent years the first year rate of disability should be adjusted to the annual basis as explained on page 106.

Adjusting the first year rates accordingly, we have the following average rates of disability for all ages combined, by policy year:

CRUDE RATES OF DISABILITY PER THOUSAND BY POLICY YEAR
FIRST YEAR RATE ADJUSTED TO ANNUAL BASIS
ALL AGES AT ISSUE COMBINED—BY POLICIES

Policy Year	Class (3)	Mutual Life (Strong)	New York Life Male		New York Life Female	
			Issues 1921-25	Issues 1926-28	Issues 1921-25	Issues 1926-28
1	4.0	3.40	4.00	5.12	7.97	7.81
2	4.5	3.52	4.55	5.46	9.18	10.27
3	5.1	4.21	4.67	5.50	8.88	8.90
4	4.9	5.00	4.79		8.69	

It appears from the foregoing that the effect of medical selection is not great, and is of short duration.

Rates of Termination among Disabled Lives

In the fraternal experiences upon which the early tables of disability were based, benefits were paid only in the event of total and permanent disability, consequently the number of recoveries was comparatively small. It was therefore assumed in constructing these tables that there were no recoveries. The only cause of termination considered was death.

Looking at the death rate among disabled lives as determined by Mr. Hunter from the fraternal experiences we note two important features:

(1) The mortality for any given age at disability decreases rapidly with duration until after the tenth year of disability in most cases.

(2) In the first disability year, the mortality decreases with age, and at the younger ages continues to do so in several of the subsequent disability years. These characteristics will be seen from the following table:

HUNTER'S DEATH RATE PER 1,000 AMONG DISABLED LIVES

Age at Disability	Year of Disability									
	1	2	3	4	5	6	7	8	9	10 & over
15	523	269	203	152	120	97	80	63	49	37
25	400	183	122	76	58	49	40	34	28	24
35	327	145	97	64	48	43	39	35	31	27
45	290	140	112	83	59	52	47	45	43	44
55	248	147	120	100	83	78	76	75	76	77

The high mortality at the younger ages is due partly to diseases like tuberculosis, which are more prevalent among the young than the old. The decrease in mortality with advancing duration of disability is due to two distinct types of impairment. One type consists of those who are seriously sick from some disease causing a high mortality, and the other of those who have become blind or have both legs amputated, or have some other affliction, not seriously decreasing their chances of longevity. The former class would probably have a decreasing mortality, the strongest of them outliving their impairment, while

the latter would probably have a mortality slightly higher than the normal. The combined mortality of these two types, after several years following disablement, would not be greatly different from that among the population in general.

In the ordinary mortality table, when aggregate instead of select rates of mortality are used, annuities at the younger ages are increased on account of the diminution in the rate of mortality from the inclusion of select lives entering at older ages. The reverse is true with disabled lives since the termination rate for the first few years following disability is high, the use of aggregate instead of select rates resulting in annuity values at the younger ages which are low as compared with select values. To produce accurate values of annuities, therefore, select rates of mortality must be used. However, as the use of a select table of disabled life mortality involves so much labor, Mr. Hunter constructed an aggregate table excluding the experience of the first disability year. The mortality rates so obtained were as follows:

Age	Hunter's Aggregate Death Rate per 1,000 Among Disabled Lives
25	151
35	89
45	87
55	101

In the experiences with modern benefits, frequently granted for temporary disabilities, the number of recoveries is large, particularly in the early years of disability. The rate of termination as now calculated is based, therefore, on both recoveries and deaths. The relative values of the termination rate among disabled lives by year of disability, as determined by the various investigations referred to is seen from the following table; Hunter's mortality rate among disabled lives is given to facilitate comparison:

DISABILITY TERMINATION RATES PER 1,000

Age at Disability	Table	Disability Year										11 & Over	Attained Age
		1	2	3	4	5	6	7	8	9	10		
25	Hunter's Inter-Company Class (1)	400	183	122	76	58	49	40	34	28	24	24	35
	Class (2)	235	225	212	197	194	192	186	174	130	65	24	
	Class (3)	611	290	300	229	207							
	New York Life M & F, 1921-25	650	380	280	210	170	130	100	70	50	40	24	
35	Hunter's Inter-Company Class (1)	327	145	97	64	48	43	39	35	31	27	27	45
	Class (2)	265	237	206	174	143	117	104	87	66	41	27	
	Class (3)	544	409	288	206	159							
	New York Life M & F, 1921-25	586	241										
45	Hunter's Inter-Company Class (1)	610	360	250	190	150	120	100	80	60	40	27	
	Class (2)	290	140	112	83	59	52	47	45	43	44	49	55
	Class (3)	315	246	179	125	87	63	51	46	46	46	49	
	New York Life M & F, 1921-25	550	306	260	159	104							
55	Hunter's Inter-Company Class (1)	580	340	220	170	140	120	100	80	70	60	49	
	Class (2)	248	147	120	100	83	78	76	75	76	77	81	65
	Class (3)	336	230	151	104	81	71	67	71	74	77	81	
	New York Life M & F, 1921-25	540	306	258	157	114							
		550	300	180	140	110	100	90	80	80	80	81	

Hunter's rates and those for the three Inter-Company classes are based on number of policies.
 New York Life 1921-25 rates are based on amounts of insurance.

Both the Class (1) and the New York Life 1921-25 rates were graded to run into the eleventh year rates of Hunter's Analyzed Disabled Life Table (T. A. S. A., XII, 51) as neither experience had data for the longer durations of disability. In Class (1) the experience was available only for the first four years of disability, and in the New York Life 1921-25 for the first eight years by the addition of the data under its 1916-20 issues. The Class (2) rates are shown only for five years, and the Class (3) rates for only one or two years. In calculating Class (2) and Class (3) premiums, the Class (1) termination rates were used for later years.

The termination rate is very high in the early disability years under the Class (2), Class (3), and the New York Life 1921-25 experience. This is due largely to the high proportion of recoveries rather than to the reasons given above for the heavy mortality rate in the early years in Hunter's Disabled Life Table. The total terminations in the recent experiences are much greater than a comparison with Hunter's termination rates would indicate, on account of the higher rate of disability in the recent experiences. An indication of the proportion of recoveries may be seen from the following table:

CLASS (3) TERMINATIONS—ALL AGES

Disability Year	By Death	By Recovery	Total	Percentage of Total due to:	
				Death	Recovery
1	307	1,712	2,019	15	85
2	45	74	119	38	62
3	9	3	12	75	25

An interesting feature of the New York Life experience was the fact that, although the rates of disability for females were roughly double those for males, there was no significant difference in the termination rates. This feature has also been found in other experiences—see the discussion of Arthur Hunter's paper in T. A. S. A., XXXII, 483. The material for both sexes was therefore combined in determining the termination rates. Another feature of the New York Life rates, which are based on amount, is that the termination rates by amount were, in general, less than those by policies. As the rate of disability was found to be higher by amount than by policies, it is evident that there is selection against the company under the larger policies,

not only in the presentation of the claims, but also in the persistence of these claims when once approved.

At the time Hunter's tables were constructed, aggregate disabled life annuities, omitting the experience of the first disability year, were found to be satisfactory. These annuity values increase with increase in age until age 35 is reached.

As already pointed out the termination rate under contracts with a definite qualification period is much heavier in the earlier years of disability than that under policies providing for benefits in event of total and permanent disability. Accordingly satisfactory aggregate disabled life annuities are not so easily obtained. The calculation of disabled life annuities where the disabled life termination rate is high in the early years of disability is discussed in Chapter VII.

Disability Experience among Women

The rate of disability among women has been distinctly higher than among men, but there is considerable diversity of opinion with regard to the degree. The rate experienced by any company on women would partly be determined by its underwriting and administrative practice. J. S. Thompson (T. A. S. A., XXVII, 76) presented the experience of the Mutual Life Insurance Company under "Certain Phases of Disability Risks" in which he stated, "The disability rate among women in case of policies with either the monthly or yearly income benefits appears to be about 50% higher than among men so far as this experience goes."

In the discussion of Mr. Thompson's paper, H. R. Bassford stated that the Metropolitan Life Insurance Company had examined a portion of its experience on Ordinary policies from which it appeared that the female rate of disability was almost double the male rate. R. G. Hunter of the Equitable Life Insurance Company of Iowa stated that the rate of disability for women in that company had been more than twice the male rate but that claims had terminated about one-third more rapidly than male claims.

In the experience of the New York Life, previously referred to, there was available a considerable amount of data on women, the years of exposure by number of policies being 537,697 and the claims 4,265.

A comparison of the rates of disability with those for men follows:

COMPARISON OF GRADUATED FEMALE RATES OF DISABILITY
WITH CORRESPONDING MALE RATES

Age	Percentage of Male Rates	
	New York Life 1921-25 Issues	New York Life 1926-28 Issues
25	209%	226%
35	264	207
45	247	223
55	154	162

The rates of termination after disability, as stated above, were practically the same for both sexes.

In considering the rates given by the New York Life, it is well to remember the practice of the company in underwriting. Mr. Hunter states that disability benefits were granted at regular rates to single or married women, whether self-supporting or not, up to a maximum of \$2,500 of life insurance except where conditions warranted a larger amount. In life insurance it was found that, by accepting women on a broader basis, thus reducing the factor of self-selection, the aggregate mortality experience on women was improved. Such a treatment as that outlined by Mr. Hunter might therefore be expected to produce the most favorable female disability experience, since a large percentage of the women insured would take the disability coverage. The female disability rate, however, on this broadened basis was more than double the male rate at many ages.

CHAPTER V.

GRADUATION

After the crude rates of disability and of death or termination are obtained from the underlying data, they must be subjected to the process of graduation or smoothing, for the purpose of eliminating irregularities due to insufficiency of the data. The regular or graduated series should as closely as possible portray the facts—that is, a series should be produced under which the values will closely approximate the ungraduated values—and progress smoothly from age to age.

The graduation of disability data has usually been made by a summation process, by a mathematical formula, by the graphic method or by comparison with a standard table. Where the underlying law shows a progressive increase in the rate from year to year, such as in an adult mortality table, the first two methods mentioned are usually satisfactory. In the case of a disability experience, however, particularly under modern monthly income benefits, there is evidence to the effect that the rate of disability tends to decrease with increase in age at the younger ages and also to develop a hump in the neighborhood of ages 30 to 35. The summation and mathematical formula methods are not well adapted to such types of experiences. The summation method tends to flatten out a curve while the formula process, though it may contain a negative term which would allow for a decreasing rate at the beginning of the table, is not readily adapted to allow for a hump in the curve at a later point, especially when it may involve another temporary decrease in the rate of disability. By using a graphic method any fundamental characteristics of the experience may be retained, but the final form of the curve will depend largely on the individual judgment of the person making the graduation.

The methods employed in the more important of the past investigations will now be discussed.

S. H. Pipe (T. A. S. A., XI, 174) in graduating the rates of disability derived from his investigation of the Foresters' Society of Ontario found a graduation by the graphic method a simple operation up to age 63. The data did not extend beyond age 70

as disability benefits ceased at that age. He assumed that everyone would become permanently disabled at age 80, and obtained the values from 63 to 80 by interpolation. The rates of mortality among disabled lives were also graduated by the graphic method.

F. B. Mead (T. A. S. A., XI, 310), after attempting to graduate the rates of mortality among disabled lives by King's "pivotal value" formula (J. I. A., XLIII, 115, Va.) found the resulting curve very undulating. He therefore resorted to the graphic method in order to obtain the necessary smoothness.

Arthur Hunter (T. A. S. A., XII, 46) prepared an ultimate rate of disability for the combined sixth and succeeding policy years. The graduation was effected by means of Finlaison's formula, i.e., a summation in fives repeated three times. No attempt was made to obtain rates of disability beyond age 65 as the experience after that age was too limited.

Robert Henderson made the various graduations in the Inter-Company investigation of 1926. In that investigation there were two sets of rates of disability for Class (1). Each of them was derived by a modification of Hunter's p_x^{at} ; in one case the rates based on the sixth year experience were obtained by adding a constant at every age to Hunter's p_x^{at} and in the other case, in which the rates were based on the aggregate experience of the fourth, fifth, and sixth years, a constant was added to a multiple of Hunter's p_x^{at} and a percentage of the age deducted. The disability rates for Classes (2) and (3) were graded by applying Makeham's Second Law to the rate of disability, instead of the force of mortality, namely, $r_x = A - Hx + Bc^x$ where $\log_{10} c = .034$, approximately the same value as in the American Men Table of Mortality. The negative term was employed because the rate of disability tended to reach a minimum at about age 30, a characteristic of many sickness experiences. The following summary shows the formulas adopted in graduating the various disability rates of the Inter-Company investigation of 1926:

Class (1), (6th year); $r_x = p_x^{at}(\text{Hunter's}) + .001162$

Class (1), (4th, 5th and 6th years); $r_x = 1.150 p_x^{at}(\text{Hunter's}) + .0012032 - .0000135 x$

Class (2); $r_x = .0048445 - .0001337 x + 10^{\overline{1.1685} + .034 x}$

Class (3); $r_x = .0059376 - .0001145 x + 10^{\overline{1.1685} + .034 x}$

In determining the rates of termination among disabled lives, a continuance table was first constructed from the crude monthly termination experience which showed for all ages combined the proportion remaining alive and disabled at the end of each month. From the probabilities of termination thereby derived, it appeared safe to group the data for the first three months of duration, for the second quarter, and for each half year thereafter, with certain adjustments in the period of exposure of the existing. This method of grouping was applied to the data for the different groups of ages at disability.

In the Class (1) experience, data were available for four years. The assumption was made that the subsequent rates of termination would ultimately merge with those for the attained age in the eleventh year of Hunter's Analyzed Disabled Life Table (T. A. S. A., XII, 51). The Class (1) termination rates were therefore graded smoothly into this ultimate rate. This result was accomplished by assuming that the claims might be divided into two parts, one part subject to a rate of termination equal to the rate of mortality in the eleventh year of Hunter's Table and the other part subject to a relatively high constant rate of termination, so that the survivors would in a few years become negligible compared with those in the first part.

A preliminary calculation was first made of the proportion in each age group that should be assumed to follow Hunter's Ultimate Disabled Life Table in order that the rate of termination of the balance should be reasonably uniform over the four disability years of experience. The resulting proportions for the four groups of ages at disability, 15-24, 25-34, 35-44 and 45 and over, were .146, .170, .343 and .393, respectively. For this irregular series was substituted one varying uniformly with the age, the values corresponding to the mean ages of the respective groups of which were .1263, .1984, .3014 and .4147. The formula for the proportion at age x was $.0103x - .1003$. On the basis of this division the mean termination rate in the supplementary series for each age group was next determined. These rates were assumed to be those for the mean age of the group and the rates for quinquennial ages from 20 to 60, inclusive, were determined from them by the formula

$$\text{colog}_{10} p'_{[x]} = A + H(x - 20) + Bc^{x-20}.$$

The constants were $A = .02691$; $H = .010208$; $B = .18322$ and $\log_{10} c = 1.90352$. On this basis annuity values were calculated for quinquennial ages and the intermediate values interpolated.

In Class (2) only two years' termination rates could be obtained from the data, as the statistics for the third claim year were negligible. The same principle as in Class (1) was applied to the graduation of the termination rates of these two years, except that owing to the irregularity of the data it was assumed that $\log_{10} c = 1.9$, which is approximately the value in Class (1). Having determined in this way first and second year termination rates for quinquennial ages at disability, these rates were extended by assuming a supplementary geometrical series applied to the survivors calculated for Class (1). The proportions of the two series and the termination rates in the supplementary series were determined for each age so as to reproduce the first and second year rates of termination.

In Class (3) the data for the first two claim years were used in deriving termination rates, and the same method was applied to the adjustment of the termination rates of these two years as was used for Class (2). The second year termination rates were, however, so close to those of Class (1) that it was assumed that those of the third and subsequent years were the same as for that class. The experience in the third year was too small to warrant its use.

W. M. Strong (T. A. S. A., XXX, 172) in the treatment of the data of the Mutual Life also adopted Makeham's Second Law, $A - Hx + Bc^x$, to represent the form of r_x . For the rates of termination, he likewise followed the Inter-Company Committee in assuming that the ultimate rates of termination after ten years would be the ultimate values of Hunter's Analyzed Disabled Life Table. Graphic methods of graduation were employed both to graduate the rates derived from the available data and to connect such rates with Hunter's ultimate rates of mortality.

The experience of the New York Life Insurance Company under the 90 day clause for issues of 1921-1925 and 1926-1928 was presented by Arthur Hunter and J. T. Phillips (T. A. S. A., XXX, 373 and 393). Aggregate rates were prepared for all

policy years combined, excluding the first. These were graduated graphically in order to preserve certain features of the curves. Four tables were prepared, male and female for each group of issues. In each of these tables the same peculiarity in incidence was noticed and was preserved in the final graduation to a modified degree. The peculiarity, which has already been mentioned in Chapter IV but will be described again for convenience, was a rapidly increasing rate of disability from about age 20, followed by a less rapidly increasing rate for five or ten years. In the female 1921-1925 group there were indications of an actual decrease in the rate in the neighborhood of ages 35-39. This peculiarity in incidence may have been present in some other investigations but the use of mathematical formulas in graduating them may have precluded such peculiarities from being shown. There was also evidence at the younger ages of a decreasing rate of disability with advance in age.

The termination experience was almost entirely confined to the 1921-1925 issues, as there were very few terminations beyond the sixth month of disability under the later issues.

The graduation of the termination rates fell into two major divisions, (1) the graduation of annual termination rates for all years, and (2) the graduation of the monthly rates for the first two years of disability.

The annual termination rates were first graduated for each year of disability separately by means of Henderson's formula (T. A. S. A., XXV, 29), using the 4th difference formula with $n = 1$. The values so obtained were then plotted and a curve drawn for each age group so as to run into Hunter's ultimate rate after ten years. These curves in almost every case passed through the points plotted for the first four years. Values for individual ages and durations were interpolated by inspection. The monthly rates for the first two years were then graded for all ages combined by means of Henderson's formula and the curve so obtained was assumed to represent the incidence of the rates for the individual age groups, but modified in order to produce the graded annual termination rates.

CHAPTER VI.

CONSTRUCTION OF COMBINED MORTALITY AND
DISABILITY TABLE

As has been previously remarked, it is not always practicable nor advisable, when investigating the experience of a life insurance company, to determine r_x directly. The functions obtained from such an investigation may be

- (a) r_x , the probability of an active life aged " x " becoming disabled within a year, or
- (b) r'_x , the absolute annual rate of disability among active lives aged " x " at the beginning of the year, or
- (c) p_x^d , the probability of an active life aged " x " becoming disabled and surviving to the end of the year.

Whichever of these probabilities is available, it must be combined with the probability of death among disabled lives and with the probabilities from the total mortality table, assuming the Hamza process to be used. Having determined the rate of mortality among the mixed and the disabled lives (it is assumed there are no recoveries, see page 90), the actual process of constructing the combined mortality and disability table depends upon which rate of disability is available.

(a) *When r_x is given.* The values of l_x and d_x are taken from the total mortality table, usually the American Experience Table of Mortality or the American Men Table, and the values r_x and q_x^d obtained by investigation. From the relations existing among these functions, the other functions may be determined. Thus, from the following equation, i_x may be obtained:

$$(12) \quad i_x = r_x \cdot l_x^{aa}$$

The number of disabled lives living at age x comprises the survivors from the number of disabled lives living at age $x-1$ increased by the number of survivors from those becoming disabled between ages $x-1$ and x . On the assumption of an even distribution of disability and of mortality throughout the year, we have the relation

$$(13) \quad d_x'' = q_x^d (l_x'' + \frac{1}{2} i_x)$$

$$(14) \quad l_{x+1}'' = l_x'' + i_x - d_x''$$

As the sum of the number of active lives and the number of disabled lives is the number of lives according to the total mortality table, and as the sum of the number of deaths among active lives and the number of deaths among disabled lives is the number of deaths according to the total mortality table, the following relations obtain,

$$(15) \quad l_x^{aa} = l_x - l_x''$$

$$(16) \quad d_x^{aa} = d_x - d_x''$$

It will be observed that decrements to the l_x^{aa} column are of two kinds, namely deaths among the active lives and lives becoming disabled. Translated into symbols

$$(17) \quad l_x^{aa} - d_x^{aa} - i_x = l_{x+1}^{aa}$$

At the initial age of the table, usually age 15, $l_x^{aa} = l_x$ and $l_x'' = 0$. By means of equations (12) and (13), i_x and d_x'' may be obtained for the initial age; and by means of equations (14) and (15), l_x'' and l_x^{aa} for the next higher age are obtained. The process is repeated until the entire table is completed. The values of d_x^{aa} may then be obtained by means of equation (16).

The only way the student will become thoroughly familiar with the method of constructing a table is to work out the various values for a few ages.

(b) *When r'_x is given.* As in the previous case the values l_x and d_x are taken from the total mortality table and the values r'_x and q_x' derived from the investigation. Making the assumption that deaths among active lives are evenly distributed throughout the year,

$$(18) \quad i_x = r'_x (l_x^{aa} - \frac{1}{2} d_x^{aa}).$$

In this equation there are two unknown decrements to the active table, i_x and d_x^{aa} . It is necessary to obtain an equation involving only one of these unknowns.

To eliminate either i_x or d_x^{aa} from equation (18) we have the relation, on the assumption that deaths among disabled lives are evenly distributed throughout the year,

$$(19) \quad (l_x'' + \frac{1}{2} i_x) q_x' = d_x'' = d_x - d_x^{aa}$$

since $d_x = d_x^{aa} + d_x''$

From (19) we have either,

$$(20) \quad i_x = \frac{d_x - d_x^{aa} - q_x' \cdot l_x''}{\frac{1}{2} q_x'}$$

or

$$(21) \quad d_x^{aa} = d_x - (l_x'' + \frac{1}{2} i_x) q_x'$$

Substituting in (18) the value of i_x from equation (20) and simplifying we have

$$(22) \quad d_x^{aa} = \frac{d_x - q_x' \cdot l_x'' - \frac{1}{2} r_x' \cdot q_x' \cdot l_x^{aa}}{1 - \frac{1}{4} r_x' \cdot q_x'}$$

Substituting in (18) the value of d_x^{aa} from equation (21) and simplifying we have

$$(23) \quad i_x = r_x' \frac{l_x^{aa} + \frac{1}{2} q_x' \cdot l_x'' - \frac{1}{2} d_x}{1 - \frac{1}{4} r_x' \cdot q_x'}$$

At the commencement of the table, r_x' and q_x' being known, d_x^{aa} may be obtained by means of formula (22) as l_x'' is zero and l_x^{aa} equal to l_x . Substituting the value of d_x^{aa} so found in equation (18) i_x is obtained. We can get d_x'' from the relation $d_x'' = d_x - d_x^{aa}$. If we use formula (23), instead of formula (22), we obtain i_x first then obtain d_x^{aa} from equation (18) and hence d_x'' .

(c) *When p_x^{at} is given.* At the initial age of the table $l_x^{aa} = l_x$ and $l_x'' = 0$. As p_x^{at} is the probability of a life aged x becoming disabled and surviving to the end of the year, at the initial age, say 15, $p_{15}^{at} \cdot l_{15}^{aa} = l_{16}''$, then as $l_{16}^{aa} = l_{16} - l_{16}''$, the value of l_{16}^{aa} for the next higher age may be obtained. To obtain l_{17}'' we must consider not only the survivors of those becoming disabled in the year beginning at age 16, but also the survivors of l_{16}'' . In other words

$$(24) \quad l_{17}'' = p_{16}^{at} \cdot l_{16}^{aa} + (1 - q_{16}') l_{16}''$$

In general

$$(25) \quad l_x'' = p_{x-1}^{at} \cdot l_{x-1}^{aa} + (1 - q_{x-1}') l_{x-1}''$$

and

$$(26) \quad l_x^{aa} = l_x - l_x''$$

The process is repeated until the table is completed.

When the function p_x^{at} is used in the construction of a combined mortality and disability table, it is usual to tabulate under the heading "Number Becoming Disabled in Course of the Year" only those who become disabled during the year and are alive at the end of the year. This number for any age x is the product of p_x^{at} and l_x^{aa} .

If a column of "Deaths among Active Lives" is shown, it would comprise for each age all deaths during the year among lives active at the beginning of the year. A life active at the beginning of the year becoming disabled and subsequently dying within the year would be included among the deaths among lives active at the beginning of the year. We have used the symbol d_x^{aa} for the number dying while active between ages x and $x+1$. The symbol d_x^a may properly be used for the deaths between ages x and $x+1$ among lives active at age x . The column d_x^a may be completed from the relation

$$d_x^a = d_x - l_x^{ii} \cdot q_x^i$$

The construction of the table when p_x^{ai} is given does not directly produce d_x^{aa} , d_x^{ii} or i_x . These values, however, may be approximated as shown below.

Students who refer to Hunter's table (T. A. S. A., XII, 66) should notice that the values shown in the column headed "Probability of Dying in Active State" are equal at any age to $\frac{d_x^a}{l_x^{aa}}$. They are really values of q_x^a where q_x^a is the probability of a life active at age x dying within the year. Since Hunter's table was published, the Actuarial Society has adopted an official disability notation, and to avoid confusion the student should make the following changes in the headings of Hunter's table:

$$\begin{aligned} r_x &\text{ should be changed to } p_x^{ai} \\ q_x^{aa} &\text{ should be changed to } q_x^a \\ l_x^{aa} \cdot r_x &\text{ should be changed to } l_x^{aa} \cdot p_x^{ai} \end{aligned}$$

Hunter's table is published in a convenient form for most purposes. The table shows values of l_x^{aa} , p_x^{ai} and q_x^a and if it is desired to obtain a table giving values of i_x , d_x^{ii} , r_x and d_x^{aa} it may readily be done by the use of the following approximate formulas. On the assumption that disability occurs, on the average, in the middle of the year, we have

$$(27) \quad l_x^{aa} \cdot p_x^{ai} = i_x (1 - \frac{1}{2} q_x^i)$$

$$(28) \quad i_x = \frac{l_x^{aa} \cdot p_x^{ai}}{1 - \frac{1}{2} q_x^i}$$

$$(29) \quad d_x^{ii} = (l_x^{ii} + \frac{1}{2} i_x) q_x^i$$

$$(30) \quad r_x = \frac{i_x}{l_x^{aa}} = \frac{p_x^{af}}{1 - \frac{1}{2} q_x^i}$$

and from (16)

$$d_x^{aa} = d_x - d_x^{af}$$

In an investigation where only p_x^{af} is obtainable, the statement in equation (27) is only an approximation as q_x^i is determined from claims which have survived the policy year in which the claim occurred. On account of the high mortality rate among disability claims in the period immediately following the occurrence of the claim, the q_x^i of the investigation understates the value which should be used in equation (27). The effect would be to understate the value of i_x and therefore of d_x^{af} .

At one time there was considerable discussion as to whether there should be a separate table for each age at entry, so that the probability of survivorship of an active entrant should be the same as the probability of survivorship according to the total mortality table; or whether there should be only one table beginning with a single initial age, so that for the succeeding ages the probability of survivorship of the active and disabled lives combined should be in accordance with that of the total mortality table. In the case of a single table the probability of survivorship of an active life is not the same as that in accordance with the total mortality table, but is slightly greater on the assumption that the probability of survivorship according to the total table is the probability of survivorship of mixed lives, some of whom are disabled (see page 91). Arthur Hunter stated (T. A. S. A., XII, 58) that he had calculated premiums and reserves for the waiver of premium benefit first on the basis of a separate table for each age at entry and then on the basis of a table beginning with a single initial age and that there was no appreciable difference in the results.

It will be recalled that, in the construction of a table of mortality and disability, there are three methods of determining the rate of mortality among active lives (see page 90). The first method described was where an investigation of the mortality among active lives had been made and the rates of mortality among active lives directly obtained therefrom. An advantage of this method is that there is definitely established at each age the rate of mortality among active lives. If, on this basis, a single table of mortality and disability commencing, for example,

at age 15 is subsequently constructed, together with separate tables for each age at entry, it is important to notice that the net premiums and reserves are identical, whether the single table or the separate tables are employed.

Approximate Methods Used in Recent Investigations

It has already been shown, in several recent investigations, notably the Inter-Company investigation of 1926, that the exposed to risk of disability has been obtained from the valuation data producing approximately the absolute rate of disability r'_x . This procedure was but a temporary expedient as most of the companies had not prepared disability statistical cards. This situation has now been corrected in many companies and undoubtedly future investigations will be based upon correct exposures which will directly produce r_x or p_x^{ai} . Where r'_x was obtained from the past investigations, r_x was determined from the relation $r_x = (1 - \frac{1}{2} q_x^{aa}) r'_x$. No values of q_x^{aa} , however, were determined, the assumption being made that the mixed life functions could be used in place of the active life functions without any great loss in accuracy. It should be noted here that the pension fund method of calculation was used in these investigations, so that the "ii" functions, l_x^{ii} , d_x^{ii} , etc., were not needed. The effect of the foregoing assumption on premiums and the correction for it in the premium formulas will be discussed later.

CHAPTER VII.

COMMUTATION COLUMNS FOR THE HAMZA
METHOD OF COMPUTATION—ANNUITIES

It is generally stipulated in life insurance contracts that benefits dependent upon total and permanent disability will not be granted should disability occur after a fixed age, say 55, 60, or 65. In such cases it is customary, in using any of the formulas given below, to eliminate the values of r_x , r'_x or p_x^{at} for ages 55, 60, or 65 and for higher ages.

In what follows it will be assumed that the disability benefit becomes payable only in the event of disability occurring before age y , and the formulas will be based on a combined table of mortality and disability, commencing at (say) age 15, in which the probability or rate of disability is taken as zero at y and all subsequent ages.

The commutation columns frequently used in the calculation of premiums and reserves for disability benefits are those based upon l_x , l_x^{aa} and l_x^{ii} , respectively.

The student should have clearly in mind the distinction between l_x^{ii} and l_x^i . The number of disabled persons living at age x according to a combined mortality and disability table is represented by l_x^{ii} , whereas the number of persons living at age x out of a certain number of disabled lives selected as a radix at the initial age of a table of disabled life mortality is represented by l_x^i . Commutation columns are formed in the usual way, as follows:

$$\begin{array}{ll}
 D_x^{aa} = v^x \cdot l_x^{aa} & C_x^{aa} = v^{x+1} \cdot d_x^{aa} \\
 N_x^{aa} = \sum_{n=0}^{n=\omega-x} D_{x+n}^{aa} & M_x^{aa} = \sum_{n=0}^{n=\omega-x} C_{x+n}^{aa} \\
 S_x^{aa} = \sum_{n=0}^{n=\omega-x} N_{x+n}^{aa} & R_x^{aa} = \sum_{n=0}^{n=\omega-x} M_{x+n}^{aa} \\
 D_x^{ii} = v^x \cdot l_x^{ii} & C_x^{ii} = v^{x+1} \cdot d_x^{ii} \\
 N_x^{ii} = \sum_{n=0}^{n=\omega-x} D_{x+n}^{ii} & M_x^{ii} = \sum_{n=0}^{n=\omega-x} C_{x+n}^{ii} \\
 S_x^{ii} = \sum_{n=0}^{n=\omega-x} N_{x+n}^{ii} & R_x^{ii} = \sum_{n=0}^{n=\omega-x} M_{x+n}^{ii} \\
 D_x^i = v^x \cdot l_x^i & C_x^i = v^{x+1} \cdot d_x^i \\
 N_x^i = \sum_{n=0}^{n=\omega-x} D_{x+n}^i & M_x^i = \sum_{n=0}^{n=\omega-x} C_{x+n}^i \\
 S_x^i = \sum_{n=0}^{n=\omega-x} N_{x+n}^i & R_x^i = \sum_{n=0}^{n=\omega-x} M_{x+n}^i
 \end{array}$$

Annuities

The notation and the formulas for annuities will now be given.

a_x^{aa} = the value of an annuity-due on an active life now aged x payable during activity.

$a_{x:\overline{n}|:\overline{y-x}|}^{aa}$ = the value of a temporary annuity-due on an active life now aged x payable for n years or for $y - x$ years, whichever is the shorter, and during activity.

a_x^d = the value of an annuity-due on a disabled life now aged x payable until death or prior recovery.

a_x^a = the value of a life annuity-due on an active life now aged x payable during survival, active or invalid.

a_x^{af} = the value of an annuity payable at the end of each year provided an active life now aged x is then alive but disabled.

$a_{x:\overline{n-1}|}^{af}$ = the value of a temporary annuity payable at the end of each year for the next $n - 1$ years provided an active life now aged x is then alive but disabled; $n - 1$ is used because this is the term which would be applicable in the waiver of an annual premium in an n -payment life or an n -year endowment.

There are two elements which are limited by the term period, (1) the period during which the disability may occur and (2) the period during which the disability annuity is payable. The last of the above annuities is sometimes viewed as having a disability coverage period of n years (under, say, an n -payment life) even though the disability annuity period is for only $n - 1$ years and is written

$$a_{x:\overline{n-1}|}^{a \text{ (} \overline{t:\overline{n}} \text{)}}.$$

The annuity, however, is usually written as $a_{x:\overline{n-1}|}^{af}$ where the periods of the coverage and annuity payments are coterminous. The other notation is useful in cases where they are not. For example, an n -year endowment policy may provide for a life disability annuity in event of disability occurring within the endowment period. The benefit would therefore be represented by $a_{x:\overline{t:\overline{n}}|}^a$.

Similarly for the annuity represented by a_x^{af} , disability is usually presumed to occur before a fixed age, say y . The disability tables are constructed accordingly. Therefore, although

such an annuity should more correctly be represented by $a_x^{a(t:\overline{v-x})}$, it is customary to merely write a_x^t , the limitation in the coverage being understood.

${}_n|a_{x:\overline{n}}^{a(t:\overline{n})}$ = the value of a deferred temporary annuity payable at the beginning of each of t years, first payment deferred n years, provided an active life now aged x is then alive but disabled and that the disability occurred during the n years.

$$(31) \quad a_x^{aa} = \frac{N_x^{aa}}{D_x^{aa}} \quad (32) \quad a_{x:\overline{n}}^{aa} = \frac{N_x^{aa} - N_{x+n}^{aa}}{D_x^{aa}}$$

$$(33) \quad a_x^t = \frac{N_x^t}{D_x^t} \quad (34) \quad a_{x:\overline{n}}^t = \frac{N_x^t - N_{x+n}^t}{D_x^t}$$

Of the l_x^{tt} disabled persons living at age x according to a combined mortality and disability table there will be ${}_np_x^t \cdot l_x^{tt}$ alive n years hence; therefore, of the l_x^{aa} active persons living at age x there will be $l_{x+n} - {}_np_x^t \cdot l_x^{tt}$ persons alive n years hence, some of whom may be disabled, the others active. Where ${}_np_x^a$ represents the probability that an active life age x will survive n years,

$$(35) \quad {}_np_x^a = \frac{l_{x+n} - {}_np_x^t \cdot l_x^{tt}}{l_x^{aa}}$$

$$(36) \quad {}_np_x^a = \frac{l_x}{l_x^{aa}} \cdot {}_np_x - \frac{l_x^{tt}}{l_x^{aa}} \cdot {}_np_x^t$$

Hence

$$\begin{aligned} \Sigma v^n \cdot {}_np_x^a &= \frac{l_x}{l_x^{aa}} \Sigma v^n \cdot {}_np_x - \frac{l_x^{tt}}{l_x^{aa}} \Sigma v^n \cdot {}_np_x^t \\ (37) \quad a_x^a &= \frac{l_x}{l_x^{aa}} \cdot a_x - \frac{l_x^{tt}}{l_x^{aa}} \cdot a_x^t \end{aligned}$$

and

$$(38) \quad a_{x:\overline{n}}^a = \frac{l_x}{l_x^{aa}} \cdot a_{x:\overline{n}} - \frac{l_x^{tt}}{l_x^{aa}} \cdot a_{x:\overline{n}}^t$$

By definition

$$(39) \quad a_x^{at} = a_x^a - a_x^{aa}$$

and

$$(40) \quad a_{x:\overline{n-1}}^{at} = a_{x:\overline{n-1}}^a - a_{x:\overline{n-1}}^{aa}$$

hence

$$(41) \quad a_x^{at} = \frac{l_x}{l_x^{aa}} \cdot a_x - \frac{l_x^{tt}}{l_x^{aa}} \cdot a_x^t - a_x^{aa}$$

$$(42) \quad a_x^{at} = a_x - a_x^{aa} + \frac{l_x^{tt}}{l_x^{aa}} (a_x - a_x^t)$$

and

$$(43) \quad a_{x:\overline{n-1}|}^{at} = a_{x:\overline{n-1}|} - a_{x:\overline{n-1}|}^{aa} + \frac{l_x^{tt}}{l_x^{aa}} (a_{x:\overline{n-1}|} - a_{x:\overline{n-1}|}^t)$$

The value of a_x^{at} as expressed by equation (41) may be deduced directly by first principles. If l_x lives have an annuity of 1 per annum ceasing at death, they will receive in the aggregate each year the same amount as l_x^{aa} active lives and l_x^{tt} disabled lives receiving similar annuities. Assuming no recoveries among disabled lives, this would be in symbols

$$l_x \cdot a_x = l_x^{aa} \cdot a_x^a + l_x^{tt} \cdot a_x^t$$

hence

$$l_x^{aa} \cdot a_x^a = l_x \cdot a_x - l_x^{tt} \cdot a_x^t$$

but as

$$a_x^a = a_x^{aa} + a_x^{at}$$

$$l_x^{aa} (a_x^{aa} + a_x^{at}) = l_x \cdot a_x - l_x^{tt} \cdot a_x^t$$

i.e.,

$$l_x^{aa} \cdot a_x^{at} = l_x \cdot a_x - l_x^{aa} \cdot a_x^{aa} - l_x^{tt} \cdot a_x^t$$

or

$$a_x^{at} = \frac{l_x}{l_x^{aa}} \cdot a_x - \frac{l_x^{tt}}{l_x^{aa}} \cdot a_x^t - a_x^{aa}$$

Convenient formulas may be deduced from formula (41) as follows:

$$a_x^{at} = \frac{l_x}{l_x^{aa}} \cdot a_x - \frac{l_x^{tt}}{l_x^{aa}} \cdot a_x^t - a_x^{aa}$$

$$\begin{aligned} D_x^{aa} \cdot a_x^{at} &= D_x \cdot a_x - D_x^{tt} \cdot a_x^t - D_x^{aa} \cdot a_x^{aa} \\ &= N_x - D_x^{tt} \cdot a_x^t - N_x^{aa} \end{aligned}$$

Hence

$$(44) \quad a_x^{at} = \frac{N_x^{tt} - D_x^{tt} \cdot a_x^t}{D_x^{aa}}$$

and

$$(45) \quad a_{x:\overline{n-1}|}^{at} = \frac{N_x^{tt} - N_{x+n}^{tt} - D_x^{tt} \cdot a_{x:\overline{n-1}|}^t}{D_x^{aa}}$$

Calculation of Disabled Life Annuity Values

As previously indicated, a_z^t represents the value of an annuity-due on a disabled life, payable until death or prior recovery. If we are dealing with aggregate disabled life data such as that used by Mr. Hunter, a_z^t is equal to

$$\frac{\sum_{n=0}^{n=\omega-x} v^n l_{x+n}^t}{l_x^t}$$

and the monthly annuity $a_z^{t(12)}$, for evaluating monthly income benefits, can be satisfactorily obtained from the relation

$$a_z^{t(12)} = a_z^t - \frac{11}{24}$$

When our data are obtained from an experience under a clause with a fixed waiting period, such as 90 days, the termination rate among disabled lives is high during the early years of disability. The disabled life termination rates given on page 117 for Class (3), for example, illustrate that the values vary much more by duration of the claim than by age for a given duration. This condition is exactly opposite to that assumed in an aggregate table, viz., that the values will be dependent only on the age, irrespective of the duration. As a result, select disabled life annuities are really imperative when computing premiums or reserves for benefits under clauses containing a waiting period to determine a presumption of permanent disability. In the Class (1) experience, based on a clause without a fixed qualification period, the variation in values by duration of the claim was not so great as that in Class (3). On this account we find approximate aggregate annuities satisfactorily obtained for Class (1) by J. D. Craig (T. A. S. A., XXVII, 47). The method followed by Mr. Craig was to obtain an hypothetical p_z^t which would reproduce the select disabled life annuities $a_{[x]+1}^t$, the formula for p_z^t being

$$p_z^t = \frac{(1+i) a_{[x-1]+1}^t}{a_{[x]+1}^t}$$

By means of the values of the hypothetical aggregate p_z^t , an aggregate table was constructed which would produce life annuity values equal to the select life annuity values. Select temporary annuity values were not of course reproduced by the hypothetical table, but the resulting aggregate values, though excessive, were not too high to affect materially the premiums. A similar method, to reproduce $a_{[x]}^t$, was tried by J. T. Phillips for the New York Life experience (T. A. S. A., XXX, 413) but

was not very satisfactory, as the variation of the disabled life termination rate by duration was considerably greater than in the Class (1) experience. Detailed illustrations of the effect of the foregoing hypothetical aggregate tables on the values of annuities, premiums and reserves may be seen by turning to the two references just mentioned. The student is also referred to F. F. Davis' discussion of Mr. Phillips' paper.

The great variation in the termination rate in the early years of disability not only makes select annuities necessary, but it also makes it advisable to use some other adjustment than that already stated in order to obtain monthly disabled life annuities. The practice has been to calculate the annuity directly from the monthly termination rates for the first year and to make the ordinary monthly adjustment on the deferred portion of the annuity. Expressed in symbols the relation is

$$a_{[x]}^{(12)} = \frac{1}{12} \sum_{n=0}^{n-11} v^{\frac{n}{12}} \cdot \frac{n}{12} p'_{[x]} + a'_{[x]} - \frac{11}{24} v p'_{[x]}$$

Actual monthly rates could be taken, of course, for a longer period than one year, but tests have shown that the ordinary adjustment for monthly payments is sufficiently satisfactory for that portion of the disabled life annuity applicable to the second and subsequent years.

On account of the relatively constant value of the first year termination rates under an experience such as that for a 90 day waiting period, the adjustment for monthly payments in respect to the first year of disability does not vary greatly by age. Because of this feature W. M. Strong used an adjustment constant for all ages in calculating the disabled life annuities under the Mutual Life experience (T. A. S. A., XXX, 180). Mr. Phillips, for the New York Life experience, used values for the expression

$$\frac{1}{12} \sum_{n=0}^{n-11} v^{\frac{n}{12}} \cdot \frac{n}{12} p'_{[x]} - \frac{11}{24} v p'_{[x]}$$

which varied from .440 at age 18 to .462 at age 47.

In the ordinary adjustment for monthly payments we have

$$a_x^{(12)} = a_x - \frac{11}{24} \\ \text{or} = a_x + \frac{13}{24}$$

The fraction $\frac{13}{24} = .542$, which compares with the above adjustments of from .440 to .462.

CHAPTER VIII.

CALCULATION OF PREMIUMS BY THE HAMZA METHOD

Waiver of Premium Benefit

An example of this benefit appears on page 207.

As has been previously stated, when a premium is waived the amount of the waived premium is not charged against the policy, but the policy and all guarantees in it are the same as though premiums had been paid each year. This benefit is usually limited to the occurrence of total and permanent disability before the insured attains the age of 65 or 60, or even an earlier age. These statements do not apply to the modified form of waiver benefit granted occasionally in the past under which the insurance is reduced by the amount of premiums waived for disabilities occurring after a certain age.

The premium, which has been waived, has sometimes been the net insurance premium and sometimes the gross premium. When the latter is waived the dividends payable to the insured while disabled should be the same as though he were active and paying premiums. The Insurance Commissioners' Standard Disability provisions require in effect the waiver of the gross premium, as they stipulate that the dividends shall be the same whether the insured is active or disabled. The student will find the question of dividends to disabled policyholders discussed in Part I, Chapter VIII. The regular commissions are usually paid on waived premiums if the gross premium is assumed to be waived. If no commissions or reduced commissions are paid but full dividends allowed, the premium waived in the calculations would be the gross premium less the savings in commissions. As premium taxes would usually not be payable on premiums waived the gross premium waived could also be reduced because of that saving in expense. The usual practice, however, is to omit any correction on that account.

The formulas that follow apply to the benefit of waiving either the gross, a modified gross, or the net premium, P , in the event

of disability occurring before age y , where premiums for the benefit cease at age y . In practice the combined mortality and disability table is adjusted so that beyond y the rate of disability is zero, and therefore N_y^{aa} in the following formulas will be equal to zero.

a_x^{at} calculated on the basis of a combined mortality and disability table, adjusted in the manner described in the preceding paragraph, therefore represents the present value of one per annum payable during the disability of a life aged x , now active, provided disability occurs before age y .

Ordinary Life policy. At the inception of the policy the value of the waiver of premium benefit is

$$(46) \quad P \cdot a_x^{at}$$

and remembering that premiums for the waiver of premium benefit cease at age y , the annual premium for the benefit is

$$(47) \quad \frac{P \cdot a_x^{at}}{a_{x:\overline{y-x}}^{aa}} \text{ or } P \cdot a_x^{at} \cdot \frac{D_x^{aa}}{N_x^{aa} - N_y^{aa}}$$

Using formula (44) the net annual premium for the waiver of premium benefit is

$$(48) \quad \frac{P(N_x^{at} - D_x^{at} \cdot a_x^t)}{N_x^{aa} - N_y^{aa}}$$

Limited payment life and endowment policies. The value of the waiver of premium benefit is

$$(49) \quad P \cdot a_{x:\overline{n-1}}^{at},$$

where n is the number of payments under a limited payment life policy, or the term of the endowment under an n -payment n -year endowment. Where $x + n = \text{or} < y$ the premium for the waiver of premium benefit is payable for n years and the net annual premium is

$$(50) \quad \frac{P \cdot a_{x:\overline{n-1}}^{at}}{a_{x:\overline{n}}^{aa}}$$

but where $x + n > y$, the premium for the benefit ceases at age y , and the net annual premium for the benefit is

$$(51) \quad \frac{P \cdot a_{x:\overline{n-1}}^{at}}{a_{x:\overline{y-x}}^{aa}}$$

Using formula (45) the net annual premium for the waiver of premium benefit, where $x + n =$ or $< y$, is

$$(52) \quad \frac{P(N_x'' - N_{x+n}'' - D_x'' \cdot a_x' : \overline{n}|)}{N_x^{aa} - N_{x+n}^{aa}}$$

or where $x + n > y$,

$$(53) \quad \frac{P(N_x'' - N_{x+n}'' - D_x'' \cdot a_x' : \overline{n}|)}{N_x^{aa} - N_y^{aa}}$$

The premium for the waiver benefit is assumed payable up to and including the last anniversary on which a premium is subject to waiver. The policyholder generally has the right to discontinue the benefit at any time and might do so just before the payment of the last annual premium, as the benefit would become valueless as soon as that premium has been paid. The active life annuity in the denominator of the above formulas, therefore, might properly be for a period one year less. For example, in (47) and (51) we would use $a_{x:\overline{y-x-1}|}^{aa}$ instead of $a_{x:\overline{y-x}|}^{aa}$ and in (50) $a_{x:\overline{n-1}|}^{aa}$ instead of $a_{x:\overline{n}|}^{aa}$. The premium waived, however, is often paid semi-annually, quarterly or monthly, so that the benefit has a value until the last premium instalment is paid. On the other hand, the waiver benefit has been often combined with an income benefit which continues beyond the date of the last premium. In such cases there would be no inducement to cancel the benefit before the last disability premium had been paid.

A similar situation arises under certain income and instalment annuity benefits in connection with endowment policies. For a further discussion of this subject the student is referred to page 148.

Disability Annuity Benefit

If the annuity is payable annually with the first payment due on the next policy anniversary after the receipt of proof of disability, formula (47) would apply to an Ordinary Life policy upon substituting " K " the amount of the annual disability annuity payment for " P ". This provides for a life disability annuity. The same formula would apply to an n -payment

life policy where $x + n \leq y$. If $x + n \geq y$ the formula for the net annual premium is

$$(54) \quad \frac{K \cdot a_x^{at}}{a_{x:n}^{aa}}$$

(Note that for $x + n = y$ either formula may be used).

The clauses in endowment policies may provide an annuity which would cease at the maturity of the endowment or they may provide a life disability annuity. In either event, disability must occur within the endowment period.

For the former benefit the net premium under an n -year endowment is

$$(55) \quad \frac{K \cdot a_{x:n}^{at}}{a_{x:n}^{aa}} \quad \text{if } x + n \leq y$$

$$(56) \quad \text{and} \quad \frac{K \cdot a_{x:n}^{at}}{a_{x:y-x}^{aa}} \quad \text{if } x + n \geq y$$

The temporary period of the annuity in the numerator is n , which assumes that the last annuity payment is due at the end of the endowment period. Under the corresponding waiver formula the last premium waiver is due one year earlier, assuming premiums are payable annually.

For a life annuity benefit under an n -year endowment the net premium formulas become

$$(57) \quad \frac{K \cdot a_x^{a(t;\overline{n})}}{a_{x:n}^{aa}} \quad \text{for } x + n \leq y$$

$$(58) \quad \text{and} \quad \frac{K \cdot a_x^{a(t;\overline{y-x})}}{a_{x:y-x}^{aa}} \quad \text{for } x + n \geq y$$

The annuity in the numerator of formula (58) is ordinarily written as a_x^{at} since the annuities and other monetary values would be calculated on the basis that the rate of disability above age y is zero.

These formulas may be expressed in commutation form similar to (48), (52) and (53). If the annuity is payable at any other time than the policy anniversary, or if it begins at an anniversary subsequent to the anniversary immediately following the occurrence of disability, the foregoing commutation col-

umns, however, are not directly applicable. This is also true of the instalment type of benefit considered below. The valuation of the benefit can be made by a summation method which is analogous to the pension fund method. For example, if the annuity should begin one year after the receipt of proof of disability, the value of the benefit at inception of the contract for an Ordinary Life policy would be

$$(59) \quad \frac{1}{D_z^{aa}} \cdot K \sum_{t=z}^{z-v-1} v^{1/2} \cdot D_z^{aa} \cdot r_z \cdot a'_{z+1/2}$$

and the corresponding net annual premium ceasing at age y or prior disability is

$$(60) \quad \frac{K}{N_z^{aa} - N_y^{aa}} \sum_{t=z}^{z-v-1} v^{1/2} \cdot D_z^{aa} \cdot r_z \cdot a'_{z+1/2}$$

The student will find it a useful exercise to develop the formulas for other plans and for other benefits, such as an annuity to begin one year after the anniversary next succeeding receipt of proof of disability.

*Payment of Face Amount in Instalments during Disability,
Balance, if any, Payable at Death or Maturity*

The foregoing discussion refers to disability annuity payments which are not deducted from the face amount of insurance. Many forms, however, were issued which provided that the face amount of the policy would be reduced by any income payments made. In some cases the income payments ceased when their sum equalled the face amount of insurance; in others the annuity continued for the duration of total disability even after the face amount had been reduced to zero. The payments under the former type are frequently designated instalment payments and those under the latter, income payments reducing the face.

The amount of the annual payment was usually one-tenth or one-twentieth of the face amount. An example of a clause for this type of disability benefit will be found on page 208.

The instalment types of benefit are not of great interest, as such forms are no longer issued. However, problems still arise in connection with them because of the large amount of insurance still in force containing such benefits. The student should, therefore, be familiar with the nature of the premium and

reserve calculations which are peculiar to these benefits. Since the first edition of the disability textbook, which described the calculations for instalment benefits, is now out of print, the formulas contained therein are given below; the corresponding formulas according to the pension fund notation are given in Chapter IX.

The following formulas are applicable to a benefit with instalment payments, the first instalment becoming due one year after the anniversary next succeeding the receipt of proof of total and permanent disability. The symbols used may be defined thus:

k represents the number of instalments, $1/k$ being the amount of each instalment,

n represents the number of years premiums are payable under a limited payment life policy, or the term of an endowment policy (only an n -payment n -year endowment policy being considered),

d represents the discount on 1 payable one year hence.

Ordinary Life policy. By paying a part of the policy before death the company loses interest on the instalment paid from the date of payment to the date of death. The value of that interest is measured by the mortality among disabled lives.

In the case of a life aged $z+1$ which has become disabled during the preceding year an instalment of $1/k$ will be paid on attaining age $z+2$. The value of the interest lost by making this advance payment is an annuity of d/k so that the value of the benefit, on account of the first instalment, is

$$\frac{d}{k} \cdot \frac{N'_{z+2}}{D'_{z+1}}$$

Similarly, the value of the benefit at the end of the year in which disability occurs on account of the second advance payment is

$$\frac{d}{k} \cdot \frac{N'_{z+3}}{D'_{z+1}}$$

As there are k possible instalments, the value at the end of the year in which disability occurs on account of the prepayment of all possible instalments is

$$\frac{d}{k} \cdot \frac{1}{D'_{z+1}} \{N'_{z+2} + N'_{z+3} + \dots + N'_{z+k+1}\}$$

$$(61) \quad = \frac{d}{k} \left(\frac{S'_{i+2} - S'_{i+k+2}}{D'_{i+1}} \right)$$

and at the inception of the contract the value of the benefit is

$$(62) \quad \frac{1}{D'_z} \sum_{i=z}^{x-y-1} v \cdot D'^a_i \cdot p'^a_i \cdot \frac{d}{k} \left(\frac{S'_{i+2} - S'_{i+k+2}}{D'_{i+1}} \right)$$

As premiums for the benefit are payable until age y the net annual premium for the benefit is

$$(63) \quad \frac{1}{N'^a_z - N'^a_y} \sum_{i=z}^{x-y-1} v \cdot D'^a_i \cdot p'^a_i \cdot \frac{d}{k} \left(\frac{S'_{i+2} - S'_{i+k+2}}{D'_{i+1}} \right)$$

Limited payment life policy. At the beginning of the contract the value of this benefit is the same as that under an Ordinary Life policy issued at the same age. When $x+n \leq y$ the annual premium for the benefit is also the same as under an Ordinary Life policy, because in this case the annual premium for the benefit will cease at age y . Where $x+n < y$ the net annual premium for the benefit is

$$(64) \quad \frac{1}{N'^a_z - N'^a_{x+n}} \sum_{i=z}^{x-y-1} v \cdot D'^a_i \cdot p'^a_i \cdot \frac{d}{k} \left(\frac{S'_{i+2} - S'_{i+k+2}}{D'_{i+1}} \right)$$

Endowment policy. Under an n -year endowment policy issued at age x , as the face amount of the policy would be payable either at death or at maturity, the temporary annuity $\frac{N'_i - N'_{i+n}}{D'_i}$ measures the value of the loss of interest from the date an instalment is paid until death or maturity. In the case of a life aged $z+1$ which has become disabled during the preceding year, an instalment of $1/k$ will be paid on attaining age $z+2$. Since the policy matures at age $x+n$ the last age at which an instalment can be paid is $x+n-1$. The value at the end of the policy year in which disability occurs of the loss of interest on

account of the first advance payment will be $\frac{d}{k} \cdot \frac{N'_{i+2} - N'_{i+n}}{D'_{i+1}}$;

on account of the second instalment payment, the value will be

$\frac{d}{k} \cdot \frac{N'_{i+3} - N'_{i+n}}{D'_{i+1}}$; and on account of the t^{th} payment,

$\frac{d}{k} \cdot \frac{N'_{i+t+1} - N'_{i+n}}{D'_{i+1}}$.

If disability occurs before age $x + n - k - 1$, there are k possible instalment payments. In this case the total value at the end of the year in which disability occurs, on account of the k possible payments will be

$$\frac{d}{k} \left[\frac{N'_{z+2} + N'_{z+3} + \dots + N'_{z+k+1} - k \cdot N'_{x+n}}{D'_{z+1}} \right]$$

or $\frac{d}{k} \left[\frac{S'_{z+2} - S'_{z+k+2} - k \cdot N'_{x+n}}{D'_{z+1}} \right]$ where $x + n \leq z + k + 2$.

Where disability occurs after age $x + n - k - 1$ there will be less than k instalments payable, the last instalment being paid at age $x + n - 1$. The highest age at which disability can occur for an instalment to be payable is $x + n - 3$. The value of the interest lost on account of the last possible instalment payment will be

$$\frac{d}{k} \cdot \frac{N'_{x+n-1} - N'_{x+n}}{D'_{x+1}} \text{ and the total value on account of the } x + n - z - 2 \text{ possible payments will be}$$

$$\frac{d}{k} \left[\frac{N'_{z+2} + N'_{z+3} + \dots + N'_{x+n-1} - (x + n - z - 2) N'_{x+n}}{D'_{z+1}} \right]$$

or

$$\frac{d}{k} \left[\frac{S'_{z+2} - S'_{x+n} - (x + n - z - 2) N'_{x+n}}{D'_{z+1}} \right] \text{ where } x + n < z + k + 2.$$

Where $x + n \geq y$, the annual premium for the benefit, being payable for n years, will be

(65)

$$\frac{1}{N_x^{aa} - N_{x+n}^{aa}} \left\{ \sum_{t=x}^{t=x+n-k-2} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S'_{z+2} - S'_{z+k+2} - k \cdot N'_{x+n}}{D'_{z+1}} \right] \right. \\ \left. + \sum_{t=x+n-k-1}^{t=x+n-3} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S'_{z+2} - S'_{x+n} - (x + n - z - 2) N'_{x+n}}{D'_{z+1}} \right] \right\}$$

The expression may be written with only one summation as follows:

(65a)

$$\frac{1}{N_x^{aa} - N_{x+n}^{aa}} \left\{ \sum_{t=x}^{t=x+n-3} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S'_{z+2} - S'_{t+m+1} - (m-1) N'_{x+n}}{D'_{z+1}} \right] \right\}$$

where m equals $k + 1$ or $x + n - z - 1$ whichever is the less.

The student must be careful in applying these formulas to particular cases. The following example will illustrate the modification sometimes necessary: If we are dealing with a

Ten Year Endowment providing instalment payments of one-tenth the face amount, the first payment falling due one year after the anniversary next succeeding receipt of proof of disability, then the first half of formula (65) will be eliminated as the upper age limit of the summation will be less than the age at issue. Also, in the second half of the formula the lower age limit of the summation should be the issue age.

Where $x + n > y$, the annual premium ceases at age y , and the benefit is not received unless disability occurs before age y .

The premium for the benefit is

(66)

$$\frac{1}{N_x^{aa} - N_y^{aa}} \left\{ \sum_{t=x}^{t=x+n-k-2} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S_{t+2}^t - S_{t+k+2}^t - k \cdot N_{t+n}^t}{D_{t+1}^t} \right] \right. \\ \left. + \sum_{t=x+n-k-1}^{t=y-1} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S_{t+2}^t - S_{t+n}^t - (x+n-z-2) N_{t+n}^t}{D_{t+1}^t} \right] \right\}$$

There are two cases where one or the other of the summations in the above formulas would be eliminated, leaving only the remaining summation:

Case I. Where $x + n - k - 2 \geq y - 1$, the insured would receive all his instalment payments before maturity and the second summation would be eliminated, the formula reading

$$(66a) \frac{1}{N_x^{aa} - N_y^{aa}} \sum_{t=x}^{t=y-1} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S_{t+2}^t - S_{t+k+2}^t - k \cdot N_{t+n}^t}{D_{t+1}^t} \right]$$

Case II. Where $n \leq k + 1$, the greatest possible number of instalment payments would be $k - 1$. The first summation of formula (66) would be eliminated and the formula would be,

(66b)

$$\frac{1}{N_x^{aa} - N_y^{aa}} \left\{ \sum_{t=x}^{t=y-1} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S_{t+2}^t - S_{t+n}^t - (x+n-z-2) N_{t+n}^t}{D_{t+1}^t} \right] \right\}$$

Generally, equation (66) may be written with only one summation as follows:

(66c)

$$\frac{1}{N_x^{aa} - N_y^{aa}} \sum_{t=x}^{t=y-1} v \cdot D_t^{aa} \cdot p_t^{at} \cdot \frac{d}{k} \left[\frac{S_{t+2}^t - S_{t+m+1}^t - (m-1) N_{t+n}^t}{D_{t+1}^t} \right]$$

where $m = k + 1$ or $x + n - z - 1$ whichever is the less.

As there will be no instalments payable unless disability occurs more than two years prior to maturity, the summation in equation (66) ceases when $z = x + n - 3$. Accordingly when $x + n = y + 1$ the summation is $\sum_{i=x}^{i=y-2}$ instead of $\sum_{i=x}^{i=y-1}$, the formula remaining otherwise unchanged. However, formula (66c) may be used in either case, as when $x + n = y + 1$ the last term in the summation is zero, i.e., when $z = y - 1$.

Maturity Instalment Benefit

This benefit provides, at disability, for the payment during a term certain of instalments the present value of which, at a given rate of interest, is the face of the policy. If death should occur before the term certain period expires, the present value of future instalments is paid in one sum. Since interest as well as principal is included in the instalments, this is equivalent to maturing the policy at disability instead of at death or maturity. Thus, in addition to the waiver of premiums, the benefit to be provided for is a loss of interest on the face amount from the first instalment due date to the date of death or maturity.

The following formulas are applicable to a maturity instalment benefit under which the instalments are payable annually, the first payment being made at the end of the policy year in which disability occurs.

Ordinary Life policy. In the case of a life becoming disabled between ages z and $z + 1$, the first instalment becomes due at age $z + 1$ and the company loses interest on the face amount from age $z + 1$ to the date of death. The present value, at the date of the first payment, of the interest lost per unit of face amount is an annuity-due of d so that the value of the benefit is

$$d \cdot \frac{N_{z+1}^I}{D_{z+1}^I}$$

At the inception of the contract the value of the benefit is

$$(67) \quad \frac{1}{D_x^{aa}} \sum_{i=x}^{i=y-1} v \cdot D_i^{aa} \cdot p_i^{at} \cdot d \cdot a_{i+1}^t$$

As premiums are payable until age y the net annual premium for the benefit is

$$(68) \quad \frac{1}{N_x^{aa} - N_y^{aa}} \sum_{i=x}^{i=y-1} v \cdot D_i^{aa} \cdot p_i^{at} \cdot d \cdot a_{i+1}^t$$

Limited payment life policy. Except where $x + n < y$, the value of the benefit and the net annual premium are the same as for an Ordinary Life policy at the same age at issue. Where $x + n < y$ the net annual premium for the benefit is

$$(69) \quad \frac{1}{N_x^{aa} - N_{x+n}^{aa}} \sum_{z=x}^{z=y-1} v \cdot D_z^{aa} \cdot p_z^{at} \cdot d \cdot a_{z+1}^t$$

Endowment policy. The value of the benefit is a loss of interest on the face amount from the first instalment due date to the maturity of the policy or the death of the insured should he die within the endowment period. The net annual premium for the benefit is,

if $x + n > y$,

$$(70) \quad \frac{1}{N_x^{aa} - N_y^{aa}} \sum_{z=x}^{z=y-1} v \cdot D_z^{aa} \cdot p_z^{at} \cdot d \cdot \frac{N_{z+1}^t - N_{x+n}^t}{D_{z+1}^t}$$

and, if $x + n \leq y$,

$$(71) \quad \frac{1}{N_x^{aa} - N_{x+n}^{aa}} \sum_{z=x}^{z=x+n-2} v \cdot D_z^{aa} \cdot p_z^{at} \cdot d \cdot \frac{N_{z+1}^t - N_{x+n}^t}{D_{z+1}^t}$$

The maturity instalment benefit is issued at the present time under group life contracts. As such contracts are usually issued on the one year renewable term plan, it may be assumed that the life insurance coverage ceases when disability occurs. In such event there would be no waiver of premium in the disability benefit. The benefit would, therefore, be equivalent only to maturing the face amount of the policy on receipt of proof of disability. The premium for the benefit would be the premium to provide at disability a single payment equal to the face amount of the policy. This is the assumption usually made.

*Total Premium for Disability Benefits. Period
during which Payable*

When a policy provides for an annuity or for the payment of the face amount in instalments during disability, it generally also covers waiver of premiums during disability. The total premium for the disability benefits will then be the sum of the premium for the waiver benefit and the premium for the annuity or instalment benefit. In calculating the premium for

the waiver benefit the premium to be waived is considered to be the premium for the regular insurance exclusive of the disability benefit, because the premium for the disability benefit is calculated on the assumption that it will cease in event of disability. Policies are frequently issued with a double indemnity accidental death benefit, the premium for which is usually waived along with the insurance premium. As the double indemnity premium is small so that the disability charge for waiving this premium is about one or two cents per \$1,000 of insurance, it is customary to make no charge for waiving the double indemnity premium.

The waiver of premium benefit contained in a limited payment life or an endowment policy ceases when the last premium is paid. The benefit considered on pages 141 and 148 ceases unless disability occurs prior to one year before the maturity of an endowment policy; the benefit considered on pages 144 to 147 ceases unless disability occurs prior to two years before the maturity of an endowment policy. Where $x + n =$ or $< y$ premiums for these benefits are calculated on the assumption that they will be received for n years, unless the insured dies or becomes disabled before that time. Accordingly it is assumed that premiums will be received after the benefit ceases. Although many policies grant the insured the privilege of canceling at any time the disability feature and reducing the premium by the amount of the premium for the disability benefits, yet it is not believed that many policyholders will take advantage of this condition.

CHAPTER IX.

CALCULATION OF PREMIUMS BY PENSION
FUND METHODS*A. Pension Fund Method Applicable to Early Types
of Benefits*

The student will find two distinct forms of pension fund notation. One form is applicable to the earlier types of benefits. A discussion and development of it will be found in J. F. Little's paper (T. A. S. A., XIV, 55) the main features of which will be covered in this study. The other form of pension fund notation is merely a development of the first form to make it applicable to the present day benefits.

The great advantage of the pension fund method is the facility with which it can be applied to different types of benefits. Pension fund commutation columns can be computed by employing either p_x^{at} or r_x to provide benefits beginning at any time after disablement and terminating at any desired time. The columns can be developed for either annual or monthly payments and are applicable to either aggregate or select rates of disability and of claim termination. Separate sets of columns are necessary, however, for different rates of disability, for different commencement dates of benefits, etc.

The student in reading Mr. Little's paper should substitute p_x^{at} for r_x . The principal symbols used therein and the formulas based thereon are as follows:

The single premium to provide 1 payable at the end of the policy year in which disability occurs is

$$(72) \quad \frac{\sum_{n=0}^{n=\omega-x} v^{n+1} \cdot {}_n p_x^{aa} \cdot p_{x+n}^{at}}{\sum_{n=0}^{n=\omega-x} v \cdot D_{x+n}^{aa} \cdot p_{x+n}^{at}} = \frac{D_x^{aa}}{D_x^{aa}}$$

$$(73) \quad = \frac{{}^r M_x^{at}}{D_x^{aa}}$$

where

$$v \cdot D_x^{aa} \cdot p_x^{at} = {}^r C_x^{at}$$

and

$$\sum_{n=0}^{n=\omega-x} {}^r C_{x+n}^{at} = {}^r M_x^{at}$$

In the various formulas now to be developed the variable will be taken as " z " instead of " n " where $z = x + n$. " n " will be used to denote the premium period in limited payment plans and also the endowment period in coterminous endowments. For limited payment endowments, " t " will denote the endowment period.

If the benefit is payable only in event of the disability occurring before age y , the single premium becomes

$$(74) \quad \frac{{}^rM_x^{at} - {}^rM_y^{at}}{D_x^{aa}}$$

If the tables are constructed with disability coverage ceasing at age y , the value of ${}^rM_y^{at}$ would be zero.

The annual premium ceasing at age y for the latter benefit is

$$(75) \quad \frac{{}^rM_x^{at} - {}^rM_y^{at}}{N_x^{aa} - N_y^{aa}}$$

and when premiums are payable for n years, $x + n \geq y$, is

$$(76) \quad \frac{{}^rM_x^{at} - {}^rM_y^{at}}{N_x^{aa} - N_{x+n}^{aa}}$$

The single premium to provide a disability annuity of 1 per annum to commence at the policy anniversary following disablement is

$$(77) \quad \frac{\sum_{t=x}^{t=\infty} v \cdot D_x^{aa} \cdot p_x^{at} \cdot a_{x+1}^t}{D_x^{aa}} = \frac{{}^{ra}M_x^{at}}{D_x^{aa}}$$

where

$$v \cdot D_x^{aa} \cdot p_x^{at} \cdot a_{x+1}^t = {}^{ra}C_x^{at}$$

and

$$\sum_{t=x}^{t=\infty} {}^{ra}C_x^{at} = {}^{ra}M_x^{at}$$

Note that ${}^{ra}C_x^{at} = {}^rC_x^{at} \cdot a_{x+1}^t$; the small letter " a " is added to the upper left hand corner of the basic symbol to indicate that the disability benefit is an annuity.

Assuming disability coverage to age y ,

$$\frac{{}^{ra}M_x^{at} - {}^{ra}M_y^{at}}{D_x^{aa}}$$

is the commutation expression for the annuity represented by a_x^{at} .

The foregoing provide at disablement for a single payment or for a life disability annuity. In the case of the waiver benefit

under limited payment life, term and endowment contracts we must provide for a temporary disabled life annuity benefit.

The single premium for a temporary disabled life annuity-due of 1 per annum applicable to the waiver of premium benefit under an n -payment life policy, where $x + n \leq y$, is

$$\begin{aligned} & \frac{1}{D_x^{aa}} \sum_{t=x}^{x+n-2} {}^tC_x^{at} \cdot a_{x+1:\overline{x+n-2-1}|} \\ &= \frac{1}{D_x^{aa}} \sum_{t=x}^{x+n-2} {}^tC_x^{at} \cdot \frac{N_{x+1}^t - N_{x+n}^t}{D_{x+1}^t} \\ (78) \quad &= \frac{{}^xM_x^{at} - {}^xM_{x+n-1}^{at} - N_{x+n}^t ({}^xM_x - {}^xM_{x+n-1})}{D_x^{aa}} \end{aligned}$$

where

$$\frac{{}^tC_x^{at}}{D_{x+1}^t} = {}^tC_x$$

and

$$\sum_{t=x}^{x+n-2} {}^tC_x = {}^xM_x$$

The formula may also be written

$$(79) \quad \frac{{}^xM_x^{at} - {}^xM_{x+n}^{at} - N_{x+n}^t ({}^xM_x - {}^xM_{x+n})}{D_x^{aa}}$$

since ${}^xM_{x+n-1}^{at} = {}^xM_{x+n}^{at} + N_{x+n}^t$

If $x + n > y$ the summation becomes

$$\begin{aligned} & \frac{1}{D_x^{aa}} \sum_{t=x}^{x+y-1} {}^tC_x^{at} \cdot a_{x+1:\overline{x+n-2-1}|} \\ (80) \quad &= \frac{{}^xM_x^{at} - {}^xM_y^{at} - N_{x+n}^t ({}^xM_x - {}^xM_y)}{D_x^{aa}} \end{aligned}$$

When $x + n \leq y$ the formula represents the commutation expression for $a_{x:\overline{n-1}|}^{a(t;\overline{n})}$ and, when $x + n > y$, the expression for $a_{x:\overline{n-1}|}^{a(t;\overline{y-x})}$.

The annual premium for the benefit, where $x + n \leq y$, is

$$(81) \quad \frac{{}^xM_x^{at} - {}^xM_{x+n}^{at} - N_{x+n}^t ({}^xM_x - {}^xM_{x+n})}{N_x^{aa} - N_{x+n}^{aa}}$$

and, where $x + n > y$, the annual premium ceasing at y is

$$(82) \quad \frac{{}^xM_x^{at} - {}^xM_y^{at} - N_{x+n}^t ({}^xM_x - {}^xM_y)}{N_x^{aa} - N_y^{aa}}$$

If instead of the waiver benefit we were to consider an annual

income benefit under an endowment policy, the first income payment to become due at the policy anniversary following the receipt of proof of disability, the annuity continuing during disability until the maturity of the endowment, then the last annuity payment would fall due on the maturity date of the policy. The annuity involved would therefore be for a period one year longer than that for the waiver benefit in the foregoing formulas. The period of exposure to disability would also be for one year longer as a benefit would be payable in event of disability occurring in the last policy year. The single premium for this benefit, $x + n \geq y$, is

$$(83) \quad \frac{{}^aM_x^{at} - {}^aM_{x+n}^{at} - N_{x+n+1}^t ({}^aM_x - {}^aM_{x+n})}{D_x^{aa}}$$

Various combinations of the various periods involved can be made; a general formula, however, can be given which will illustrate the modifications in the formula. Take the case of an n -payment t -year endowment, $x + n < y < x + t$. The annual premium for an annual income disability benefit in event of disability before y , the income to commence on the policy anniversary following receipt of proof of disability and continuing during disability until the maturity of the endowment is

$$(84) \quad \frac{{}^aM_x^{at} - {}^aM_y^{at} - N_{x+t+1}^t ({}^aM_x - {}^aM_y)}{N_x^{aa} - N_{x+n}^{aa}}$$

The period of the " M " functions depends on the period of coverage. As the coverage ceases at age y , the final " C " in the summation is for $y-1$, i.e., the subtractive " M " symbol is for age y .

The N^t symbol gives the period of the disability annuity. As the last annuity payment is on the maturity of the endowment, that is at age $x + t$, the annuity ceases at age $x + t + 1$. Therefore we have N_{x+t+1}^t in the formula.

The N^{aa} symbols give the premium paying period. The last premium is payable at age $x + n - 1$ and ceases at age $x + n$. Therefore the subtractive symbol is N_{x+n}^{aa} .

If the student keeps clearly in mind these three periods—the coverage period, the benefit period and the premium payment

period, and the symbols affected by them, he should have little difficulty with the various combinations.

Instalment Benefits

Life policy. In the calculation of premiums for the instalment type of benefit, that is, where the face amount is reduced by any disability payments made, it was shown on page 142 that the Hamza method could not be used without recourse to a pension fund summation method. If we use, with a slight modification, the pension fund methods and notation as in Mr. Little's paper, additional commutation columns may be developed for the instalment type of benefit. In the case of a disability instalment annuity of $\frac{1}{k}$ th of the face amount, the first payment due one year after the policy anniversary following receipt of notice of claim, the single premium under a life policy for the benefit in event of disability occurring before age y , is

$$\begin{aligned}
 & \frac{d}{k \cdot D_x^{aa}} \sum_{z=x}^{z=y-1} {}^rC_z^a \frac{S_{z+2}^i - S_{z+k+2}^i}{D_{z+1}^i} \\
 &= \frac{d}{k \cdot D_x^{aa}} \sum_{z=x}^{z=y-1} {}^{ra}C_z (S_{z+2}^i - S_{z+k+2}^i) \\
 (85) \quad &= \frac{d}{k} \cdot \frac{{}^{ra}M_x^{at} - {}^{ra}M_y^{at}}{D_x^{aa}}
 \end{aligned}$$

where $S_{z+2}^i - S_{z+k+2}^i$ is formed for the single value of k required and

$${}^{ra}C_z (S_{z+2}^i - S_{z+k+2}^i) = {}^{ra}C_z^a$$

and

$$\sum_{z=x}^{z=y-1} {}^{ra}C_z^a = {}^{ra}M_x^{at}$$

and d , as in the previous chapter, represents the discount on 1 payable one year hence. The adjustments in formula (85) to obtain the annual premium for Ordinary Life and limited payment life policies are obvious.

If the instalment payments commence on the anniversary next succeeding receipt of proof of disability, $S_{z+1}^i - S_{z+k+1}^i$ must be substituted for $S_{z+2}^i - S_{z+k+2}^i$ in the above formula so that expression (85) becomes

$$(85a) \quad \frac{d}{k} \cdot \frac{{}^{ra}M_x^{at} - {}^{ra}M_y^{at}}{D_x^{aa}}$$

where ${}^{ra}C_z (S_{z+1}^i - S_{z+k+1}^i) = {}^{ra}C_z^a$ and $\sum_{z=x}^{z=y-1} {}^{ra}C_z^a = {}^{ra}M_x^{at}$.

Mr. Little used the symbols nC_z instead of ${}^nC_z^{at}$ and nM_z instead of ${}^nM_z^{at}$. Mr. Little's notation has been changed in order to facilitate a distinction being made between the expressions ${}^nC_z(S_{z+1}^t - S_{z+k+1}^t)$ and ${}^nC_z(S_{z+2}^t - S_{z+k+2}^t)$. At the same time such a change, although making the symbols somewhat more complex, seems more consistent with the notation,

$${}^{Ja}C_z^{at} = {}^rC_z^{at} \cdot \frac{S_{z+2}^t}{D_{z+1}^t} = {}^nC_z \cdot S_{z+2}^t \quad \text{and} \quad {}^{Ja}C_z^{at} = {}^rC_z^{at} \cdot \frac{S_{z+1}^t}{D_{z+1}^t} = {}^nC_z \cdot S_{z+1}^t,$$

appearing below.

Endowment policy. The pension fund formula for an instalment benefit under an n -payment n -year endowment is obtained from formulas (65) and (66) which are in pension fund summation form. The annual premium for an instalment annuity of $\frac{1}{k}$ -th the face amount, the first payment due one year after the policy anniversary following receipt of proof of disability, according to formula (65), for $x+n \geq y$ is, in pension fund form,

$$\begin{aligned} & \frac{d}{k} \cdot \frac{1}{N_x^{aa} - N_{x+n}^{aa}} \left\{ \sum_{z=x}^{x+n-k-2} \frac{{}^rC_z^{at}}{D_{z+1}^t} (S_{z+2}^t - S_{z+k+2}^t - k \cdot N_{z+n}^t) \right. \\ & \quad \left. + \sum_{z=x+n-k-1}^{x+n-3} \frac{{}^rC_z^{at}}{D_{z+1}^t} [S_{z+2}^t - S_{z+n}^t - (x+n-z-2)N_{z+n}^t] \right\} \\ & = \frac{d}{k} \cdot \frac{1}{N_x^{aa} - N_{x+n}^{aa}} \left\{ \sum_{z=x}^{x+n-k-2} [{}^nC_z(S_{z+2}^t - S_{z+k+2}^t) - k \cdot N_{z+n}^t \cdot {}^nC_z] \right. \\ & \quad \left. + \sum_{z=x+n-k-1}^{x+n-3} [{}^rC_z^{at} \cdot \frac{S_{z+2}^t}{D_{z+1}^t} - {}^nC_z(S_{z+n}^t + x+n-2 \cdot N_{z+n}^t) + z \cdot {}^nC_z \cdot N_{z+n}^t] \right\} \\ & = \frac{d}{k} \cdot \frac{1}{N_x^{aa} - N_{x+n}^{aa}} \left\{ {}^{Ja}M_x^{at} - {}^{Ja}M_{x+n-k-1}^{at} - k \cdot N_{x+n}^t ({}^nM_x - {}^nM_{x+n-k-1}) \right. \\ & \quad + ({}^{Ja}M_{x+n-k-1}^{at} - {}^{Ja}M_{x+n-2}^{at}) - (S_{x+n}^t + x+n-2 \cdot N_{x+n}^t) ({}^nM_{x+n-k-1} \\ & \quad \quad - {}^nM_{x+n-2}) + [(x+n-k-2)({}^nM_{x+n-k-1} - {}^nM_{x+n-2}) \\ & \quad \quad \left. + ({}^nR_{x+n-k-1} - {}^nR_{x+n-2} - k-1 \cdot {}^nM_{x+n-2})] N_{x+n}^t \right\} \\ & \text{where } {}^nR = \sum {}^nM, \quad {}^{Ja}C_z^{at} = {}^rC_z^{at} \cdot \frac{S_{z+2}^t}{D_{z+1}^t} \quad \text{and} \quad {}^{Ja}M_z^{at} = \sum {}^{Ja}C_z^{at}. \end{aligned}$$

Rearranging, we have

$$\frac{d}{k} \cdot \frac{1}{N_x^{aa} - N_{x+n}^{aa}} \left\{ {}^rI_k^a M_x^{at} - {}^rI_k^a M_{x+n-k-1}^{at} + {}^rI_a M_{x+n-k-1}^{at} - {}^rI_a M_{x+n-2}^{at} \right. \\ \left. - S_{x+n}^t ({}^rM_{x+n-k-1} - {}^rM_{x+n-2}) - N_{x+n}^t [k ({}^rM_x - {}^rM_{x+n-k-1}) \right. \\ \left. + (x+n-2) ({}^rM_{x+n-k-1} - {}^rM_{x+n-2}) \right. \\ \left. - (x+n-k-2) ({}^rM_{x+n-k-1} - {}^rM_{x+n-2}) \right. \\ \left. - ({}^rR_{x+n-k-1} - {}^rR_{x+n-2} - \overline{k-1} {}^rM_{x+n-2}) \right\}$$

Canceling out appropriate terms, we obtain, for the premium,

$$(86) \quad \frac{d}{k} \cdot \frac{1}{N_x^{aa} - N_{x+n}^{aa}} \left\{ {}^rI_k^a M_x^{at} - {}^rI_k^a M_{x+n-k-1}^{at} + {}^rI_a M_{x+n-k-1}^{at} - {}^rI_a M_{x+n-2}^{at} \right. \\ \left. - S_{x+n}^t ({}^rM_{x+n-k-1} - {}^rM_{x+n-2}) - N_{x+n}^t (k \cdot {}^rM_x - {}^rR_{x+n-k-1} + {}^rR_{x+n-1}) \right\}$$

Similarly, formula (66) may be written, for $x+n > y$,

$$(87) \quad \frac{d}{k} \cdot \frac{1}{N_x^{aa} - N_y^{aa}} \left\{ {}^rI_k^a M_x^{at} - {}^rI_k^a M_{x+n-k-1}^{at} + {}^rI_a M_{x+n-k-1}^{at} - {}^rI_a M_y^{at} \right. \\ \left. - S_{x+n}^t ({}^rM_{x+n-k-1} - {}^rM_y) - N_{x+n}^t [k \cdot {}^rM_x + (y-x-n+1) {}^rM_y \right. \\ \left. - {}^rR_{x+n-k-1} + {}^rR_y] \right\}$$

The student will find it excellent practice to develop the formulas for other forms of instalment benefits such as the one in which the first instalment payment is due on the anniversary following receipt of proof. The student may check his results in the suggested case with those developed by J. F. Little in T. A. S. A., XIV, 66 and, for the endowment form, in the same volume, page 384. It will be noted in the foregoing solution that we have the symbol

$${}^rI_a C_x^{at} = {}^rC_x^{at} \cdot \frac{S_{x+2}^t}{D_{x+1}^t}$$

In Mr. Little's paper an annuity-due is considered so that he uses the symbol

$${}^rI_a C_x^{at} = {}^rC_x^{at} \cdot \frac{S_{x+1}^t}{D_{x+1}^t}$$

Maturity Instalment Benefit

Since, under this benefit, the first instalment usually becomes due at the end of the policy year in which disability occurs, the

same pension fund symbols are applicable as those previously derived for the waiver of premium benefit. The net annual premium, per unit of face amount, is:

Ordinary Life policy

$$(88) \quad d \cdot \frac{{}^{ra}M_x^{at} - {}^{ra}M_y^{at}}{N_x^{aa} - N_y^{aa}}$$

This will also be the expression for the net annual premium under an n -payment life policy if $x+n \geq y$. If $x+n < y$ the denominator in the above expression would become $N_x^{aa} - N_{x+n}^{aa}$.

Endowment policy

for $x+n \geq y$

$$(89) \quad d \cdot \frac{{}^{ra}M_x^{at} - {}^{ra}M_y^{at} - N_{x+n}^t ({}^{ra}M_x - {}^{ra}M_y)}{N_x^{aa} - N_y^{aa}}$$

for $x+n < y$

$$(90) \quad d \cdot \frac{{}^{ra}M_x^{at} - {}^{ra}M_{x+n-1}^{at} - N_{x+n}^t ({}^{ra}M_x - {}^{ra}M_{x+n-1})}{N_x^{aa} - N_{x+n}^{aa}}$$

The latter formula may also be written

$$(91) \quad d \cdot \frac{{}^{ra}M_x^{at} - {}^{ra}M_{x+n}^{at} - N_{x+n}^t ({}^{ra}M_x - {}^{ra}M_{x+n})}{N_x^{aa} - N_{x+n}^{aa}}$$

B. Pension Fund Method Applicable to Present Day Benefits

The pension fund calculations and notation already discussed involved the rate of disability represented by p_x^{at} and assumed that aggregate or ultimate disabled life annuities were to be used. The above method could also be employed with an r_x rate of disability provided different values were assigned to the symbols. However, with the advent of the modern contract wherein disability is defined by means of a definite waiting period, usually fairly short, and with the introduction of a monthly disability income benefit, select disabled life annuities must be used requiring a modified pension fund notation.

The new pension fund notation will be found in two papers, one by Robert Henderson (T. A. S. A., XXX, 463), and one by J. T. Phillips (T. A. S. A., XXX, 393). The notation is as follows:

The net single premium to provide 1 at date of disability, in event of disability occurring before age y , is

$$(92) \quad \frac{\sum_{z=x}^{z=y-1} v^{\frac{1}{2}} \cdot D_z^{aa} \cdot r_z}{D_x^{aa}} = \frac{{}_yM_x^r}{D_x^{aa}}$$

where $v^{\frac{1}{2}} \cdot D_z^{aa} \cdot r_z = C_z^r$

and $\sum_{z=x}^{z=y-1} C_z^r = {}_yM_x^r$

The net single premium for a monthly income benefit commencing when the insured qualifies for benefits, in event of disability occurring before age y and on the assumption of a uniform distribution of claims during the policy year, is

$$(93) \quad \frac{\sum_{z=x}^{z=y-1} v^{\frac{1}{2}} \cdot D_z^{aa} \cdot r_z \cdot a_{[z+\frac{1}{2}]}^{(12)}}{D_x^{aa}} = \frac{{}_yM_x^r}{D_x^{aa}}$$

where $v^{\frac{1}{2}} \cdot D_z^{aa} \cdot r_z \cdot a_{[z+\frac{1}{2}]}^{(12)} = {}^\omega C_z^r$

and $\sum_{z=x}^{z=y-1} {}^\omega C_z^r = {}^\omega M_x^r$

The expression ${}^\omega M_x^r$ corresponds to ${}^a M_x^{at} - {}^a M_y^{at}$ of the earlier form. The student should carefully note the significance of the small letters at the various corners of the basic symbol. The letter "r" is placed at the upper right corner to indicate that the rate of disability involved is r_x and to distinguish it from the former notation which had "at" in that position as its distinguishing symbol. The "ω" at the upper left indicates the terminating age of the disabled life annuity which in this case is a whole life disabled life annuity. No letter appears in this position of the commutation column symbol for a single payment benefit at disablement. The letter at the lower left indicates the period of the summation, i.e., the period of disability coverage. If no letter appears here it indicates that the summation is complete, i.e., the coverage period and the annuity period are equal. In a life annuity the summation would be to age ω, if no letter appears at the lower left corner, but ordinarily "y" would appear there, where y is the limiting age for the rate of disability, y being less than ω. In the case of a temporary dis-

ability annuity ceasing at an age u where u is not greater than the limiting age y , the complete summation would cease at age u . As the commutation symbols show the terminal age of the disability annuity involved, the limits of the summations in these formulas will be expressed in terms of ages, rather than in terms of policy years as heretofore.

Formula (93) applies to the evaluation of a monthly income benefit. The corresponding single premium for the waiver of premium benefit under an annual premium Ordinary Life policy, issued in conjunction with a monthly income benefit, is

$$(94) \quad \frac{\sum_{t=x}^{t=y-1} v \cdot D_x^{aa} \cdot r_x \cdot \frac{1}{2} p_{[x+\frac{1}{2}]}^t \cdot a_{[x+\frac{1}{2}]+1}^t}{D_x^{aa}}$$

The rate r_x and not p_x^{af} , is the rate of disability directly obtained from an experience under which benefits (income) commence on receipt of proof of disability. Even a waiver benefit alone does not in practice always produce p_x^{af} because premiums are frequently payable other than annually. Therefore, in many cases the disabled life does not have to survive as such to the end of the policy year of disablement in order to receive a benefit. J. D. Craig (T. A. S. A., XXVII, 47), dealing with aggregate disabled life annuities, made the assumption that

$$(95) \quad v^{\frac{1}{2}} \cdot \frac{1}{2} p_{x+\frac{1}{2}}^t \cdot a_{x+1}^t = a_{x+\frac{1}{2}}^{t(12)}$$

so that the same commutation columns could be used for both the income and the waiver benefit. The same assumption was made by W. M. Strong (T. A. S. A., XXX, 172) in dealing with select disabled life annuities. On account of the very high but rapidly decreasing termination rate in the early months of disability under modern policies, the above approximation slightly overstates the charge for the waiver benefit, especially under limited payment life and endowment plans (see J. T. Phillips, T. A. S. A., XXX, 407). The overstatement, however, is small and it is frequently the practice today to treat the waiver benefit as equivalent to the waiver of one-twelfth of the annual premium each month.

There is another slight overstatement in Mr. Craig's assumption due to the fact that the mean date of each twelve of the

monthly payments of the monthly annuity would fall one-half month prior to the policy anniversary if we assume that the date of disability will occur, on the average, in the middle of a policy year. This small error may be corrected by using a continuous annuity instead of a monthly annuity. Using select disabled life functions, Mr. Craig's assumption would be written:

$$(96) \quad v^{\frac{1}{2}} \cdot {}_M p_{[x+\frac{1}{2}]}^i \cdot a_{[x+\frac{1}{2}]+\frac{1}{2}}^i = a_{[x+\frac{1}{2}]}^{i(12)} \text{ approximately.}$$

Substituting a continuous annuity for the monthly annuity, we have:

$$(97) \quad v^{\frac{1}{2}} \cdot {}_M p_{[x+\frac{1}{2}]}^i \cdot a_{[x+\frac{1}{2}]+\frac{1}{2}}^i = \bar{a}_{[x+\frac{1}{2}]}^{i(12)} - \frac{1}{2}d \text{ approximately.}$$

This correction was introduced in the premium formula used in calculating the Class (3) disability premiums given by Mr. Henderson in T. A. S. A., XXX, 463. It will be assumed, in the formulas which follow, either that a monthly disability annuity is applicable to both the disability annuity and the waiver of premium benefit or that a continuous annuity is used for the waiver benefit.

The net single premium for a monthly income benefit under an n -year endowment, $x+n=u$, the income ceasing at maturity is, for disability occurring before age y , $u \geq y$,

$$(98) \quad \frac{\sum_{t=x}^{t=u-1} v^{\frac{1}{2}} \cdot D_x^{aa} \cdot r_x \cdot a_{[x+\frac{1}{2}]:u-x-\frac{1}{2}}^{i(12)}}{D_x^{aa}} = \frac{{}_u M_x^r}{D_x^{aa}}$$

where $v^{\frac{1}{2}} \cdot D_x^{aa} \cdot r_x \cdot a_{[x+\frac{1}{2}]:u-x-\frac{1}{2}}^{i(12)} = {}^u C_x^r$

and $\sum_{t=x}^{t=u-1} {}^u C_t^r = {}^u M_x^r$

If $u > y$ the summation stops at age $y-1$ and we have

$$(99) \quad \frac{\sum_{t=x}^{t=y-1} v^{\frac{1}{2}} \cdot D_t^{aa} \cdot r_t \cdot a_{[t+\frac{1}{2}]:u-t-\frac{1}{2}}^{i(12)}}{D_x^{aa}} = \frac{{}_y M_x^r}{D_x^{aa}}$$

Many of the modern clauses have provided for retroactive benefits, i.e., upon receipt of proof of total disability lasting the required time, the disability benefits would commence as of the

beginning of disability, the monthly income being usually payable for each completed month of disability. If the waiting period is f months, the benefit under an Ordinary Life policy in event of disability occurring before age y is equal to a single payment, at the end of the waiting period, of $f - 1$ months' income and

the waiver of $\frac{f}{12}$ of an annual premium together with a monthly

disability annuity-due and waiver of future premiums. The net single premium per unit of annual benefit is

$$(100) \quad \frac{{}_vM'_x \cdot \frac{f-1}{12} + {}_wM'_x}{D_x^{aa}} \text{ for the income, and}$$

$$(100a) \quad \frac{{}_vM'_x \cdot \frac{f}{12} + {}_wM'_x}{D_x^{aa}} \text{ for the waiver of premiums.}$$

The standard provisions which were adopted in 1930 required that the minimum waiting period should be four months, that premiums falling due after disablement and within at least six months from notice of claim should be waived, and that no income should become payable until the end of the fourth month. Under a standard four months' clause the benefit would be equal to waiver of one-third of an annual premium at the end of the four months' qualification period together with a monthly disability annuity-due and waiver of future premiums. As the waiver benefit only is retroactive, the premiums should be calculated for the waiver and income portions separately. As a matter of fact this is usually done in the premium calculations for all benefits since the waiver benefit is frequently sold by itself as well as in conjunction with the income benefit.

The student should have no difficulty in determining the corresponding annual premiums; two examples, however, are now given.

The annual premium payable under an Ordinary Life contract to provide a monthly annuity during disability in event

of disability before age y , the disability premium ceasing at age y , is

$$(101) \quad \frac{{}_y^u M'_z}{N_x^{aa} - N_y^{aa}}$$

The annual premium payable under an n -year endowment to provide a monthly disability annuity ceasing at maturity, in event of disability before age y , is

for $u \leq y$, where $x + n = u$,

$$(102) \quad \frac{{}_y^u M'_z}{N_x^{aa} - N_u^{aa}}$$

(the disability premium ceases at maturity), and

for $u > y$,

$$(102a) \quad \frac{{}_y^u M'_z}{N_x^{aa} - N_y^{aa}}$$

(the disability premium ceases at age y).

CHAPTER X.

APPROXIMATIONS IN CALCULATING PREMIUMS FOR
MODERN BENEFITS

As already stated the same commutation columns are used in present day calculations for both the waiver of premium and monthly income benefits. In addition to any error involved in the assumption of (96) or (97) used in connection with the waiver under Ordinary Life policies, there is also an overstatement under endowment and limited payment policies due to the monthly temporary disabled life annuity running for one-half year longer than is necessary, if premiums are payable annually.

The correct expression for the single premium to provide the waiver benefit under an n -payment policy, $x + n = u \approx y$, is

$$(103) \quad \frac{\sum_{z=x}^{x=u-2} v \cdot D_z^{aa} \cdot r_z \cdot {}_{1/2}p_{[z+1/2]}^t \cdot a_{[z+1/2]+1/2}^{t(u-z-1)}}{D_x^{aa}}$$

Assuming that the relation in (96) is true, (103) becomes

$$(104) \quad \frac{\sum_{z=x}^{x=u-2} v {}_{1/2} \cdot D_z^{aa} \cdot r_z \cdot a_{[z+1/2]}^{t(12)} : u-z-1]}{D_x^{aa}}$$

The corresponding single premium for the income disability annuity is repeated again to facilitate comparison

$$(105) \quad \frac{\sum_{z=x}^{x=u-1} v {}_{1/2} \cdot D_z^{aa} \cdot r_z \cdot a_{[z+1/2]}^{t(12)} : u-z-1]}{D_x^{aa}}$$

The comparison of formulas (104) and (105) shows that the waiver expression does not include any coverage for the n th policy year and provides, in event of disability in the m th year, for the waiver of only $n - m$ premiums. The income expression, when used to evaluate the waiver benefit, provides coverage for the n th policy year and, when disability occurs in the m th year, for the waiver of $n - m + 1/2$ premiums. For example, in event of disability in the $(n - 1)$ th year, formula (104) includes the waiver of 1 premium and (105) the waiver of $1 1/2$ premiums. For disability occurring in the n th year the former provides no waiver and the latter the waiver of $1/2$ premium.

When we consider, however, that premiums are frequently payable semi-annually, quarterly and even monthly, it seems reasonable to assume that in such cases half a year's waiver benefit may be obtained, on the average, in event of disability in the n th or last policy year. The practical and conservative considerations prevail so that even in endowment and limited payment policies, the income annuity functions, sometimes with the continuous annuity adjustment, are applied to the waiver benefit. (The student is referred to the discussion on page 140 concerning the period during which the premium should be charged for the waiver benefit).

When a continuous annuity is used for evaluating the waiver benefit, we have

$$\begin{aligned} & \sum_{z=x}^{x-v-1} C_z^w \cdot \bar{a}_{z+\frac{1}{2}}^i \\ &= \sum_{z=x}^{x-v-1} C_z^w (a_{z+\frac{1}{2}}^{i(12)} - \frac{1}{24}) \text{ approximately} \\ &= {}_vM_x^w - \frac{1}{24} \cdot {}_vM_x^w \end{aligned}$$

The deduction from the basic formula to provide for the use of a continuous instead of a monthly annuity is therefore, under the Ordinary Life plan per \$1 of annual premium waived,

$$(106) \quad \frac{1}{24} \cdot \frac{{}_vM_x^w}{N_x^{aa} - N_y^{aa}}$$

The use of select disabled life annuities in the calculation of premiums for modern benefits increases greatly the number of commutation values required under policies with temporary disability benefits. However, the number of commutation values with temporary disabled life annuities has been limited by tabulating them according to terminal age of the temporary disabled life annuity and only for every fifth such age. Clauses with temporary disability benefits, such for example as the waiver of premium under an endowment or a limited payment policy, would have the temporary benefit expiring at the same age for any given age at issue irrespective of the year in which disability occurred. With commutation values prepared for all ages at disability and every fifth terminal age of the disability annuity, premiums may be calculated for every fifth age at issue. The values for intervening ages at issue are then obtained by interpolation.

CHAPTER XI.

USE OF MIXED LIFE TABLE INSTEAD OF ACTIVE
LIFE TABLE

There is another important assumption which has been made in recent years. The Hamza process assumes a standard mortality table to represent the mortality for the total group of mixed lives. The separation of the mixed lives into active and disabled is effected by an analysis of the disabled lives. Knowing the rate of disability and the rate of death among disabled lives, the "aa" and "ii" tables are compiled. In the pension fund notation it is obvious that the "aa" tables are necessary in order to provide D_x^{aa} for the computation of the pension fund values. However, as there were not sufficient data to obtain death and recovery rates among disabled lives in the earlier investigations of the experience under modern contracts, premiums were calculated on the assumption that the l_x values could be used, instead of l_x^{aa} , to get the exposure to disability in the premium formula as well as in obtaining the present value of the premiums payable. Various corrections were then introduced in the premium formula to offset the inaccuracy in this assumption. The student should note their significance.

Let us consider the expression for the annual premium to provide a monthly disabled life annuity under an Ordinary Life contract in event of disability occurring before age y . The true expression is

$$(107) \quad \frac{\sum_{x=z}^{x=y-1} v^{\frac{1}{12}} \cdot D_x^{aa} \cdot r_x \cdot a_{[z+\frac{1}{12}]}^{(12)}}{N_x^{aa} - N_y^{aa}}$$

Using mixed life functions instead of active life functions the expression becomes

$$(108) \quad \frac{\sum_{x=z}^{x=y-1} v^{\frac{1}{12}} \cdot D_x \cdot r_x \cdot a_{[z+\frac{1}{12}]}^{(12)}}{N_x - N_y}$$

This formula was used by J. D. Craig in T. A. S. A., XXVII, 39, except that aggregate disabled annuities were employed instead of select values.

There are two errors involved in (108):

(1) D_x in the numerator assumes that the mixed lives are exposed to risk of disability instead of only the active lives. This overstates the number assumed to become disabled and therefore overstates the premium.

(2) N_x in the denominator assumes that the mixed lives will pay the disability premium whereas only the active lives pay the disability premium. This results in an understatement of the disability premium.

The net effect may be determined empirically. Tests have shown that the net result is a slight overstatement (J. T. Phillips, T. A. S. A., XXX, 408).

The two errors apply only where a true rate of disability is determined. If the rate of disability is obtained from valuation data so that the exposures to risk of disability contain disabled as well as active lives, the resulting rate of disability should properly be applied against a mixed life function. In such a case, only the error in the denominator would be present.

An adjustment was introduced in the denominator of the premium formula appearing in the Inter-Company investigation of 1926. The Inter-Company expression was

$$(109) \quad \frac{\sum_{z=x}^{x-y-1} v^z \cdot D_z \cdot r_z \cdot a_{[z+\frac{1}{2}]}^{(12)}}{N_x - N_y - \sum_{z=x}^{x-y-1} v^z \cdot D_z \cdot r_z \cdot a_{[z+\frac{1}{2}]}^{(12)}}$$

This correction allows for the non-receipt of the disability premium from disabled lives (see below) by assuming that the waiver benefit covers the waiver of the disability premium as well as the insurance premium. It will be noted, however, that the disability premium is assumed to be waived for the balance of the insurance contract instead of to age y only.

As the payment of the disability premium ceases at age y , the Inter-Company formula contains too great a deduction in the denominator. H. S. Beers (T. A. S. A., XXVII, 279), in the following expression, allows for the fact that the disability premium ceases at age y :

$$(110) \quad \frac{\sum_{z=x}^{x-y-1} v^z \cdot D_z \cdot r_z \cdot a_{[z+\frac{1}{2}]}^{(12)}}{N_x - N_y - \sum_{z=x}^{x-y-1} v^z \cdot D_z \cdot r_z \cdot a_{[x+\frac{1}{2}]}^{(12)} \cdot \frac{1}{v^{x-y}}}$$

This can be seen as follows:

Let P_x^D be the disability premium, payable during activity

only, for a given disability benefit. The value of the disability premiums payable to age y , whether the insured is active or disabled, is

$$P_z^D \cdot \frac{N_z - N_y}{D_z}$$

The value of the disability premiums payable during disability is

$$P_z^D \cdot \frac{\sum_{t=z}^{x-y-1} v^t \cdot D_t \cdot r_t \cdot a_{[z+y] : \overline{y-z-M}}^{(12)}}{D_z}$$

using mixed life functions and a monthly annuity for the waiver evaluation. The value of the disability premiums payable during activity only is, therefore,

$$P_z^D \cdot \frac{N_z - N_y - \sum_{t=z}^{x-y-1} v^t \cdot D_t \cdot r_t \cdot a_{[z+y] : \overline{y-z-M}}^{(12)}}{D_z}$$

Equating this expression to the value of the benefit, we obtain expression (110) for the value of P_z^D . The student will note the analogy to the calculations for premium return benefits.

The effect of these various expressions on the final premium values is shown by the table now submitted, assuming the rate of disability involved is a true r_x . The premiums are per thousand of insurance and provide for the waiver of a gross premium equal to the American 3% net, loaded one-third, and the payment of a monthly income of \$10 in event of total disability occurring before age 60, with a three months' waiting period.

NET DISABILITY PREMIUMS—ORDINARY LIFE

Age	Premiums				Difference in Premiums			Ratio of Premiums		
	Form. (107) True	Form. (108) Craig	Form. (109) Inter-Company	Form. (110) Beers	(2)-(1)	(3)-(1)	(4)-(1)	(2)+(1)	(3)+(1)	(4)+(1)
	(1)	(2)	(3)	(4)				%	%	%
15	\$1.84	\$1.84	\$1.87	\$1.86	\$0.00	\$0.03	\$0.02	100.0	101.6	101.1
25	2.31	2.32	2.36	2.35	.01	.05	.04	100.4	102.2	101.7
35	3.11	3.12	3.19	3.17	.01	.08	.06	100.3	102.6	101.9
45	4.58	4.59	4.73	4.67	.01	.15	.09	100.2	103.3	102.0
55	7.39	7.40	7.71	7.53	.01	.32	.14	100.1	104.3	101.9

If the rate of disability involved has been determined from exposures containing disabled lives then Beers' formula, (110), becomes the true expression.

CHAPTER XII.

ADJUSTMENTS FOR VARIATION IN THE WAITING PERIOD

The standard provisions which were adopted in 1930 required a minimum waiting period of four months. As the principal experiences available upon which premium rates for the new clause could be based were for benefits with 90 day waiting periods, it was necessary to make adjustments in the premiums for the change in waiting period. Two methods may be used.

FIRST METHOD

ADJUSTMENT IN RATE OF DISABILITY AND CLAIM ANNUITY VALUES

The first method consists of obtaining adjusted rates of disability and adjusted claim annuities. Let r_x be the rate of disability derived from an experience with a waiting period of f months and r'_x the required rate of disability applicable to an experience with a waiting period of g months. If $g - f = m$ and if ${}_m q_{[x]}^f$ represents the rate of termination of the claims, either by recovery or death, in the first m months after the claim has been approved, then

$$r'_{x+\frac{1}{12}} = (1 - {}_m q_{[x+\frac{1}{12}}^f) r_x \text{ approximately}$$

(111) or ${}_m p_{[x+\frac{1}{12}}^f \cdot r_x$

A similar adjustment is necessary in the claim annuities. If accented symbols are used to distinguish functions applicable to the g months' waiting period, we have

$$(112) \quad a_{[x+\frac{1}{12}]}^{g(12)} = \frac{a_{[x+\frac{1}{12}}^{f(12)} - \frac{1}{12} \sum_{t=0}^{t=m-1} v^{\frac{t}{12}} \cdot {}_t p_{[x+\frac{1}{12}}^f}{v^{\frac{m}{12}} \cdot {}_m p_{[x+\frac{1}{12}}^f}$$

There is little variation in the value of claim annuities by age at disability and also little difference in the rate of disability for any two successive ages. We may therefore say, without any great loss in accuracy, that

$$\text{and} \quad r_{x+m}^* = r_x^* \\ a_{[x+\frac{1}{2}+\frac{m}{12}]}^{*t(12)} = a_{[x+\frac{1}{2}]}^{*t(12)}$$

With the adjusted rates of disability and adjusted claim annuity values, new commutation columns may be constructed applicable to a benefit with a g months' waiting period.

When the difference between the old and new waiting period is small, say less than one year, other approximations, as described in the following paragraphs, may safely be made which will eliminate the necessity of calculating new commutation columns.

During the early months of disability the monthly termination rate is very high at first but decreases rapidly with increase in duration of the claim. On the other hand the termination rate in these early months is fairly constant with age at disability. The assumption may therefore be made that ${}_m p_{[x+\frac{1}{2}]}^t$ is constant for all ages. Also where m is small the interest element may be neglected as its effect would likely be less than the variation in the values of ${}_m p_{[x]}^t$ from the constant value chosen. If the effect of the interest factor is appreciable it may be retained, however, without affecting the method of approximation.

As the claim annuity value also varies very little by age at disability, it is possible to determine an average ratio between $a_{[x]}^{t(12)}$ and $a_{[x]}^{*t(12)}$ which may be used for all ages at disability. A ratio might also apply to temporary annuities, though perhaps with not the same degree of accuracy.

If h is the constant value taken for ${}_m p_{[x]}^t$ and k the constant value for the ratio $\frac{a_{[x]}^{*t(12)}}{a_{[x]}^{t(12)}}$ for all values of x and n , the new commutation columns would be related as follows:

$$\begin{aligned} C_x^{*r} &= v^x \cdot D_x^{*a} \cdot r_x^* \text{ approximately} \\ &= v^x \cdot D_x^{*a} \cdot h \cdot r_x \\ &= h \cdot C_x^r \\ {}_v M_x^{*r} &= \sum C_x^{*r} \\ &= \sum h \cdot C_x^r \\ &= h \cdot {}_v M_x^r \\ {}^u C_x^{*r} &= C_x^{*r} \cdot a_{[x+\frac{1}{2}]}^{*t(12)} \\ &= h \cdot k \cdot {}^u C_x^r \end{aligned}$$

In the same way

$${}_vM''_z = h \cdot k \cdot {}_vM'_z$$

$${}_vC''_z = h \cdot k \cdot {}_vC'_z$$

$$\text{and } {}_vM''_z = h \cdot k \cdot {}_vM'_z$$

Premium formula (101) applicable to a similar benefit but with a g months' waiting period instead of a period of f months would be

$$\begin{aligned} & \frac{{}_vM''_z}{N_z^{aa} - N_v^{aa}} \\ &= \frac{h \cdot k \cdot {}_vM'_z}{N_z^{aa} - N_v^{aa}} \end{aligned}$$

$= h \cdot k \cdot$ premium for the benefit with the f months' waiting period.

If the benefit involved is of the type covered by formula (100a) the net annual premium formula applicable to the g months benefit is

$$\begin{aligned} & \frac{{}_vM''_z \cdot \frac{g}{12} + {}_vM''_z}{N_z^{aa} - N_v^{aa}} \\ &= h \cdot \frac{{}_vM'_z \cdot \frac{g}{12} + k \cdot {}_vM'_z}{N_z^{aa} - N_v^{aa}}. \end{aligned}$$

SECOND METHOD

ADJUSTMENT IN PREMIUM FORMULA

In the second method of adjustment for change in waiting period, the same commutation columns are retained throughout, the adjustment being made in the premium formulas, based on the difference in benefits within the old and new waiting periods.

Let us assume, as before, that we have commutation tables based on an experience under a clause with an f months' waiting period and that it is desired to obtain premiums for a clause with a g months' waiting period, $g - f = m$.

It is also assumed that we are dealing with a monthly income and waiver benefit. The adjustments for the new premium will depend on whether or not any of the benefits are to be retro-active to the beginning of the waiting period.

For convenience the premium for non-retroactive benefits with an f months' waiting period will be assumed to be the basic premium. The basic net annual premium for the Ordinary Life plan, per unit of annual benefit, is

$$\frac{{}_v M'_x}{N_x^{aa} - N_v^{aa}} \quad \text{for the income benefit}$$

$$\text{and} \quad \frac{{}_v M'_x - \frac{1}{24} \cdot {}_v M'_x}{N_x^{aa} - N_v^{aa}} \quad \text{for the waiver benefit,}$$

using the continuous annuity to evaluate the premiums waivable. The waiting period adjustments applied to these basic premiums will also be on the assumption that a continuous annuity is applicable to the waiver benefit. The adjustments are as follows:

(1) *Non-retroactive benefit, m months' longer waiting period.*

If the new benefit contains no retroactive features, i.e., if the benefit provided is monthly income and waiver beginning as of the end of g months of total disability, the adjustment is quite simple. It is a deduction from the basic annual premium for the benefit received during m months in respect to all entrants, i.e., those qualifying for benefits at the end of f months.

The decrease in the income benefit per disabled life entrant for the income benefit for any policy year is

$$\frac{1}{12} \sum_{t=0}^{t=m-1} v^{t/12} \cdot {}_t p_{[x+1/12]}^{(12)} \\ = a_{[x+1/12]:\overline{m}|}^{(12)}, \text{ at date of disability}$$

under an f months' waiting period. The deduction from the basic annual premium per unit of annual income benefit is

$$(113) \quad \frac{\sum_{z=x}^{x+m-1} a_{[z+1/12]:\overline{m}|}^{(12)} \cdot C'_z}{N_x^{aa} - N_v^{aa}}$$

If $a_{[z+1/12]:\overline{m}|}^{(12)}$ be assigned the approximate, constant value a , the adjustment becomes

$$(114) \quad a \cdot \frac{{}_v M'_x}{N_x^{aa} - N_v^{aa}}$$

This adjustment is applicable to the income payments which are assumed to fall due on dates an integral number of months from the end of the waiting period. A continuous annuity is

used for the waiver benefit. The deduction from the basic annual premium for waiver per unit of annual premium waived is

$$(115) \quad \frac{\sum_{x=z}^{x=y-1} \bar{a}_{[x+1/2]:\overline{m}}^{(1)} \cdot C_x^r}{N_x^{aa} - N_y^{aa}} = a' \cdot \frac{{}_yM_x^r}{N_x^{aa} - N_y^{aa}}$$

where a' is the approximate, constant value of $\bar{a}_{[x+1/2]:\overline{m}}^{(1)}$

$$= \frac{\bar{a}_{[x+1/2]:\overline{m}}^{(12)} + a_{[x+1/2]:\overline{m}}^{(12)}}{2} \text{ approximately.}$$

(2) *Income and waiver benefits retroactive to commencement of disability.*

When either one or both of the benefits are made retroactive to commencement of disability additional adjustments are required. In order to obtain the annual premium for a monthly income benefit payable for each completed month of disability, for example, we must increase the basic premium, adjusted as in (114), by the annual premium to provide $g - 1$ monthly payments payable as a single benefit at the end of the waiting period of g months, in addition to the annuity payment then due, to those who complete that many months of total disability. The addition to the annual premium per unit of annual benefit is

$$(116) \quad \frac{\frac{g-1}{12} \cdot \sum_{x=z}^{x=y-1} v^{\overline{m}} \cdot {}_m p_{[x+1/2]}^{(1)} \cdot C_x^r}{N_x^{aa} - N_y^{aa}}$$

If $v^{\overline{m}} \cdot {}_m p_{[x+1/2]}^{(1)}$ be assigned the approximate, constant value b , the addition becomes

$$(117) \quad \frac{g-1}{12} \cdot b \cdot \frac{{}_yM_x^r}{N_x^{aa} - N_y^{aa}}$$

The similar adjustment applicable to the waiver benefit, if it is retroactive to commencement of disability, is an addition of

$$(118) \quad \frac{g}{12} \cdot b \cdot \frac{{}_yM_x^r}{N_x^{aa} - N_y^{aa}}$$

The reason for the difference in adjustment for retroactive income and waiver benefits is clear. If, at the end of a g

months' waiting period, the income benefit is payable for each completed month of disability, $g-1$ payments will be made in addition to the one payment then due under a non-retroactive clause. On the other hand if the waiver benefit is retroactive, waiver of premiums for the g months is given in addition to waiver of premiums falling due after the g months, as provided by the non-retroactive clause.

Alternative Treatment

Instead of the above adjustments under (1) and (2) we may consider the adjustments applied to the basic premium to consist of the following:

(i) A deduction for benefits in the m months in respect to those who complete f months of total disability but do not complete g months, i.e., those whose disabilities terminate in the period between the old and new waiting periods,

(ii) An addition for $f-1$ months' benefit in the case of a retroactive income and for f months' benefit in the case of a retroactive waiver of premium, payable in a single sum at the end of g months to all who complete g months of total disability, and

(iii) A deduction for m months' benefit in respect to those who complete g months of total disability for any benefit which commences only as of the end of the g months, i.e., for any benefit which is not retroactive.

The net of these three adjustments differs slightly from that given under (1) and (2) by the interest on the m payments made to those surviving the g months, i.e., those completing g months of total disability. In the alternative treatment it is assumed that, in respect to those surviving g months, the $g-1$ retroactive monthly income payments, for example, will be payable as follows: $f-1$ of them in one sum at the end of g months, the remaining m payments, having been paid monthly beginning at the end of f months. This involves a slight overstatement of the premium to the extent of the interest on these m payments as both the $f-1$ and the m payments are payable in a single sum of $m+f-1$, or $g-1$ payments at the end of g months. This difference in interest is unimportant as in actual practice the

interest element is ignored entirely. On the other hand, by the alternative treatment, only the necessary net additions or deductions are made. For example, if the waiver benefit is retroactive, using the adjustments given under (1) and (2) eliminates all benefits for the m months and then adds back the waiver benefit in those months for the survivors at end of g months. With adjustments (i), (ii) and (iii) we deduct benefits in the m months only for terminations in the m months, then make an additional deduction in the same period for any non-retroactive benefit in respect to the survivors at the end of the g months.

In practice these various adjustments are often made by the use of so-called continuance tables without any interest element as the difference in waiting periods is usually small. Monthly disabled life continuance tables are constructed by taking a radix of 1,000 or 10,000 lives who have just completed f months of total disability, then applying the monthly termination rate among disabled lives to obtain the monthly terminations and the number of disabled life survivors at the end of successive months. From such a table the adjustment factors are readily obtained. The table on page 176 is based upon the experience under a clause with a three months' waiting period (i.e., $f = 3$) and with non-retroactive benefits. With retroactive benefits the rate of disability is usually higher and the termination rate heavier. The saving through the heavier termination rate would not ordinarily be sufficient to offset the additional cost under the higher rate of disability so that, if premiums are to be calculated for a retroactive benefit, care should be taken to make some allowance for the higher claim rate under such a clause.

As the trend has been towards a proportionately greater number of claims being presented as benefits are made more liberal, the reverse might hold when the waiting period is increased. The greater waiting period should tend to reduce the cost over and above any adjustment suggested above. This reduction would be a partial offset to any increase in cost for the addition of retroactive benefits when the basic data is from a non-retroactive experience. It may be considered sufficient when the waiver benefit only is retroactive. If the income benefit is retroactive it would be necessary to use data based on a retroactive experience or to make allowance therefor.

There is another element in the basic experience which should be considered. Many policies provided benefits either for permanent disability or for total disability lasting, say, three months. Different companies treated such a clause in different ways. In some companies a large percentage of their claims were approved on a "permanent" basis before the expiration of the waiting period. In others almost all of their claims were approved on a "total" basis. As the relative proportions of total and of permanent claims would have an effect on the rate of disability and the value of the disabled life annuity, the proportions existing in the basic data should be known. If the proportion covered by the experience differed greatly from that anticipated under the new benefit an adjustment might be considered advisable. For example, if, in the experience covered by the basic data with a three months' waiting period, one-half of the claims were approved on a permanent basis with benefits granted from date of disablement, and if the new clause were to provide no benefits prior to the end of the waiting period, the basic disabled life annuities would overstate the cost by about three months' benefits in one-half the cases. It would not be safe to assume, however, that the disabled life annuities could be reduced by the value of $1\frac{1}{2}$ months' benefit as some of those approved on a permanent basis would die before the expiration of the waiting period. It would probably be necessary to further analyze the basic material for a measure of the correction necessary. The effect on the rate of disability would be to treat some of the claims as occurring at an earlier age. This would slightly overstate the cost of the benefits under the new clause but, as the usual experience of the companies has been that disability premiums have nearly always been inadequate, an element which slightly overstates the cost may be disregarded.

A sample disabled life continuance table based on an experience under a non-retroactive benefit with a three months' waiting period is now given for all ages at disability combined, together with the calculation of various values obtained from the table:

DISABLED LIFE CONTINUANCE TABLE (from a three months' non-retroactive experience)

(1) Month of disability after expiration of 3 months' waiting period	(2) Monthly termination rate among disabled lives	(3) Number of disabled lives surviving in contingence table at beginning of nth month of disability	(4) Number of lives terminating from contingence table in nth month of disability	(5) $\sum_{n=1}^m \frac{l_{m-1}^t}{12}$	(6) $\sum_{n=2}^{m+1} \frac{l_{n-1}^t}{12}$	(7) $a_{\overline{m} }^{(12)} = \frac{1}{12} \cdot \frac{(5)^{**}}{10,000}$	(8) $a_{\overline{m} }^{(12)} = \frac{1}{12} \cdot \frac{(5)^{**}}{10,000}$	(9) $\bar{a}_{\overline{m} }^t = \frac{(7)+(8)^{**}}{2}$
m	$\left \frac{d_{m-1}^t}{l_{m-1}^t} \right $	$\frac{l_{m-1}^t}{12}$	d_{m-1}^t	$\sum_{n=1}^m \frac{l_{n-1}^t}{12}$	$\sum_{n=2}^{m+1} \frac{l_{n-1}^t}{12}$	$a_{\overline{m} }^{(12)} = \frac{1}{12} \cdot \frac{(5)^{**}}{10,000}$	$a_{\overline{m} }^{(12)} = \frac{1}{12} \cdot \frac{(5)^{**}}{10,000}$	$\bar{a}_{\overline{m} }^t = \frac{(7)+(8)^{**}}{2}$
1	.095	10,000	950	10,000	9,050	.0833	.0754	.0794
2	.094	9,050	851	19,050	17,249	.1588	.1437	.1513
3	.093	8,199	763	27,249	24,685	.2271	.2057	.2164
4	.091	7,436	677	34,685	31,444	.2890	.2620	.2755
5	.086	6,759	581	41,444	37,622	.3454	.3135	.3295
6	.079	6,178	488	47,622	43,312	.3968	.3609	.3789
7	.073	5,690	415	53,312	48,587	.4443	.4049	.4246
8	.067	5,275	353	58,587	53,509	.4882	.4459	.4671
9	.061	4,922	300	63,509	58,131	.5292	.4844	.5068
10	.057	4,622	263	68,131	62,490	.5678	.5208	.5443
11	.053	4,359	231	72,490	66,618	.6041	.5552	.5797
12	.050	4,128	206	76,618	70,540	.6385	.5878	.6132

** No allowance for interest.

* $f = 3$

Values of the approximate constants referred to in the various adjustments above, according to the foregoing continuance table, are now given for new waiting periods of from 4 to 12 months.

APPROXIMATE CONSTANTS FOR WAITING PERIOD ADJUSTMENTS

Excess, in months, of new waiting period over original waiting period of 3 months m	Constant ($f=3$; $g=f+m$)			
	Income Benefit		Waiver Benefit	
	a	$\frac{g-1}{12} \cdot b$	a'	$\frac{g}{12} \cdot b$
1	.0833	.2263	.0794	.3017
2	.1588	.2733	.1513	.3417
3	.2271	.3099	.2164	.3718
4	.2890	.3380	.2755	.3943
5	.3454	.3604	.3295	.4119
6	.3969	.3794	.3789	.4268
7	.4443	.3956	.4246	.4396
8	.4882	.4102	.4671	.4512
9	.5292	.4237	.5068	.4622

where

$$a = a_{[s+M] : \overline{m}}^{(12)}$$

$$a' = a_{[s+M] : \overline{m}}^{(1)}$$

$$b = v^{\overline{m}} \cdot m p_{[s+M]}^f$$

An illustration of the details of calculating these constants is now given:

Let us take the constants a and a' for $m=2$. a represents the value in any policy year of the deduction on account of monthly income payments not received in the interval between three months and five months after disablement. Among 10,000 entrants under the three months' waiting period 10,000 claims lose one monthly payment for the fourth month of disability and 9,050 of them lose a payment for the fifth month. Therefore the value per unit of annual income per entrant is

$$\frac{\frac{1}{12} \times 10,000 + \frac{1}{12} \times 9,050}{10,000} = .1588$$

a' represents the corresponding deduction under the waiver benefit. The 950 claims terminating in the fourth month after disablement and the 851 claims terminating in the fifth month lose, on the average, only one-half month's waiver benefit in the

month in which the claim terminates. Therefore the value per unit of annual premium per entrant is

$$\frac{\frac{1}{12}(10,000 - \frac{1}{2} \times 950) + \frac{1}{12}(9,050 - \frac{1}{2} \times 851)}{10,000} = .1513$$

The same result is obtained from the expression,

$$\frac{\frac{1}{2}(\frac{1}{12} \times 10,000 + \frac{1}{12} \times 9,050) + \frac{1}{2}(\frac{1}{12} \times 9,050 + \frac{1}{12} \times 8,199)}{10,000}$$

which directly corresponds to the column $\bar{a}_{\overline{m}|}$ in the table on page 176.

The student will find it instructive to reproduce other values given in the table.

Some examples of waiting period adjustments, based on the above tables, will now be given.

The net annual premium to provide, under an Ordinary Life policy, waiver of premium and a monthly income in event of total disability occurring before age 60, lasting for four months, the waiver benefit only to be retroactive to the commencement of disability, is ($f = 3$, $g = 4$, $m = 1$)

$$(P+I) \frac{{}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}} + \left\{ -\frac{P}{24} - a \cdot I - a' \cdot P + \frac{4}{12} \cdot b \cdot P \right\} \frac{{}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}}$$

($-\frac{P}{24}$ being the adjustment to the monthly annuity to give a continuous annuity for the waiver benefit)

$$= (P+I) \frac{{}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}} + \left\{ -.0417P - .0833I - .0794P + .3017P \right\} \frac{{}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}}$$

$$(119) = P \cdot \frac{{}_{60}M'_x + .1806 {}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}} + I \cdot \frac{{}_{60}M'_x - .0833 {}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}}$$

when P is the annual premium waived and I is the yearly disability income.

The above formulas compare with those given by Robert Henderson in T. A. S. A., XXX, 463, for the 3% premiums, viz.,

$$(120) = P \cdot \frac{{}_{60}M'_x + .1835 {}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}} + I \cdot \frac{{}_{60}M'_x - .0833 {}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}}$$

the values of N_x according to the American Experience table being used for N_x^{aa} .

Mr. Henderson's constant of .1835 is based upon a disability termination rate of .085 in the first month of disability instead of the .095 given in the table on page 176. It is left to the student to verify Mr. Henderson's formula.

The net annual premium for a similar benefit but with a six months' waiting period, only the waiver benefit being retroactive, as above, the income starting at the end of the waiting period, is ($f = 3, g = 6, m = 3$)

$$(121) \quad (P+I) \frac{{}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}} + \left\{ -\frac{P}{24} - a \cdot I - a' \cdot P + \frac{6}{12} b \cdot P \right\} \frac{{}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}} \\ = P \cdot \frac{{}_{60}M'_x + .1137 {}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}} + I \cdot \frac{{}_{60}M'_x - .2271 {}_{60}M'_x}{N_x^{aa} - N_{60}^{aa}}$$

The premium formula used by many companies to obtain net premiums for the benefit which was introduced in 1932, with a six months' waiting period and coverage to age 55, was

$$(122) \quad P \cdot \frac{{}_{55}M'_x + .1108 {}_{55}M'_x}{N_x^{aa} - N_{55}^{aa}} + I \cdot \frac{{}_{55}M'_x - .2250 {}_{55}M'_x}{N_x^{aa} - N_{55}^{aa}}$$

This benefit is the same as that covered by formula (121) except for the lower age limit on the coverage. The difference in the constants of the two formulas is due to formula (122) being based on a monthly termination rate of .1060 for the first month of disability, .0981 for the second month and .0910 for the third month (see the interpolation of the Class (3) termination rates given by J. T. Phillips in T. A. S. A., XXXIII).

The student will find it a useful exercise to develop formula (122) from the termination rates given, and to determine formulas for other plans. For example, the net premium for the retroactive waiver of premium benefit, with a six months' waiting period and coverage to age 55, under an n -year endowment, $x + n = u < 55$, is

$$(123) \quad P \cdot \frac{{}_{55}M'_x + .1108 ({}_{55}M'_x - {}_{55}M'_u)}{N_x^{aa} - N_u^{aa}}$$

Mixed life functions were used in the calculation of premiums based upon formulas (122) and (123) in the same manner as was indicated in connection with formula (120).

*Valuation of Claims and Comparison of Actual
and Expected Claims*

The second method of determining premiums for a benefit with a waiting period different from that in the basic data, does not directly produce a modified rate of disability or a modified claim annuity value. Although such modified values are not required in order to determine the premiums, it may be necessary that modified claim annuity values be obtained for the subsequent valuation of claims thereunder, while a modified rate of disability should be calculated in order to see how the claim rate experienced compares with that assumed in the premium calculation.

The modified values of $a_{[x]}^{(12)}$ and r_x may be determined in the manner described under the first method of adjusting for variation in waiting period.

Even if such modified values are determined, it is usually preferable to make the premium calculations by the second method, thus avoiding the recalculation of the values for the various basic commutation columns.

CHAPTER XIII.

RESERVES

GENERAL DISCUSSION—RESERVES ON ACTIVE LIVES

GENERAL DISCUSSION

In the discussion of Methods of Accounting (Part I, Chapter XII) it was shown that various reserve items such as active life reserves, disabled life or claim reserves, reserves for unreported claims, and for claims in course of settlement are carried on account of disability benefits. The first two reserves are the principal disability liability items and are ascertained by applying tabular reserve factors to the amount of insurance with disability benefits or to the amount of the benefit. It is with the determination of these reserve factors that we are now concerned.

It has already been pointed out (Part II, Chapter I) that the active and disabled life reserves could be determined in one item by using a sickness table method. The convention annual statement calls, however, for the separation of disability reserves into

“Extra reserve for total and permanent disability benefits . . .” } (the active life reserve)
and

“The present value of amounts incurred but not yet due for total and permanent disability benefits . . .” } (the claim reserve)

In order to enable the student to obtain a clear idea of the distinction between these two reserves and to readily understand the principles underlying them, the following demonstration, presented by Arthur Hunter in T. A. S. A., XII, 63, is given, in connection with a policy containing the waiver of premium benefit (only net premiums being waived):

“For clearness of analysis, let us imagine that there are three separate and independent companies carrying on the successive stages of this kind of insurance, as follows:

"Company A, which is a plain life insurance company and has nothing to do with the 'waiver of premium' feature;

"Company B, which guarantees to each person when he insures in Company A that if he shall thereafter become disabled it will purchase for him in Company C an annuity equal to the net premium which he pays to Company A;

"Company C, which, whenever an insured in Company A becomes disabled, receives from Company B the price of the annuity.

"We first ask the question: What reserve should Company C hold on the disabled lives which compose its entire membership? It is evident that the reserve it should hold on any such life must be equal to the single premium for the annuity, upon the disabled life mortality table.

"Next comes the question: What reserve should Company B hold on its membership? It would seem that such reserve should be the present value of a risk equal to the single premium which it would have to pay to Company C in case the life should become disabled in any subsequent year, less the present value of the net premium it must charge the insured for this indemnity."

The reserve to be held by Company A is the regular life insurance reserve only, i. e., American 3 per cent, $3\frac{1}{2}$ per cent or whatever reserve is carried. The life insurance reserve is based on a mixed life table and is the same in practice whether the life be active or disabled. In other words, the disability status has no effect upon the life insurance reserve. This is still true under the instalment type of benefit, where the face amount is reduced by the amount of instalments paid, except that the regular life insurance reserve is usually carried on the reduced amount only.

RESERVE ON ACTIVE LIVES

As indicated above, the disability reserve on active lives may be viewed prospectively, thus:

$$(124) \quad \left\{ \begin{array}{l} \text{Present value} \\ \text{of benefit} \end{array} \right\} \text{ minus } \left\{ \begin{array}{l} \text{Present value of} \\ \text{future net premiums} \end{array} \right\}$$

When two or more benefits are combined in the same policy, such as the waiver of premium and a disability annuity, the reserve to be held is the sum of the reserves under each of the disability benefits.

Another general expression frequently used for the active life reserve is:

$$(125) \quad \text{Present value of } \left\{ \begin{array}{l} \text{(Future net premiums, at attained} \\ \text{age, for benefit)} \\ \text{minus} \\ \text{(Future net premiums payable)} \end{array} \right\}$$

or in symbols $(P_{+t} - P) a_{+t}$.

The formulas now to be given for reserves on active lives do not have any meaning when a duration of the policy is reached which makes it impossible for an active life to become entitled to the disability benefit. The terminal reserve in such cases should be taken as zero.

Formulas According to Hamza Method Notation

Waiver of premium benefit. The terminal reserve under an Ordinary Life policy at the end of the t th policy year for the waiver of premium benefit in event of total and permanent disability before age y is, assuming premiums payable annually,

$$(126) \quad P \cdot a_{x+t}^{at} - P_x^D \cdot a_{x+t}^{aa} \cdot \overline{v-x-1}$$

where P is the premium waived and P_x^D the net disability premium.

Substituting the value of a_{x+t}^{at} according to formula (44) the above formula, expressed in the Hamza method notation, becomes

$$(127) \quad P \cdot \frac{N_{x+t}^{tt} - D_{x+t}^{tt} \cdot a_{x+t}^t}{D_{x+t}^{aa}} - P_x^D \cdot \frac{N_{x+t}^{aa} - N_y^{aa}}{D_{x+t}^{aa}}$$

The same result may be obtained retrospectively, the active life reserve being equal to

$$(128) \quad \left\{ \begin{array}{l} \text{Accumulated value} \\ \text{of premiums} \\ \text{received} \end{array} \right\} \text{ minus } \left\{ \begin{array}{l} \text{Accumulated value of} \\ \text{benefits or claims} \\ \text{incurred} \end{array} \right\}$$

or in symbols

$$(129) \quad P_x^D \cdot \frac{N_x^{aa} - N_{x+t}^{aa}}{D_{x+t}^{aa}} - P \cdot \frac{\sum_{z=x}^{x+t-1} v \cdot D_z^{aa} \cdot p_z^{at} \cdot a_{z+1}^t}{D_{x+t}^{aa}}$$

It is left to the student to prove the equivalence of formulas (127) and (129).

Formula (126) may also be expressed in another form which is suitable for use in the calculation of reserves. Substituting for a_{x+t}^{aa} the value according to (42) and using $\frac{D_{x+t}^{aa}}{D_{x+t}^{aa}}$ in place of $\frac{D_{x+t}^{aa}}{D_{x+t}^{aa}}$ the reserve expression becomes

$$(130) \quad P \left[(a_{x+t} - a_{x+t}^{aa}) + \frac{D_{x+t}^{aa}}{D_{x+t}^{aa}} (a_{x+t} - a_{x+t}^t) \right] - P_x^D \cdot a_{x+t}^{aa} : \overline{y-x-t}$$

The reserve formulas for limited payment life and endowment policies corresponding to formula (127) for the Ordinary Life plan are as follows, where n is the number of years for which premiums are payable under the limited payment or endowment policy, y the limiting age of the coverage and P_x^D the net disability premium:

for $x + n < y$,

$$(131) \quad \frac{P(N_{x+t}^{aa} - N_{x+n}^{aa} - D_{x+t}^{aa} \cdot a_{x+t}^t : \overline{n-t}) - P_x^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}$$

for $x + n \geq y$,

$$(132) \quad \frac{P(N_{x+t}^{aa} - N_{x+n}^{aa} - D_{x+t}^{aa} \cdot a_{x+t}^t : \overline{n-t}) - P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

The active life reserve for the waiver of premium benefit disappears, of course, when the insured attains the earlier of the ages $x + n$ and y ; or prior thereto, should he be disabled before attaining such age. These reserves are frequently negative and when this happens the company holds for valuation purposes one-half of the net annual premium for the benefit.

Annuity during disability.

Let P_x^D be the net annual premium for the benefit.

Let K be the annuity; that is, the amount payable each year.

Let n be the number of years premiums are payable under a limited payment life policy, or the term of an endowment policy, an n -payment n -year endowment only being considered, and

Let t be the duration for which the reserve for the disability benefit is desired.

Where the annuity is payable annually and the first payment falls due one year after the receipt of proof of disability, the

active life reserve on an Ordinary Life policy, at the end of t years in force, is as follows:

$$(133) \quad \frac{K (\sum_{i=0}^{t-1} v^i \cdot D_{x+i}^{aa} \cdot r_x \cdot a_{x+i}^t) - P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

For a limited payment life policy, where $x+n \leq y$, the premium and the benefit are precisely the same as under an Ordinary Life policy, consequently the reserve is the same; but where $x+n < y$ the reserve is

$$(134) \quad \frac{K (\sum_{i=0}^{t-1} v^i \cdot D_{x+i}^{aa} \cdot r_x \cdot a_{x+i}^t) - P_x^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}$$

When t is equal to or greater than n the latter part of this expression vanishes.

For an endowment policy with an income benefit which ceases when the policy matures, the active life reserve, where $x+n \leq y$, is

$$(135) \quad \frac{K (\sum_{i=0}^{t-1} v^i \cdot D_{x+i}^{aa} \cdot r_x \cdot a_{x+i}^t : x+n-1) - P_x^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}$$

where $x+n > y$,

$$(136) \quad \frac{K (\sum_{i=0}^{t-1} v^i \cdot D_{x+i}^{aa} \cdot r_x \cdot a_{x+i}^t : x+n-1) - P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

Payment of face amount in instalments, balance, if any, at death or maturity. The following formulas are applicable when the first instalment becomes due one year after the anniversary next succeeding the receipt of proof of disability.

Let k represent the number of annual instalments, $1/k$ being the amount of each instalment.

Let P_x^D represent the net annual premium for the benefit.

Let n be the number of years premiums are payable under a limited payment life policy or the term of an endowment policy, an n -payment n -year endowment only being considered.

Let d represent the discount on 1 payable one year hence, and

Let t be the duration for which the reserve is desired.

The reserve on an active life at the end of the t th policy year is as follows, for an Ordinary Life policy:

$$(137) \quad \frac{\left[\sum_{i=0}^{t-1} v^i \cdot D_{x+i}^{aa} \cdot p_x^{a'} \cdot \frac{d}{k} \left(\frac{S_{x+i+2}^t - S_{x+i+2}^t}{D_{x+i+1}^{aa}} \right) \right] - P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

For a limited payment life policy, when $x + n \geq y$, the reserve is the same as under an Ordinary Life policy; but when $x + n < y$, the reserve is

$$(138) \quad \frac{\left[\sum_{i=x+t}^{z=y-1} v \cdot D_i^{aa} \cdot p_i^{at} \cdot \frac{d}{k} \left(\frac{S_{i+2}^t - S_{i+k+2}^t}{D_{i+1}^t} \right) \right] - P_x^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}$$

For an endowment policy, when $x + n \geq y$, the reserve is

$$(139) \quad \frac{\left[\sum_{i=x+t}^{z=y+n-3} v \cdot D_i^{aa} \cdot p_i^{at} \cdot \frac{d}{k} \left(\frac{S_{i+2}^t - S_{i+m+1}^t - (m-1)N_{i+n}^t}{D_{i+1}^t} \right) \right] \frac{1}{D_{x+t}^{aa}} - \frac{P_x^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}}{D_{x+t}^{aa}}$$

where $m = k + 1$ or $x + n - z - 1$ whichever is less. When $x + n > y$, the reserve is

$$(140) \quad \frac{\left[\sum_{i=x+t}^{z=y-1} v \cdot D_i^{aa} \cdot p_i^{at} \cdot \frac{d}{k} \left(\frac{S_{i+2}^t - S_{i+m+1}^t - (m-1)N_{i+n}^t}{D_{i+1}^t} \right) \right] \frac{1}{D_{x+t}^{aa}} - \frac{P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}}{D_{x+t}^{aa}}$$

Where $x + n = y + 1$ the summation in formula (140) ceases with the term $z = x + n - 3$. See paragraph following formula (66c) in Part II, Chapter VIII.

Maturity instalment benefit. The same assumptions as in the previous section are applicable except that the first instalment is taken as becoming due at the end of the policy year in which disability occurs.

The active life reserve at the end of t years for an Ordinary Life policy is as follows:

$$(141) \quad \frac{[\sum_{i=x+t}^{z=y-1} v \cdot D_i^{aa} \cdot p_i^{at} \cdot d \cdot a_{i+1}^t] - P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

For a limited payment life policy the active life reserve is the same as under an Ordinary Life policy when $x + n \geq y$; but when $x + n < y$ it is

$$(142) \quad \frac{[\sum_{i=x+t}^{z=y-1} v \cdot D_i^{aa} \cdot p_i^{at} \cdot d \cdot a_{i+1}^t] - P_x^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}$$

The reserve on active lives for an endowment policy is, when $x + n \geq y$,

$$(143) \frac{\left[\sum_{i=x+t}^{x+n-2} v \cdot D_z^{aa} \cdot p_z^{at} \cdot d \left(\frac{N_{z+1}^t - N_{z+n}^t}{D_{z+1}^t} \right) \right] - P_z^D (N_{z+t}^{aa} - N_{z+n}^{aa})}{D_{z+t}^{aa}}$$

when $x + n > y$,

$$(144) \frac{\left[\sum_{i=x+t}^{y-1} v \cdot D_z^{aa} \cdot p_z^{at} \cdot d \left(\frac{N_{z+1}^t - N_{z+n}^t}{D_{z+1}^t} \right) \right] - P_z^D (N_{z+t}^{aa} - N_y^{aa})}{D_{z+t}^{aa}}$$

Formulas According to Pension Fund Notation

Pension fund notation applicable to older types of benefits.
Using the general formula (124) the reserve on an active life at the end of t years for the waiver of premium benefit under an Ordinary Life policy is

$$(145) \frac{P(r^a M_{z+t}^{at} - r^a M_y^{at}) - P_z^D (N_{z+t}^{aa} - N_y^{aa})}{D_{z+t}^{aa}}$$

This corresponds to formula (127).

The reserve formulas for limited payment life and endowment policies corresponding to formulas (131) and (132) are, where $x + n < y$,

$$(146) \frac{P\{r^a M_{z+t}^{at} - r^a M_{z+n}^{at} - N_{z+n}^t (r^a M_{z+t} - r^a M_{z+n})\} - P_z^D (N_{z+t}^{aa} - N_{z+n}^{aa})}{D_{z+t}^{aa}}$$

where $x + n \leq y$,

$$(147) \frac{P\{r^a M_{z+t}^{at} - r^a M_y^{at} - N_{z+n}^t (r^a M_{z+t} - r^a M_y)\} - P_z^D (N_{z+t}^{aa} - N_y^{aa})}{D_{z+t}^{aa}}$$

The reserve formulas for the annuity benefit may also be expressed entirely in commutation form, thus eliminating the summation expressions used in formulas (133) et seq. The reserve on an active life, at the end of t years, for a disability annuity benefit under an Ordinary Life policy, the first payment of the annuity to be made on the policy anniversary following the receipt of proof of disability, is

$$(148) \frac{K(r^a M_{z+t}^{at} - r^a M_y^{at}) - P_z^D (N_{z+t}^{aa} - N_y^{aa})}{D_{z+t}^{aa}}$$

The formulas for the annuity benefit under endowment policies are now given, as the student may easily develop those

applicable to limited payment life policies. The formula for a temporary income ceasing when the policy matures is, where $x + n \equiv y$,

(149)

$$\frac{K \{ {}^r M_{x+t}^{at} - {}^r M_{x+n}^{at} - N_{x+n}^t ({}^r M_{x+t} - {}^r M_{x+n}) \} - P_z^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}$$

where $x + n > y$,

$$(150) \quad \frac{K \{ {}^r M_{x+t}^{at} - {}^r M_y^{at} - N_{x+n}^t ({}^r M_{x+t} - {}^r M_y) \} - P_z^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

The reserve formula for the instalment benefit reducing the face amount, the first payment falling due one year after the policy anniversary next succeeding receipt of proof of disability, is as follows for the Ordinary Life plan:

$$(151) \quad \frac{d/k ({}^r I_k M_{x+t}^{at} - {}^r I_k M_y^{at}) - P_z^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

This formula is also applicable to the limited payment life plan when adjustment is made for the premium paying period.

For the endowment plan, the formulas are,

when $n - t > k + 1$ and $x + n \equiv y$,

$$(152) \quad \frac{d/k \{ {}^r I_k M_{x+t}^{at} - {}^r I_k M_{x+n-k-1}^{at} + {}^r I_k M_{x+n-k-1}^{at} - {}^r I_k M_{x+n-2}^{at} - S_{x+n}^t ({}^r M_{x+n-k-1} - {}^r M_{x+n-2}) - N_{x+n}^t (k \cdot {}^r M_{x+t} - {}^r R_{x+n-k-1} + {}^r R_{x+n-1}) \} \frac{1}{D_{x+t}^{aa}} - \frac{P_z^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}}{D_{x+t}^{aa}}$$

when $n - t > k + 1$ and $x + n > y$,

$$(153) \quad \frac{d/k \{ {}^r I_k M_{x+t}^{at} - {}^r I_k M_{x+n-k-1}^{at} + {}^r I_k M_{x+n-k-1}^{at} - {}^r I_k M_y^{at} - S_{x+n}^t ({}^r M_{x+n-k-1} - {}^r M_y) - N_{x+n}^t [k \cdot {}^r M_{x+t} + (y - x - n + 1) {}^r M_y - {}^r R_{x+n-k-1} + {}^r R_y] \} \frac{1}{D_{x+t}^{aa}} - \frac{P_z^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}}{D_{x+t}^{aa}}$$

when $n - t \equiv k + 1$ and $x + n \equiv y$,

$$(154) \quad \frac{d/k \{ {}^r I_k M_{x+t}^{at} - {}^r I_k M_{x+n-2}^{at} - S_{x+n}^t ({}^r M_{x+t} - {}^r M_{x+n-2}) + N_{x+n}^t [{}^r R_{x+t} - {}^r R_{x+n-1} - (n - t - 1) {}^r M_{x+t}] \} \frac{1}{D_{x+t}^{aa}} - \frac{P_z^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}}{D_{x+t}^{aa}}$$

when $n - t \equiv k + 1$ and $x + n > y$,

$$(155) \quad d/k \left\{ {}^tI_a M_{x+t}^{at} - {}^tI_a M_y^a - S_{x+n}^t ({}^tM_{x+t} - {}^tM_y) \right. \\ \left. + N_{x+n}^t [{}^tR_{x+t} - {}^tR_y - (n-t-1){}^tM_{x+t} + (x+n-y-1){}^tM_y] \right\} \frac{1}{D_{x+t}^{aa}} \\ - \frac{P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

For the maturity instalment benefit the reserve under an Ordinary Life policy is as follows:

$$(156) \quad \frac{d({}^tM_{x+t}^{at} - {}^tM_y^a) - P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

With the exception of an adjustment for the premium paying period when $x + n < y$, the formula is the same for a limited payment life policy.

The formulas for an endowment policy are, when $x + n \equiv y$,

$$(157) \quad \frac{d[{}^tM_{x+t}^{at} - {}^tM_{x+n}^{at} - N_{x+n}^t ({}^tM_{x+t} - {}^tM_{x+n})] - P_x^D (N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}$$

when $x + n > y$,

$$(158) \quad \frac{d[{}^tM_{x+t}^{at} - {}^tM_y^a - N_{x+n}^t ({}^tM_{x+t} - {}^tM_y)] - P_x^D (N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}$$

Pension fund notation applicable to modern benefits. In the new notation the expression corresponding to formula (145) for the waiver of premium reserve at the end of t years on an Ordinary Life policy is

$$(159) \quad P \cdot \frac{{}_yM_{x+t}^r}{D_{x+t}^{aa}} - P_x^D \cdot \frac{N_{x+t}^{aa} - N_y^{aa}}{D_{x+t}^{aa}}$$

The various reserve formulas applicable to other plans for the waiver of premium benefit and also for annuity benefits can easily be obtained. Two examples are given:

The active life reserve at the end of t years for the waiver of premium benefit under an n -payment life policy, $x + n = u < y$, is

$$(160) \quad P \cdot \frac{{}_yM_{x+t}^r}{D_{x+t}^{aa}} - P_x^D \cdot \frac{N_{x+t}^{aa} - N_u^{aa}}{D_{x+t}^{aa}}$$

The active life reserve for the monthly disability income benefit on the same plan, K being the amount of the annuity yearly, is

$$(161) \quad K \cdot \frac{{}_yM_{x+t}^r}{D_{x+t}^{aa}} - P_x^D \cdot \frac{N_{x+t}^{aa} - N_u^{aa}}{D_{x+t}^{aa}}$$

When $t \leq n$, both terms in expression (160) and the second term in expression (161) become equal to zero.

The foregoing formulas assume that the commutation functions involve monthly disabled life annuities—see formula (93)—and that no adjustment is made for variation in waiting period. If a continuous disabled life annuity is to be used for the evaluation of the waiver of premium benefit, then formula (160), for example, would become

$$(162) \quad P \cdot \frac{{}^u M'_{x+t} - \frac{1}{24} \cdot {}^u M'_{x+t}}{D_{x+t}^{aa}} - P_x^D \cdot \frac{N_{x+t}^{aa} - N_u^{aa}}{D_{x+t}^{aa}}$$

The deduction in the first term being in accordance with the relationship $\bar{a}_{[x]} = a_{[x]}^{(12)} - \frac{1}{24}$, the value of P_x^D in formula (162) would, of course, be slightly different from the value of P_x^D in formula (160).

If adjustments are made for variation in waiting period, as described in the second method of Part II, Chapter XII, the numerator of the first term of the reserve factor should be modified accordingly.

In practice many reserves are calculated according to the general formula (125). Let us consider, for simplicity, the waiver of premium benefit under a limited payment life or endowment policy. Let ${}^u P_x^d$ be the net disability premium to provide the waiver benefit per \$1.00 of premium for an n -payment life or endowment policy issued at age x , $x + n = u$. Values of ${}^u P_x^d$ are determined for all values of x and u , i.e., for all values of x and n . On account of the labor involved in calculating complete tables of select disabled life annuities and commutation columns based thereon, values of ${}^u P_x^d$ have usually been determined for all values of x but only for quinquennial values of u , the remaining values being easily obtained by interpolation. If P is the premium waivable, the reserve on an active life at the end of t years is, for $x + n \leq y$,

$$(163) \quad P ({}^u P_{x+t}^d - {}^u P_x^d) \frac{N_{x+t}^{aa} - N_u^{aa}}{D_{x+t}^{aa}}$$

If ${}^u P_x^D$ is the net disability premium for this policy, then ${}^u P_x^D = P \cdot {}^u P_x^d$

For $x + n > y$, N_u^{aa} in formula (163) would become N_y^{aa} .

Accumulation formulas for active life reserves. Active life reserves have sometimes been calculated by means of an accumulation method. In life insurance we have the relation

$$(164) \quad {}_{t+1}V_x = ({}_tV_x + P_x) u_{x+t} - k_{x+t}$$

where

$$u_x = \frac{D_x}{D_{x+1}}$$

and

$$k_x = \frac{C_x}{D_{x+1}}$$

If ${}_tV_x^I$ be the t th year active life reserve and P_x^I the net premium for the disability income benefit and ${}_tV_x^W$ and P_x^W the corresponding functions for the waiver of premium benefit, we have the following relation for the income benefit:

$$l_{x+t}^{aa} ({}_tV_x^I + P_x^I) (1+i) = l_{x+t+1}^{aa} \cdot {}_{t+1}V_x^I + K \cdot l_{x+t}^{aa} \cdot r_{x+t} \cdot a_{[x+t+1]}^{(12)} (1+i)^{\frac{1}{2}}$$

or

$$(165) \quad {}_{t+1}V_x^I = \frac{l_{x+t}^{aa}(1+i)}{l_{x+t+1}^{aa}} ({}_tV_x^I + P_x^I) - K \cdot \frac{l_{x+t}^{aa} \cdot r_{x+t} \cdot a_{[x+t+1]}^{(12)} (1+i)^{\frac{1}{2}}}{l_{x+t+1}^{aa}}$$

$$= ({}_tV_x^I + P_x^I) u_{x+t}^{aa} - K \cdot {}^\omega k_{x+t}^r$$

where

$$\frac{{}^\omega C_x^r}{D_{x+1}^{aa}} = {}^\omega k_x^r$$

and

$$\frac{D_x^{aa}}{D_{x+1}^{aa}} = u_x^{aa}$$

and K = amount of income yearly.

The corresponding equation for the waiver of premium benefit under the Ordinary Life plan, if monthly annuities are used for evaluating the benefit, is

$$(166) \quad {}_{t+1}V_x^W = ({}_tV_x^W + P_x^W) u_{x+t}^{aa} - P \cdot {}^\omega k_{x+t}^r$$

The relation for the waiver of premium benefit under an n -payment life policy, $x+n=u < y$ (the limiting age for coverage), is

$$(167) \quad {}_{t+1}V_x^W = ({}_tV_x^W + P_x^W) u_{x+t}^{aa} - P \cdot {}^\omega k_{x+t}^r$$

where

$$\frac{{}^\omega C_x^r}{D_{x+1}^{aa}} = {}^\omega k_x^r$$

The foregoing formulas apply where the benefits are not retroactive to the beginning of the waiting period and where

there are no adjustments for variation in such period. If the benefits are retroactive or adjustments have been made on account of variation of waiting period the above formulas should be modified. For example, if the benefit provided is a retroactive income under an f months' waiting period, there are $f-1$ monthly income payments falling due at date of disability in addition to the initial payment of the monthly disability annuity—see formula (100). The relation in (165) would then become

$$(168) \quad {}_{t+1}V'_x = \frac{l_{x+t}^{aa} (1+i)}{l_{x+t+1}^{aa}} (V'_x + P'_x) - K \cdot \frac{l_{x+t}^{aa} \cdot r_{x+t} \left(a_{|x+t+1|}^{f(12)} + \frac{f-1}{12} \right) (1+i)^x}{l_{x+t+1}^{aa}}$$

$$= ({}_tV'_x + P'_x) u_{x+t}^{aa} - K \left({}^o k'_{x+t} + \frac{f-1}{12} \cdot k'_{x+t} \right)$$

where $\frac{C'_x}{D'_{x+1}} = k'_x$

In the same way the formulas may be modified on account of adjustments for variation in waiting period and where continuous annuities are used for the evaluation of the waiver of premium benefit.

Since the adjustments usually involve factors which are taken as constant for all ages, values of the compound function may be tabulated. For example, in formula (168) values of

$$\left({}^o k'_{x+t} + \frac{f-1}{12} \cdot k'_{x+t} \right) \text{ would be tabulated. (See discussion of}$$

adjustments in premium formulas—Part II, Chapters IX and XII).

Modern active life reserve practice. The foregoing reserve formulas in terms of pension fund notation applicable to modern benefits, contain D_x^{aa} and N_x^{aa} symbols. As pointed out in Part II, Chapter XI, under present day calculations mixed life functions are employed instead of those based on active lives. In practice, therefore, formulas (159) et seq. would be modified accordingly.

The reserves discussed in this chapter have been designated as reserves on active lives. They are the reserves held by Com-

pany B of the illustration at the beginning of the chapter. Under a total and permanent disability clause, in which no recoveries from disability are contemplated, the company should cease to carry the active life reserve when the policyholder becomes disabled. When recoveries enter the experience Company B should hold reserves not only on lives then active, but also on disabled lives in respect to benefits which may be received on account of any disability arising subsequent to recovery from the present disability. For theoretical accuracy, a combined mortality and disability table should be constructed with recoveries taken into account. The active lives would then include re-entries into that group on account of recoveries among the disabled lives. The active life reserves so obtained would be carried only on lives active at date of valuation. However, because of the complications involved in forming a combined mortality and disability table which takes recoveries into account, the construction of such a table has not been effected for the purpose of the evaluation of active life reserves. Furthermore, there is little need for such a table on account of the fairly general practice of the companies of carrying active life reserves on all policies containing disability benefits, whether the lives be active or disabled.

The practice of carrying active life reserves on both active and disabled lives began very early in the history of disability benefits. In the beginning, the benefits provided were very inexpensive so that the active life reserve on an individual policy was small. As a matter of convenience, therefore, many companies did not determine for active life valuation purposes the insurance with disability benefits which was in force on active lives. Even today, with the active life reserves considerably larger than formerly, the companies continue to carry active life reserves on the total insurance in force with disability benefits. In fact, one large company, which formerly made a subdivision of the insurance with disability benefits, decided to discontinue the practice in regard to new issues with benefits containing a waiting period. This was done principally on account of the great number of entries and reversals which otherwise would be necessary to determine the total insurance in force on active lives, due to the high rates of disability and of claim termination.

The additional reserve carried may be viewed, as stated in Part I, Chapter XII, as a partial offset to the probable inadequacy of active life reserves in the light of recent experience.

With the adoption of the Class (3) table as the basis of premium calculations, a more suitable basis has been provided for active life reserves. In the meantime most companies have tried various methods of approximation in order to obtain satisfactory reserves. An indication of the situation existing in 1929, when Mr. Hunter presented his paper on disability premiums and reserves to the Actuarial Society, may be obtained from the following quotation therefrom (T. A. S. A., XXX, 386): "The majority of companies doing business in New York State hold reserves for active lives under their modern contracts based on Hunter's reserves increased by either a flat or varying percentage, the increase being from 10% to 100%. A number of companies add a constant per \$1,000 of the face of the policy, while several use a combination of these two methods. Two or three companies add a correction to Hunter's based on either Hezlett's table or the AHJ section of the Manchester Unity experience. A number of companies, especially Canadian, hold 50% of the total premiums received, either with or without interest. This great diversity of methods reveals not only the general opinion that an addition should be made to rates based on Hunter's table, but also the pressing need of a new and adequate experience on which to base the active life reserves under modern policies."

When the Class (3) table was adopted in 1930 by many companies for premium and active life reserves on new issues to male risks, there was also generally adopted a different scale of premium rates for women, equal, in many prominent companies, to double the male scale. In such cases this was assumed to be equivalent to using a rate of disability for women equal to twice that applicable to men. The corresponding active life reserves were double the male reserves. On the other hand, when premiums based on 150% and 165% of Class (3) with additional percentage increases above age 45 were adopted in 1932 for male risks, it was planned to calculate the corresponding active life reserves anew on account of the additional percentage increases applied to the rate of disability above age 45.

Approximate methods of valuation. Many companies use approximate methods of determining the active life reserve. Most frequently the approximation involves the elimination of the age element. One company, for example, finds for each plan and year of issue, the average active life reserve thereunder and the age at issue corresponding to such average reserve. The reserve according to such average age is used in the annual valuation for a period of years, after which the average is revised. Another approximation adopted is to calculate the reserves, either by the above approximation or otherwise, on the principal plans, then to increase the resulting reserve by a pre-determined percentage to obtain the total for all plans. The student will find various methods of approximation described by F. A. Shailer in T. A. S. A., XXV, 80. The discussion of Mr. Shailer's paper should also be read.

Mean reserves. The reserve formulas given in this chapter are for terminal reserves, i. e., the reserve at the end of a specified policy year. For valuation purposes a mean reserve is usually used. On the assumption of an even distribution of policy anniversaries throughout the year, the mean reserve for the n th policy year under a premium paying policy is

$$(169) \quad \frac{{}_{n-1}V_x^D + P_x^D + {}_nV_x^D}{2}$$

where ${}_nV_x^D$ is the terminal reserve at the end of the n th policy year and P_x^D is the net premium for the benefit. ${}_{n-1}V_x^D + P_x^D$ is called the initial reserve.

For a paid-up policy the formula becomes

$$(170) \quad \frac{{}_{n-1}V_x^D + {}_nV_x^D}{2} \quad \text{or} \quad \frac{A_{x+n-1}^D + A_{x+n}^D}{2}$$

where A_{x+n}^D is the net single premium at age $x+n$ for the benefit. These formulas are the same as those used for life insurance benefits.

In the calculation of mean reserves the companies generally apply to disability benefits the principles adopted for life insurance. For example, if either or both of the terminal reserves are negative, the corresponding mean reserve is taken as equal to not less than one-half the net premium.

Although the mean reserve is based on the assumption of an even distribution of issues throughout the year, the same formula is usually used even where it is definitely known that the distribution is not uniform. Take, for example, the calendar year in which a new benefit is issued. If the new benefit is introduced subsequent to January 1st, then the above mean reserves are not strictly applicable to issues of that calendar year. The general practice, however, is to make no adjustment in the mean reserve for such uneven distributions of issues.

Another point of interest is the treatment of the waiver of premium reserve under limited payment life and endowment contracts. In Part II, Chapter VIII it was stated that although the policyholder could not have a waiver benefit after paying the n th annual premium, the charge for the benefit was spread over n years instead of $n-1$ years. (For the purpose of this discussion $x+n$ is assumed not to be greater than the limiting age for the coverage). A mean reserve is held for the $(n-1)$ th policy year which, on account of the decreasing nature of the benefit, is usually one-half the net premium. On the other hand, a waiver benefit is possible after the beginning of the n th policy year if premiums are payable fractionally. A small reserve, therefore, should properly be held in the n th policy year but usually no reserve is carried.

CHAPTER XIV.

RESERVES ON DISABLED LIVES

Considering again the illustration at the beginning of Part II, Chapter XIII, it is seen that reserves must also be held on disabled lives. These are the reserves which would be carried by Company C. Such reserves are frequently called claim reserves and are equal to the value of the disability benefit at date of valuation calculated on the basis of a disabled life table.

Waiver of Premium

The claim reserve at the policy anniversary following the receipt of proof of disability under an Ordinary Life policy, t years in force, for the waiver of the premium P , including the premium then due, is

$$(171) \quad P \cdot a'_{x+t}$$

The claim reserve at the policy anniversary under an n -payment life or endowment policy is

$$(172) \quad P \cdot a'_{x+t : \overline{n-t}|}$$

Annuity during Disability

The liability of the company with reference to this benefit, when a life is disabled, is evidently the present value of the annuity calculated on the basis of a disabled life mortality or termination table.

Let K represent the amount of the annuity per annum.

Let t represent the integral number of years the policy has been in force, and

Let n represent the number of premiums under a limited payment life policy, or the term of an endowment policy.

Annual annuity. The formulas which follow are applicable when the first payment is deferred until one year after receipt of proof of disability, so that the annuity payments on a disabled life are assumed to be made, on the average, in the middle of the policy year.

Under an Ordinary Life or limited payment life policy the disability claim reserve for this benefit, at the policy anniversary succeeding receipt of proof of disability, is

$$(173) \quad K \cdot {}_1|a_{x+t}^t$$

and under an endowment policy the disability claim reserve for this benefit is

$$(174) \quad K \cdot {}_1|a_{x+t:\overline{n-t}}^t$$

if benefits cease at maturity.

The student should determine the corresponding expressions for other forms of annual income benefits.

Monthly annuity. When the disability benefit consists of a monthly annuity, the annuity usually begins on receipt of proof or is made retroactive to the commencement of disablement. Assuming that disability occurs, on the average, in the middle of the policy year, the reserve *at date of disability*, on a select basis, under an Ordinary Life or limited payment life policy for a monthly income benefit, not retroactive, is

$$(175) \quad K \cdot a_{[x+t+\frac{1}{2}]}^{(12)}$$

If the income benefit is retroactive to commencement of disability, with a waiting period of f months, then the value at date of disability, before any payments have been made, is

$$(176) \quad K \cdot a_{[x+t+\frac{1}{2}]}^{(12)} + \frac{f-1}{12} \cdot K$$

The modifications for temporary annuities are obvious.

*Payment of Face Amount in Instalments, Balance, if any,
Payable at Death or Maturity*

Ordinary Life policy. We may regard the entire contract, including the waiver of premium benefit, as changed to a series of single premium endowment insurances on a disabled life and may readily ascertain the reserve accordingly.

It is assumed that the first instalment is payable one year after the anniversary next succeeding the receipt of proof of disability.

If k be the number of instalments payable and $1/k$ the amount of each, and z the age at the beginning of the year in which disability occurs, the total value of the policy at the end of that year would be

$$(177) \quad \frac{1}{k} (A_{z+1:1}^t + A_{z+1:2}^t + \dots + A_{z+1:k}^t)$$

From another viewpoint the additional reserve required on account of the payment of instalments in advance would be, in accordance with formula (61),

$$(178) \quad \frac{d}{k} \cdot \frac{S_{t+2}^t - S_{t+k+2}^t}{D_{z+1}^t}$$

J. B. Gibb (T. A. S. A., XVIII, 123) has shown that the reserve obtained by formula (178) and the reserve for the waiver of premium benefit, together with the regular insurance reserve on the basis of disabled life mortality, are equal to the total value given by formula (177). He drew attention to a practical difficulty by pointing out that in order to comply with the accepted standards of valuation, there must be carried the regular insurance reserve of $1 - (\pi_x + d) a_{x+n}$ on all risks instead of $1 - (\pi_x + d) a_{x+n}^a$ on active risks and $1 - (\pi_x + d) a_{x+n}^d$ on disabled risks. The total reserve on a disabled risk is thus insufficient by the difference between $1 - (\pi_x + d) a_{x+n}^d$ and $1 - (\pi_x + d) a_{x+n}$. To provide the instalment or maturity benefit, the deficit of $(\pi_x + d) (a_{x+n} - a_{x+n}^d)$ must be met from the general surplus or from a special fund. If it is to be met from a special fund, Mr. Gibb suggests that it might properly be

charged to a mortality fund on the ground that the transaction is in the nature of granting a special surrender value in order to get an impaired risk off the books. It should be remembered, however, that when an insurance company carries a reserve such as that according to the American Experience Table this reserve is assumed to be sufficient for its business as a whole. Mr. Gibb shows that the regular reserve according to the American Experience Table is more than sufficient for lives whose vitality is good, but too small for disabled lives. So long as the policies on disabled lives are continued in force, the sum of the reserves found by treating the active and disabled lives separately will agree with the regular American reserve for the whole company, the deficit of which in the case of the disabled lives is offset by the excess carried on the active lives. This follows from the equations

$$l_x = l_x^{aa} + l_x^{tt}$$

$$l_x \cdot a_x = l_x^{aa} \cdot a_x^a + l_x^{tt} \cdot a_x^t$$

the reserve for an active life being $1 - (\pi_x + d) a_{x+n}^a$ and that for a disabled life $1 - (\pi_x + d) a_{x+n}^t$. Where, however, the policy is terminated at disability, this deficit must be charged off, the apparent loss being offset by the excess reserves held on business remaining in force on active lives.

As already intimated, the regular insurance reserve on a disabled life, if taken as $1 - (\pi_x + d) a_{x+n}^t$, together with the waiver of premium reserve and that found in formula (178), will be exactly sufficient to meet payment of the instalments as they fall due. Whenever an instalment is paid the amount of insurance is reduced by the amount of the instalment. Thus if an instalment of $1/k$ of a unit policy is paid, the regular insurance reserve is reduced in the proportion $1/k$, the premium to be waived becomes $[(k-1)/k] \pi_x$ and the instalment reserve is reduced to the amount necessary to provide for the payment of $k-1$ instalments of $1/k$. The total reduction in reserve on account of the payment of the first instalment of $1/k$ at the age of $x+2$ is therefore:

$$(a) \quad \frac{1}{k} (A_{x+2}^t - \pi_x \cdot a_{x+2}^t) = \text{reduction in insurance reserve,}$$

(b) $\frac{1}{k} \cdot \pi_x \cdot a_{z+2}^i =$ reduction in premium waiver reserve,

(c) $\frac{1}{k} \cdot d \cdot a_{z+2}^i = \frac{1}{k} \cdot d \cdot \frac{N_{z+2}^i}{D_{z+2}^i} =$ reduction in instalment reserve.

The sum of the values (a), (b) and (c) is $1/k$. We therefore see that when the insurance is reduced in this manner the reserve released is exactly equal in amount to the payment made.

Immediately before payment of the first instalment the additional reserve for disability instalments is

$$\frac{d}{k} \cdot \frac{S_{z+2}^i - S_{z+k+2}^i}{D_{z+2}^i}$$

of which there is canceled, after the first instalment is paid,

$$\frac{d}{k} \cdot a_{z+2}^i = \frac{d}{k} \cdot \frac{N_{z+2}^i}{D_{z+2}^i}$$

The balance represents the additional reserve, per unit of original face amount, just after the first instalment is paid, namely

$$\frac{d}{k} \cdot \frac{S_{z+3}^i - S_{z+k+2}^i}{D_{z+2}^i}$$

Following this method the additional reserve immediately after payment of the g th instalment would be

$$(179) \quad \frac{d}{k} \cdot \frac{S_{z+g+2}^i - S_{z+k+2}^i}{D_{z+g+1}^i}$$

and before payment of the $(g+1)$ th instalment,

$$(180) \quad \frac{d}{k} \cdot \frac{S_{z+g+2}^i - S_{z+k+2}^i}{D_{z+g+2}^i}$$

It may readily be shown that the amount of the reserve released on payment of the $(g+1)$ th instalment, together with the reserve on the insurance of $1/k$ and that on the waiver benefit are exactly sufficient to cover the instalment of $1/k$.

Limited payment life policy. The additional reserve for disability instalments is the same as for an Ordinary Life policy.

Endowment policy. Let n years be the term of the endowment policy. Let m equal $k+1$ or $x+n-z-1$, whichever is the less.

Immediately after payment of the g th instalment the additional reserve for disability instalments is, per unit of original face amount,

$$(181) \quad \frac{d}{k} \left(\frac{S_{z+g+2}^t - S_{z+m+1}^t - (m-g-1)N_{z+n}^t}{D_{z+g+1}^t} \right)$$

The total reserve liability when disability instalments are payable is

- (a) The regular life insurance reserve for the unpaid portion of the policy, on the assumption that premiums have not been waived;
- (b) The reserve for the future premiums waivable, premiums having been reduced proportionately after each instalment has been paid;
- (c) The reserve equal to the present value of the interest on the remaining instalments from the date when each instalment is payable until the death of the insured or the maturity of the policy.

*Payment of Face Amount in Instalments for a Term Certain,
Commuted Value of Unpaid Instalments Payable
at Death or Maturity*

Claim reserves for this type of benefit, of which the maturity instalment type is the most common example, may be determined by methods of reasoning similar to those used in determining claim reserves under the instalment reducing the face type of benefit just given. However, a common treatment is to mature the policy at disability and value it for reserve purposes as if it were a supplementary contract not involving life contingencies.

Mean Reserves

As stated in the preceding chapter, the reserves usually used in the valuation of a company's reserve liability are mean reserves. The formulas given in this chapter have been reserves

at a policy anniversary or at the date of disability. The corresponding mean reserves can easily be determined. For example, the mean reserve in the first year of disability corresponding to formula (171) is obtained as follows:

The value of the benefit at the policy anniversary at which the insured is age $x + t$, immediately after the premium due at that date has been waived, is

$$P \cdot a_{x+t}^t$$

This is termed the initial annuity value.

The value of the benefit at the next policy anniversary, before the premium due at that date is waived, is

$$P \cdot a_{x+t+1}^t$$

This is termed the terminal annuity value.

The mean reserve between these two anniversaries in respect to future benefits, is

$$(182) \quad P \cdot \frac{a_{x+t}^t + a_{x+t+1}^t}{2}$$

The mean reserve corresponding to formula (172) is

$$(183) \quad P \cdot \frac{a_{x+t:\overline{n-t-1}}^t + a_{x+t+1:\overline{n-t-1}}^t}{2}$$

If the student understands the principle involved in the above calculation, he should have no difficulty in applying adjustments for monthly annuities, or in determining the mean claim reserves for other types of benefits. For select disabled life annuities the same principles will apply, although in the case of the first year mean reserve for claims under modern benefits some adjustment may be necessary on account of the rapidly decreasing termination rate during that year. In the case of claims under modern benefits, the initial and terminal values for claim valuation purposes are determined as of the date of disability or anniversaries of that date and not as of policy anniversaries. Where benefits are retroactive to the beginning of the waiting period, the first year mean reserve excludes such retroactive payments.

Average Reserves

Reference has already been made to the incidence by age of the value of disabled life annuities based on Hunter's aggregate table. The values increase with age until they reach a maximum value at age 35, the values for higher ages decreasing with advance in age. The nature of this incidence makes it possible to have fairly constant average values for all ages combined. By reason of this feature a number of companies use average disabled life mean annuities which are independent of age. Such annuities according to Hunter's table with 3% interest, which have been approved by at least one insurance department for valuation purposes, are:

AVERAGE DISABLED LIFE MEAN ANNUITIES—HUNTER'S 3%

Number of future years' premiums to be waived or income payments to be made	Average Annuity	Number of future years' premiums to be waived or income payments to be made	Average Annuity
1	1.0	11	6.0
2	1.9	12	6.2
3	2.6	13	6.4
4	3.2	14	6.6
5	3.7	15	6.8
6	4.2	16	7.0
7	4.7	17	7.2
8	5.1	18	7.3
9	5.4	19	7.4
10	5.7	20 or over and life	7.7

Average disabled life annuities have also been prepared for the Class (3) experience. Values on a select basis appear on pages 81 and 155 of Disability Reserves published in 1930 by the Actuarial Society.

Average values are usually approved for a definite period subject to checking and revision, if necessary, at the end of that time.

Select Disabled Life Annuities

The values of disabled life annuities vary greatly according to the duration of the claim. This is true not only of claim annuities under modern benefits, but also of those under benefits

in event of total and permanent disability. The variation by duration may be seen from the following:

DISABLED LIFE ANNUITIES—3%

Age (x)	Hunter's Select		Based on Experience under 90 Day Waiting Period		Hunter's Aggregate
	$a_{[x]}$	$a_{[x-5]+5}$	$a_{[x]}$	$a_{[x-5]+5}$	
25	8.8	18.4	2.9	13.0	7.1
35	9.5	17.1	3.2	12.2	8.6
45	8.3	14.6	3.2	10.4	8.2
55	6.9	10.5	3.6	8.7	7.3

The values for Hunter's select table were given by M. Davis in T. A. S. A., XVII, 229, and those under a benefit with a 90 day waiting period are based on values given by J. T. Phillips in T. A. S. A., XXX, 430. Hunter's aggregate values have also been shown for purposes of comparison (see p. 116 for relation between select and aggregate annuities). Ordinarily the bulk of the disability claims in force would be in their early years of disability, so that a claim valuation by Hunter's aggregate is usually satisfactory for total and permanent claims. For the valuation of claims under modern benefits, Hunter's aggregate table produces excessive reserves under claims which have been in force a short time. The Committee on Disability Experience in its 1926 Report compared the Class (3) claim annuities with Hunter's and found that at date of disability the ratio was only .42. At the end of the first year of disability the ratio was .84 and at the end of the third year .96. Similar ratios were obtained for the annuities under Classes (1) and (2). The Committee therefore recommended that the following percentages of the reserves on Hunter's table be used for the valuation of claims:

	Class (1)	Class (2)	Class (3)
Unreported	70%	35%	40%
First Year	80	50	65
Second Year	100	85	100
Third Year	100	100	100

The percentages for first year claims, for example the 65% suggested for Class (3), represents the ratio for the first year mean annuity and is approximately the mean of the ratios quoted above. On the other hand the claim annuity in respect

to an unreported claim is comparable with the claim value at date of disability so that the annuity ratio at date of disability is the one which is applicable. The student will find in Arthur Hunter's paper (T. A. S. A., XXX, 387) the effect of various reserve bases on the total claim liability of an actual experience.

The student should bear in mind that practically all values of claim annuities obtained in the past have been based on a limited experience. It has not been possible to get data of sufficient duration to obtain satisfactory ultimate values. The usual procedure has been to assume that, after a certain claim duration, say ten years, the claim termination rate will ultimately run into those shown for the attained age in the eleventh year of Hunter's Analyzed Disabled Life table (T. A. S. A., XII, 51). Mr. Phillips, in his study of the New York Life experience, found, upon plotting the annual termination rates for as many years as were obtainable, that the rates might be merged with some other ultimate table with a higher mortality rate. Tests were made to see what the effect would be upon the annuity values if the ultimate termination rates were double those of Hunter's table. The effect was found to be about a 10% decrease in the select annuity value (see T. A. S. A., XXX, 405). If the ultimate rate were closer to Hunter's, the effect on the annuity values would be proportionately less. It is evident therefore that any misstatement in the premiums due to the arbitrary assumption regarding the ultimate claim termination rate is small and is probably on the safe side. The method used in the Inter-Company disability investigation of 1926 to run the termination rates from the select basis into the ultimate basis is of interest in the present discussion. The student will find a description of the method in Part II, Chapter V.

APPENDIX A

SPECIMEN DISABILITY CLAUSES

I. WAIVER OF PREMIUM ONLY.

1. *Clause issued about 1896, one of the earliest forms.*

"It is further especially agreed that if at any time upon acceptance of due and satisfactory proof of the total and permanent blindness or deafness of the insured hereunder, or that said insured has become totally and permanently incapacitated, either by accident, bodily or mental disorder, the said insured in lieu of all other benefits and advantages accruing hereunder shall be entitled to either of the following options:

First—From the date of such proof the premiums payable hereunder shall be discontinued or remitted and this policy shall thereafter be maintained subject to the conditions endorsed hereon from the equation reserve fund.

Second—Such a Disability Life Annuity in exchange for this policy as its face value will purchase at the then age of the insured, according to the Table of Disability Annuity rates endorsed hereon.

It is further agreed that the member or insured making claim under the Disability Clause aforesaid, must at the time of making such claim, specify in writing under which of the aforesaid options claim is made."

2. *Clause issued about 1906.*

"Premiums on Contract Paid by Company If Insured Is Wholly Disabled—As Follows: After one full annual premium shall have been paid and before a default in the payment of any subsequent premium, if the Insured shall furnish satisfactory proof that he has become wholly disabled and will be permanently, continuously, and wholly disabled for life, by bodily injuries or disease, from pursuing any and all gainful occupations, the Company, by an endorsement in writing upon this contract, will agree to pay for the Insured the premiums, if any, which shall thereafter become payable during the continuance of such disability. In any such case the Cash Loan and Cash Surrender values shall increase in like manner as if the premiums were being paid by the Insured. If, however, the Insured shall recover so as to be able to engage in any gainful occupation for wages or profit during the premium-paying period, the obligation on his part to make payment of premiums in accordance with this contract shall recommence, but only from date of

recovery, with the same force and effect as if this provision were not contained therein."

NOTE—The terms of later Waiver of Premium Clauses may be deduced from the illustrations given below of the more comprehensive clauses including Monthly Income Benefits.

II. INSTALMENT AND INCOME BENEFITS.

1. *Face Amount of Policy payable in Instalments upon Proof of Disability, Balance, if any, payable at Death or Maturity. Clause issued about 1913.*

(a) *With Option.*

"Total Disability Benefit—At any time after one full year's premium shall have been paid, and before default in the payment of any subsequent premium, if the assured, before attaining his sixtieth birthday, shall furnish proof satisfactory to the Company that he has become wholly and permanently disabled by bodily injury or disease, and is and will be permanently, continuously and wholly prevented thereby from performing any work for compensation or profit, or from following any gainful occupation, then:

(i) The Company by endorsement hereon will agree to pay for the assured the subsequent premiums, if any, as they shall become due, during the period of such total and permanent disability. Any premium so paid shall not be a lien on this policy, and the guaranteed values in the table attached hereto shall increase in the same manner as if the premiums had been paid in cash by the assured, and the policy shall continue to participate in profits. (ii) The Company will also, upon the written request of the assured and of the beneficiary and assignee, if any, and after payment of all indebtedness to the Company in respect of the policy, agree by endorsement hereon to pay in lieu of all other values, benefits and privileges herein provided, and in full settlement of this policy, one-twentieth of the amount assured and existing bonus additions immediately, and a like amount yearly thereafter until twenty such instalments in all shall have been paid. Should the assured die before the twenty instalments shall have been paid, the remaining instalments shall be paid to the beneficiary as they become due. Provided always that notwithstanding proof of disability may have been accepted by the Company as satisfactory, the assured shall as often as required by the Company furnish satisfactory proof of the continuance of such disability. If the assured shall fail to furnish such proof, or if he shall so far recover, as to be able to engage in any gainful occupation, then all premiums thereafter falling due must be paid as originally provided in the policy; and if option (ii) shall have been availed of, the Company's

obligation to pay any further instalments shall immediately cease, and the original policy shall be restored for its face amount, less the sum of the instalments paid, the guaranteed values in the table attached hereto being reduced proportionately, but no reduction being made in the future premiums. Without prejudice to any other cause of disability, the entire and irrecoverable loss of the sight of both eyes, or the severance of both hands above the wrists, or of both feet above the ankles, or of one entire hand and one entire foot, shall be considered as total and permanent disability within the meaning hereof."

(b) *Without Option (includes provision for Modified Waiver of Premium Benefit in event of disablement after age 60).*

"Waiver of Premiums—If, after this policy shall have been in force one full year and before default in the payment of any premium, the Company receives due proof that the insured before attaining the age of sixty years has become wholly disabled by bodily injury or disease so that he is and will be presumably, thereby permanently and continuously prevented from engaging in any occupation whatsoever for remuneration or profit, the Company shall waive payment of each premium as it thereafter becomes due during the insured's said disability. In making any settlement under this policy the Company shall not deduct any part of the premiums so waived, and the loan and cash surrender values provided for in this policy shall increase from year to year in the same manner as if the premiums so waived had been paid in cash. Under all the conditions aforesaid, except that the insured shall have attained the age of sixty years before becoming disabled, the Company shall waive payment of each premium thereafter becoming due during such disability, but the face amount of the policy shall be reduced by the amount of each such waived premium, and the loan and cash surrender values as provided for in this policy shall be based upon said reduced amount of insurance in the same manner as if the premiums for such reduced amount of insurance had been duly paid.

"Instalment Payments—In addition to waiving payment of premiums as aforesaid, if such disability shall have occurred before the insured attained the age of sixty years, the Company, one year after said proof of such disability, shall pay to the insured one-tenth of the face amount of the policy and a like amount in each insurance year thereafter during the continuance of such disability prior to the maturity of the policy; the policy must be returned to the Company for the endorsement thereon of each payment. Each instalment shall reduce to that extent the amount of insurance in force, and the loan and cash surrender values provided for in this policy shall be calculated

for the reduced amount insured on the basis provided herein. If at the time when any such instalment becomes payable there shall be an indebtedness on the policy in excess of the cash surrender value of the reduced amount of insurance, the Company shall apply such part of the instalment as may be necessary to reduce the indebtedness to the amount secured by such cash surrender value. Whenever the total amount of said instalments, together with the amount of any remaining indebtedness to the Company, shall equal the face amount of the policy, plus any paid-up dividend additions, unpaid dividends and dividends left to accumulate to the credit of the policy, then the Company's obligations under the policy shall thereby be fully satisfied and discharged without further action.

"Recovery from Disability—Should the Company accept under this policy proofs of disability, it may nevertheless at any time thereafter, and from time to time, but not oftener than once a year, demand of the insured proof of the continuance of such disability, and upon failure to furnish such proof, or, if it appears that the insured has become able to engage in any occupation whatsoever for remuneration or profit, no further premiums shall be waived and no further instalment payments will be made by the Company. But if the amount of the insurance shall then have been reduced under any of the foregoing provisions such reduced amount of insurance shall thereafter be the face amount of the policy, and the premiums thereafter falling due will be reduced in proportion to the reduced amount of insurance, and all benefits under the policy will be reduced accordingly. Without prejudice to any other cause of disability, the entire and irrecoverable loss of the sight of both eyes, or the severance of both hands above the wrists, or of both feet above the ankles, or of one entire hand and one entire foot shall be considered as total and permanent disability within the meaning of this section."

2. Annual Income, not Reducing Policy, issued about 1915.

"When Such Benefits Take Effect—If the insured, after payment of premiums for at least one full year and before default in the payment of any subsequent premium and before attaining the age of 60 years at nearest birthday and while this policy is in full force, shall furnish due proof to the Company at its Home Office that he has become totally and permanently disabled by bodily injury or disease, so that he is, and will be, permanently, continuously and wholly prevented thereby from performing any work for compensation, gain or profit, or from following any gainful occupation, and that such disability has then existed continuously for not less than 60 days, the Company will grant the following benefits.

“Benefits. 1, Waiver of Premium—The Company will, during the continuance of such disability, waive payment of each premium as it thereafter becomes due, commencing with the first premium due after receipt of said due proof of such disability. 2, Income to Insured—One year after the anniversary of the date of issue of this policy next succeeding the receipt of said due proof, the Company will pay to the insured, if then living and such disability still continue, a sum equal to one-tenth of the face amount of the policy, but not including dividend additions, and a like sum on each such anniversary thereafter, if the insured be then living and such disability still continue.

“The face amount of the policy shall not be decreased because of any premium waived or any income payment made, nor shall such waived premium or income payment be deducted in any subsequent settlement of the policy, and the loan and surrender values will increase each year in the same manner as if each premium had been paid when due instead of being waived.

“Proof of Continuance of Disability Required: Recovery From Disability—Although the proof of total and permanent disability may have been accepted by the Company as satisfactory, the insured shall at any time thereafter, and from time to time, but not oftener than once a year, on demand, furnish to the Company due proof of the continuance of such disability, and if the insured shall fail to furnish such proof, or if it shall appear to the Company that the insured is able to perform any work, or follow any occupation whatever for compensation, gain or profit, no further premiums shall be waived and no further income shall be paid.

“General Provisions—The provisions for disability benefits shall immediately terminate either, (a) if the written request of the insured for cancelation thereof be received at the Home Office of the Company together with the policy for endorsement, or (b) if the insured shall, voluntarily or involuntarily, engage in military or naval service in time of war or in work as a civilian in any capacity whatever in connection with actual warfare.

“The additional annual premium of \$ _____ in consideration of which these disability benefits are granted, is payable until the prior termination of the provision for these benefits. The premium stated on the face of the policy includes such additional premium, and the premium payable, if any, after the provision for these benefits terminates will be the premium stated on the face of the policy less the amount of such additional premium.

“The entire and irrecoverable loss of the sight of both eyes or the severance of both entire hands or both entire feet or of one entire hand and one entire foot shall be considered total and

permanent disability without prejudice to any other cause of disability."

3. *Monthly Income, not Reducing Policy, without "90 Day Clause", issued about 1920.*

"If the insured, after the first premium on this policy has been paid, shall furnish due proof to the Company, while this policy is in full force and effect and while there is no default in the payment of premium, that he, at any time after payment of such first premium, while less than sixty years of age, from any cause whatsoever shall have become permanently disabled or physically or mentally incapacitated to such an extent that he by reason of such disability or incapacity is rendered wholly and permanently unable to engage in any occupation or perform any work for any kind of compensation of financial value, the Company upon receipt of such proof will waive the payment of each premium that may become payable thereafter under this policy during such disability. Without prejudice to any other cause of disability, the permanent loss of the sight of both eyes, or loss by severance of both hands above the wrists, or of both feet above the ankles, or of one hand and one foot, shall be considered disability or incapacity within the meaning of this provision.

"Disability Before Age 60—Monthly Income to the Insured—If such disability shall occur before the insured is sixty years of age the Company will, in addition to such waiver, pay to the insured monthly as specified on the first page hereof, the sum of \$10.00 for each \$1,000 of the face amount of insurance under the policy. The first monthly payment shall be made three months after the Company shall have received such proof and subsequent payments shall be made on the first day of each month thereafter during such disability. Interest due on any indebtedness under the policy may be deducted from such monthly income payments.

"Such waiver of premiums and such monthly payment shall be additional to all other benefits and obligations under this policy and the policy shall be continued in force and the face amount of insurance, less any indebtedness, shall become due and payable at death or maturity in the same manner as if the insured had actually continued to pay the premiums.

"Disability after Age 60—Waiver of Premiums With Reduction of Face Amount of Insurance—If the disability of the insured as defined above shall occur after the insured is sixty years of age, the Company upon receipt of due proof of such disability will waive the payment of each premium that may become payable thereafter under this policy during such disability, but the face amount of insurance hereunder shall be reduced

by the amount of each premium so waived, and any loan and non-forfeiture values shall thereafter be based upon the face amount of insurance thus reduced.

“Recovery From Disability—The insured, upon demand by the Company at any time during such disability and before the Company’s liability hereunder has ceased, shall furnish due proof that he actually continues in a state of disability as defined above, and in case of his failure so to do the insured shall be deemed to have recovered from such state of disability.

“In the event that the insured recovers from such state of disability no further premiums shall be waived and no further monthly payments shall be made, and any insurance under the policy, exclusive of the accidental death benefit, at the time of such recovery shall be continued in force subject to the payment by the insured of any premiums falling due thereafter.

“Extra Premium for Total and Permanent Disability Benefits—The total and permanent disability benefits are granted in consideration of the payment of an extra annual premium of \$ which is included in the amount of the premium stated on the first page hereof and which is payable at the same time and subject to the same provisions as to payment as the regular premium under this policy, provided, however, that in no event shall said extra premium be payable on or after the anniversary of the policy next succeeding the date when the insured attains sixty years of age.”

4. *Monthly Income, with “90 Day Clause”, Income Benefit not antedated into 90 Day Period. Clause issued about 1921.*

“Definition—For the purpose of this policy: (a) disability is total when it prevents the insured from engaging in any occupation or performing any work for compensation of financial value and (b) total disability is presumably permanent only under the circumstances and from the date (herein called the effective date) as follows: (1) when due proof is received by the Company that it will presumably exist continuously during the remainder of the insured’s life—then from the date upon which such proof is received by the Company; or (2) when it has existed continuously for three months—then from the date of expiration of such three months; or, (3) when it involves the entire and irrecoverable loss of sight of both eyes, or the severance of both hands at or above the wrists, or of both feet at or above the ankles, or such severance of one entire hand and one entire foot—then from the date of such loss or severance, (c) disability resulting directly or indirectly from military or naval service in time of war is not a risk assumed by the Company.

“Benefits—Upon receipt of due proof before the expiration of one year after default in the payment of premium, or if there be no such default not later than one year from the maturity of this policy, that the insured, while this policy was in force, became totally and presumably permanently disabled as above defined due to bodily injury or disease.

“Disability Before Sixty—(1) Before the anniversary of the register date of this policy upon which the insured's age at nearest birthday is sixty years, the Company will (a) waive payment of all premiums falling due upon this policy after the effective date of such disability and during its continuance, (b) pay to the insured a monthly disability income of \$10.00 for each \$1,000 of the face amount of this policy from the effective date of such disability; the first payment to be made upon receipt of such proof and subsequent payments to be made monthly thereafter during the continuance of such disability. If such proof shows that the insured has been committed by a court as an insane person or committed as a ward or patient in an incorporated institution for mental cases or is otherwise incompetent to transact any business or to care for self, and that no legal representative of the insured's estate has been appointed and qualified, the Company at its election may pay such income to any beneficiary as trustee for the insured. Any disability income due but unpaid at the death of the insured, shall be payable to the beneficiary entitled to receive the proceeds of the policy at the death of the insured.

“Note—Any premiums so waived and any disability income so paid shall not be deducted from any amount payable in any settlement of this policy.

“Disability After Sixty—(2) After the anniversary of the register date of this policy upon which the insured's age at nearest birthday is sixty years, the Company will waive payment of all premiums falling due after the effective date of such disability; but the amount of insurance shall be reduced, as each premium is so waived, by the amount of such premium, and subsequent premiums and any loan and surrender values shall be reduced proportionately.

“Recovery From Disability—The Company shall have the right at any time during the first two years after receipt of such proof, and thereafter once a year, to require proof of the continuance of such total disability. If the insured fails to furnish satisfactory proof, or if it appears at any time that such total disability has been terminated, no further premiums will be waived and no further disability income payments will be made on account of such disability.”

5. *Monthly Income with "90 Day Clause" Income Benefit antedated into 90 Day Period. Clause issued about 1926.*

"Upon due proof that since the payment of the initial premium upon this contract, before a default in the payment of any subsequent premium, and before the anniversary of this contract nearest the sixtieth anniversary of the date of birth, the insured has become wholly disabled by bodily injuries or disease and will be continuously and wholly prevented thereby for life from engaging in any occupation or employment for wage or profit, the Company will waive the payment of any premiums which may fall due on this contract during such disability and will pay for each completed month from the commencement of such disability and during its continuance, a monthly income of one per cent. of the face amount of this policy. Provided that in a case not susceptible of proof of permanency when claim is presented, then after the insured has been wholly disabled by bodily injuries or disease and has been prevented thereby from engaging in any occupation or employment for wage or profit for a period of not less than three consecutive months, upon due proof thereof the Company will grant the aforesaid benefits from the commencement of such disability and during its continuance. The premiums so waived and the disability income so paid will not be deducted in any settlement hereunder.

"Upon proof of such disability occurring after the anniversary of this contract nearest the sixtieth anniversary of the date of birth, the Company will allow all premiums falling due thereafter during such disability to accumulate without interest as an indebtedness hereunder.

"The values in the table of non-forfeiture values incorporated in the contract shall increase during such disability in the same manner as if the premiums were being paid by the insured.

"Independently of all other causes the Company will consider as permanent total disability the entire and irrecoverable loss of the sight of both eyes, or of the use of both hands or of both feet, or of one hand and one foot.

"Failure to pay any renewal premium due under this contract during total disability which terminates in death will not be held to be a default, and if upon presentation of a valid death claim hereunder it shall appear that such disability occurred before the anniversary of the contract nearest the sixtieth anniversary of the date of birth and existed for a continuous period of not less than one month, the benefits herein described will be paid to the executors, administrators or assigns of the insured.

"Upon written request signed by the insured and upon return of this contract for proper endorsement, the Company will annul

this provision and thereafter the premiums shall be reduced by the amount charged for these benefits.

"In any event any premiums payable after the anniversary of this contract nearest the sixtieth anniversary of the date of birth of the insured shall be so reduced."

6. *Specimen Clauses from contract introduced in 1929 in which Disability is defined in terms of loss of Earned Income.*

"Definition of Disability—The insured will be regarded as totally disabled when, by reason of accidental bodily injury or by sickness, his average monthly earned income for a period of four months has not exceeded one-fourth of his former earned income (averaged monthly for the twelve months immediately preceding such four months) and such disability will be regarded as permanent while the earned monthly income of the insured, on account of such injury or sickness, does not exceed the said one-fourth."

"Definition of Earned Income—As herein used the term 'earned income' means wages, salaries, professional fees, and other amounts, received as compensation for personal services actually rendered in any profession, trade or business, not including therein amounts received as a pension or retirement allowance, or as a temporary continuance in whole or in part of customary earned income during the insured's enforced absence from business on account of accidental bodily injury or sickness."

"Misstatement of Age, Alteration of Co-existing Policy, or Change of Risk—If the age of the insured has been misstated, or if the plan or amount of the Co-existing Policy shall be changed, or if the risk hereunder shall be increased or diminished by reason of any change in the insured's profession, trade or business, or in the duties incident thereto, which would alter the Company's classification of the risk, the Premiums and benefits hereunder shall be modified accordingly, and such further adjustment, whether involving a payment by or to the Company, shall be made so that the status of this Policy thereafter shall be in all respects the same as if such modifications had been in existence from the inception of this Policy. If any payment due the Company under the provisions of this paragraph be not made within thirty days after written notice and demand therefor, mailed to the last known address of the insured and assignee, if any, this Policy will terminate. If the risk hereunder, in consequence of any change in the insured's profession, trade or business, or in the duties incident thereto, is not classified by the Company as an acceptable risk, the Company at its option, may

declare this Policy terminated and pay the Cash Surrender Value hereof."

"Reduction of Benefits—If, at the time of the approval of proofs as herein provided, the monthly income benefit to which the insured shall be entitled hereunder, and under other Supplementary Disability Policies issued to him by the Company, together with the income benefits, if any, to which he shall be entitled, by reason of disease or accident, under insurance in any other company or association of whatever kind, shall exceed in the aggregate seventy-five percentum of his former earned income, ascertained as herein provided, the monthly income provided for herein shall be reduced so that the total monthly income under this and such other insurance, if any, shall not exceed seventy-five per centum of such former earned income. In event of such reduction future Premiums hereon will be equitably reduced, and the Company will pay an equitable part of the Cash Surrender Value hereof, computed as of a date immediately preceding disability. The monthly income benefit provided for herein will not be restored to its original amount unless there shall be an election at the time of such reduction to have the Premiums remain at their original amount, and to leave with the Company the portion of the Cash Surrender Value referred to above, in which case the monthly income benefit provided for herein will be so restored, subject to all the provisions hereof, upon the insured's recovery from his then disability."

7. *Monthly Income Contract, with "four months clause", in accordance with the "Standard Provisions" recommended by the Insurance Commissioners, becoming effective in many States on July 1, 1930.*

"In consideration of the payment in advance of an additional _____ annual premium of \$_____ which is included in and payable with the premium stated in said Policy and of the payment of a like sum with each premium after the first payable under said Policy, the Company agrees to pay to the Insured _____ Dollars each month and to waive payment of premiums under said Policy upon receipt of due proof that the Insured is totally and presumably permanently disabled before _____ age 60, as hereinafter provided.

"Upon receipt by the Company at its Home Office of due proof, as hereinafter provided, that the Insured has become totally disabled by bodily injury or disease so that he is and will be thereby wholly prevented from performing any work, following any occupation or engaging in any business for remuneration or profit, and that such disability has already continued uninterruptedly for a period of at least four months (such total

disability of such duration being presumed to be permanent only for the purpose of determining liability hereunder), and provided that

- (1) such total disability began before default in payment of premium under said Policy (or, in event of default, not later than the last day of grace), and that
- (2) such total disability began before the anniversary of said Policy on which the Insured's.....age at nearest birthday is 60, and prior to the maturity of said Policy, and that
- (3) such total disability did not arise from bodily injury or disease occurring before the insurance under said Policy took effect, and known to the Insured, but not disclosed in the application for the insurance under said Policy, and that
- (4) such total disability has been continuous from the beginning of the period of disability claimed,

the Company will grant the following benefits:

(a) Waiver of Premium—The Company will waive the payment of each premium under said Policy, including the premium for this Agreement, falling due after the commencement of such total disability and during its continuance, provided, however, that no premium shall be waived which shall have fallen due more than one year prior to the date of receipt at the Home Office of the Company of written notice of claim, as hereinafter provided. The premium to be waived shall be the premium according to the mode of payment in effect when such total disability began. If such total disability began during the grace period and the Insured is in default, said Policy will be restored if the Insured pays to the Company the premium in default with interest thereon at six per cent per annum when the claim is approved. Any premium falling due after notice of claim is received and prior to approval of claim, shall be payable in accordance with the terms of said Policy, but will, if paid to the Company, be refunded upon approval of such claim.

(b) Income Payments—The Company will pay to the Insured the monthly income stated above for the fourth and each succeeding completed month of such total disability during its continuance, provided, however, that no such monthly payment shall be made for any fractional part of a month of disability nor for any period more than one year prior to the date of receipt at the Home Office of the Company of written notice of claim. If, in the opinion of the Company, disability results from or is accompanied by mental incapacity, payments may, at the option of the Company, be made to the beneficiary in lieu of the Insured.

"Independently of any other cause of disability, the total and irrecoverable loss of the sight of both eyes, or of the use of both hands or of both feet or of one hand and one foot, shall be considered total disability.

"Written notice of claim hereunder must be received by the Company at its Home Office during the lifetime and during the continuance of total disability of the Insured. Failure to give such notice within such times, shall not invalidate any such claim if it shall be shown not to have been reasonably possible to give such notice within such times and that notice was given as soon as was reasonably possible.

"Due proof of claim must be received at the Home Office of the Company before the expiration of one year after default in payment of premium under said Policy and in any event, whether or not there be a default, not later than one year from the anniversary of said Policy on which the Insured's age at nearest birthday is 60 or one year after maturity of said Policy, whichever is the earlier date, otherwise the claim shall be invalid.

"Before making any income payment or waiving any premium under said Policy, the Company may demand due proof of the continuance of total disability, but such proof will not be required oftener than once a year after such disability has continued for two full years. If such proof shall not be furnished, or if at any time the Insured shall become able to perform any work, follow any occupation, or engage in any business for remuneration or profit, no further income payments shall be made nor premiums waived and if any such payments are made or premiums waived they shall constitute an indebtedness against said Policy unless paid to the Company.

"The sum payable in any settlement of said Policy shall not be reduced by income payments made nor by premiums waived under the above provisions. Dividends, loan and surrender values shall be the same as if the waived premiums had been duly paid. Any disability benefit due but unpaid at the time of the Insured's death shall be payable to the person or persons entitled to the proceeds of said Policy.

"Disability Benefits shall not apply if the disability of the Insured shall result from self-inflicted injury or from military or naval service in time of war; nor shall these benefits apply to the Temporary Insurance or to the Paid-up Insurance provided in said Policy under 'Surrender Values', or to any Dividend Additions provided therein under 'Participation in Surplus—Dividends'.

"Any premium due on or after the anniversary of said Policy on which the age of the Insured at nearest

birthday is 60 will be reduced by the amount of premium charged for Disability Benefits. If for any reason said reduction shall not be made and said amount shall be paid to and received by the Company as a part of any premium under said Policy, the amount overpaid, with six per cent. interest thereon, will be refunded and the Company shall not incur any other or further obligation or liability.

“Upon written request of the Insured on any anniversary of said Policy and upon return of said Policy and this Agreement for proper indorsement, the Company will terminate this Agreement and thereafter the premium shall be reduced by the amount charged therefor.

“This Agreement shall automatically terminate if any premium on said Policy shall not be duly paid or if said Policy shall be surrendered.

“The Benefits and Provisions contained in the Sections ‘Miscellaneous Benefits’ and ‘Other Provisions’ of said Policy shall also apply to this Agreement, except as to the above conditions under which said Disability Benefits shall not be effective.”

III. CLAUSE CONTAINED IN A GROUP POLICY ISSUED IN 1931.

“Upon receipt by the Company of due notice and proof—both in writing—that any Employee while insured hereunder and prior to his sixtieth birthday has become totally and permanently disabled as a result of bodily injury or disease, so as to be prevented thereby from engaging in any business or occupation and performing any work for compensation or profit for the remainder of his life, and that such disability has already continued uninterruptedly for a period of at least three months, and provided:

- (A) the disability for which claim is made commenced after the Employee had been continuously insured hereunder for a period of one year, and
- (B) written notice of such disability was received by the Company not more than one year after premium payment on account of the Employee had been discontinued,

the Company shall:

1. discontinue all the insurance hereunder on the Employee, and
2. three months after receipt of such proof of such disability commence to pay to the Employee, in lieu of the payment of the Life Insurance at his death, monthly instalments determined as to number and amount by the amount of such Life Insurance in force at the date of commence-

ment of such disability, in accordance with the table below. If such disability is due to or accompanied by mental incapacity, such instalments may, at the option of the Company, be paid to the Beneficiary of record of the Employee.

“The requirement specified in proviso (A) above shall not apply to (1) any Employee in the employ of the Employer at the date of issue of this Policy provided the Employee became insured hereunder within thirty-one days after he became eligible, or (2) any Employee included in a new class or unit of not less than fifty (50) Employees who was in the employ of the Employer on the date this Policy was extended to include such class or unit and who became insured within thirty-one days after he became eligible.

“The following table illustrates the number of monthly instalments and the amount of each instalment for certain amounts of insurance:

Amount of Insurance	Number of Instalments	Amount of Each Instalment	Amount of Insurance	Number of Instalments	Amount of Each Instalment
\$500.00	20	\$25.52	\$1,000.00	40	\$26.25
600.00	24	25.66	1,100.00	44	26.40
700.00	28	25.81	1,200.00	48	26.55
750.00	30	25.88	1,250.00	50	26.63
800.00	32	25.96	1,300.00	52	26.70
900.00	36	26.10	1,400.00	56	26.85
			1,500.00	60	27.00

“For amounts of insurance in excess of \$1,500.00, the number of monthly instalments shall be sixty (60) and the amount of each such instalment shall be on the basis of \$18.00 per month for each \$1,000 of insurance.

“Proofs of Total and Permanent Disability Claims—The Company shall have the right, when a claim is made for total and permanent disability benefits, or during the period while instalment payments are being paid, to require the Employee to furnish further written proof of such disability or proof of its continuance, and to submit to examinations by its physicians.

“If it shall subsequently appear by reason of the Employee's recovery that the claim was approved in error, or, if the Employee shall fail on demand to furnish satisfactory written proof of the continuance of such disability or fail to submit to examinations by the Company's physicians, the payment of further instalments on account of the Employee shall immediately cease. If the Employee then reenters the employ of the Employer, his Life Insurance hereunder may be revived but shall be limited in amount to the commuted value at three and one-half per centum per annum, compound interest, of the instalments then remaining unpaid at the time of cessation of instalment pay-

ments, augmented by any increase to which the Employee would have been entitled had his service not been interrupted on account of disability.

“During the period of total and permanent disability, the Employee shall not have the right to receive in one lump sum the commuted value of any unpaid instalments; but if the employee dies during such period, any instalments provided herein, remaining unpaid at the date of death, shall be commuted at the rate of three and one-half per centum per annum, compound interest, and paid to the Beneficiary in one sum or in the manner set forth in Section 15.*

“The proofs required by this Section shall be upon forms furnished by the Company.”

* This section refers to Optional Modes of Settlement.

APPENDIX B
SPECIMEN DISABILITY CLAIM FORMS

INSURED'S STATEMENT RE CLAIM FOR DISABILITY BENEFITS

-
1. No. of Policy _____ Date of Policy _____ Amount, \$ _____

2. Give name in full _____
3. Date of Birth _____ Place of Birth _____
4. In what other Companies and for what amounts are you insured under policies providing for Disability Benefits or Health Benefits?

5. Permanent residence _____
6. Where can you be called on in person now? _____
7. (a) Occupation at time disability began _____
(b) Business address _____
8. (a) Are you wholly disabled at the present time? _____
(b) State cause of disability _____

9. (a) On what date did the illness begin that led up to present disability and what was the nature of the illness?

- (b) Give name and address of the first physician consulted at the beginning of that illness _____
- (c) State date on which you first consulted that physician _____
- (d) Give names of all other physicians consulted and dates of such consultations _____

10. From what date has your disability prevented you from engaging in any occupation whatsoever for remuneration or profit? _____

(month) (day) (year)
11. I expressly waive on behalf of myself and of any person who shall have or claim any interest in the above numbered policy, all provision of law forbidding any physician or any other person who has heretofore attended or examined me, from disclosing any knowledge or information which he thereby acquired.
- Insured's Signature _____
- Dated at _____ 19 _____
-

ATTENDING PHYSICIAN'S REPORT ON CLAIM FOR
DISABILITY BENEFITS

~~to~~ (To be furnished without expense to the Company)

1. Name of Insured _____
2. Residence _____
3. Is Insured now under your treatment? _____
4. Date of first consultation in present disability? _____
5. History given you at that time? _____
 - (a) Date of onset of present disability? _____
 - (b) Predisposing cause, if any, of present disability _____
 - (c) Names and addresses of any physicians previously consulted by insured _____
6. If you were consulted for any cause prior to the present disability, give dates and details of such consultations or treatments _____
7. Diagnosis and symptoms of disease, infirmity, or injury causing disability _____
8. (a) Is the insured in your opinion wholly disabled and prevented from engaging in any occupation whatsoever for remuneration or profit? _____
 - (b) If he is, from what date, to your knowledge, has he been so prevented? _____
9. Give your opinion as to whether or not the insured will be permanently, continuously and wholly prevented for life from pursuing any and all gainful occupations, by reason of this disability _____

Signature _____

Dated _____ Post Office Address _____

MEDICAL EXAMINER'S REPORT ON CLAIM FOR DISABILITY BENEFITS
If the Company's Medical Examiner has been the attending physician of the Insured, he is not to make this report.

To be sent by Cashier direct to Examiner and to be returned to Cashier by Examiner. Addressed envelope to be enclosed for that purpose.

1. (a) Name of Insured? _____
(b) Approximately: Age _____; height _____; weight _____;
color of hair _____; color of eyes _____
2. Residence? _____
3. How well do you know insured? _____
4. What is the nature of the injury, infirmity or disease of the insured for which disability is claimed? _____

5. (a) What was the condition of health of the insured for the year preceding the date of the present disability? _____

(b) When did the illness begin which led up to insured's present disability? _____

(c) When did insured first consult a physician in connection therewith? _____

(d) From what date has he been prevented, by reason of ^{his} disability, from engaging in any occupation whatsoever for remuneration or profit? _____
(e) Give full particulars and the names and addresses of all physicians consulted? _____

6. Give your opinion as to whether or not the insured will be permanently, continuously and wholly prevented thereby for life from pursuing any and all gainful occupations by reason of ^{his} disability? _____

7. After careful examination to what extent do you find insured disabled and due to what cause? _____

Examiner's Signature _____

Post Office Address _____

Dated at _____ 19 _____

INSPECTOR'S REPORT ON CLAIM FOR DISABILITY BENEFITS

Pol. No. _____

Cl. No. _____

19____

Mr. _____ (A fee of \$ _____ will be credited for this report.)

Please report carefully on:—

Present Present

Name _____ Age _____ Address _____

(In _____ 19____, he was employed as _____ at _____)

It is alleged that since _____ he has been **TOTALLY DISABLED** due to _____

District Inspector.

(Do not consult attending physician or insured in making inquiries or members of his immediate family or divulge the purpose of this inquiry.)

1. Do you personally know the person described above?

(Answer "Yes" or "No") _____

A. If answer to above is "Yes"—

How long have you known him?

B. (1) If answer to above is "No"—

From whom did you obtain in-

formation?—

(Name Occupation Address)

(2) How long have they known him? _____

2. If known personally—

When did you last see him? _____

3. Can you learn of any serious illness or accident within the past five years?

Ans. ("Yes" or "No") _____

A. If answer is "Yes"—Please indicate nature and severity—AND

WHEN _____

4. Is he now sick or disabled? (Ans. "Yes" or "No") _____

If answer is "Yes"—

A. Is he confined at home or hospital (which)? _____

B. (1) Is he totally disabled? _____

or

(2) Only partially disabled? _____

C. (1) Is disability considered permanent? _____

or

(2) Do Temporary? _____

D. In your opinion, is he able to engage in any occupation or perform any work? _____

E. IF TOTALLY disabled—Since

what DATE HAS HE BEEN

UNABLE TO DO ANY WORK?

5. Is age stated above apparently correct? ("Yes" or "No") _____

(If ans. is "No")—

About what age does he appear to be? _____

6. Is he at the present time employed or engaged in any business requiring his personal presence? ("Yes" or "No") _____

(If answer to above is "Yes"—THEN answer A-B-C below)

A. Does he devote customary full time to his work or business? ("Yes" or "No") _____

or

B. Is he only partially employed ("Yes" or "No") _____

C. If employed (either wholly or only partially) about what compensation does he receive (per month) _____

and

(1) What does he do? _____

(2) Who employs him? _____

7. If recently disabled but now improved—

A. How long was he totally disabled? _____

B. When did his total disability cease? (i. e.,—When did he return to work) _____

8. Has he ever used intoxicants to excess? ("Yes" or "No") _____

(If answer is "Yes")—

A. About how often? _____

(Once a week—month—or two or three times a year)

B. If intoxicated (out of control) what was average duration—For an evening—a day—a week—or

how long? _____

C. How long ago did he give up drinking to excess or intoxication? _____

9. Any suspicion that he has ever been addicted to the use of any drug or harmful substitute for intoxicants? ("Yes" or "No") _____

A. If answer is "Yes" what? _____

10. Is his general character and reputation good? (If "No" please comment) _____

REMARKS.—Comment here on any answer to above questions which may require explanation—and add names and addresses of any known physicians who have attended him—and if he has left your community—furnish definite present address, if possible.

(If not last employed as stated at top of this blank—

Q. 1. What was his LAST regular employment (title and kind of bus.)?

2. By whom employed at that time? (See 4 E.)

Dated at _____ 19____

(Sign Here)

(Use reverse side if necessary and write from this end on back of blank)

APPENDIX C

PROPOSED STANDARD PROVISIONS FOR TOTAL AND
PERMANENT DISABILITY BENEFITS IN
CONNECTION WITH LIFE POLICIES*

HON. ALBERT CONWAY, SUPERINTENDENT OF INSURANCE OF THE
STATE OF NEW YORK, AND THE NATIONAL CONVENTION OF
INSURANCE COMMISSIONERS.

Gentlemen:

Early in 1928 the Hon. James A. Beha, then Superintendent of Insurance of the State of New York, appointed the following committee to consider standard provisions for total and permanent disability benefits:

- Mr. John M. Laird, Vice-President
Connecticut General Life Insurance Company
- Mr. James F. Little, Associate Actuary
Prudential Insurance Company
- Mr. Arthur Hunter, Second Vice-President and Chief Actuary
New York Life Insurance Company
- Mr. Alexander T. Maclean, Second Vice-President and Actuary
Massachusetts Mutual Life Insurance Company
- Mr. James D. Craig, Chairman, Actuary
Metropolitan Life Insurance Company

On December 12, 1928, at the regular meeting of the National Convention of Insurance Commissioners, a resolution was adopted favoring uniform disability clauses in life insurance policies and the following committee was appointed to consider the general question:

- Mr. William G. Hayes, Assistant Actuary
Virginia Insurance Department
- Mr. Russell O. Hooker, Actuary
Connecticut Insurance Department
- Mr. Arthur B. Lines, Actuary
Massachusetts Insurance Department
- Mr. Walter A. Robinson, Actuary
Ohio Insurance Department
- Mr. Grady H. Hipp, Chairman, Actuary
New York Insurance Department

* Reprinted from T. A. S. A., XXX, 260.

The two committees have been in conference and have agreed upon the enclosed recommendations. This agreement was reached after conference with many company representatives and the recommendations have been largely influenced by the desire of the Commissioners for uniform provisions. This has necessitated recommending requirements that would eliminate certain benefits now being granted by a few companies which may not constitute unsound underwriting, but the elimination of which would tend to bring about the desired uniformity. In some respects the recommendations represent a compromise between opposing points of view. For example, the recommendation of a waiting period of from four months to one year, with no income payment before the end of such period, represents a compromise between the companies which did not desire any fixed period after which total disability would be presumed to be permanent and the companies which preferred a three months' waiting period with retroactive payments.

The recommendation that benefits would be allowed in the event of disability occurring prior to age sixty instead of age sixty-five was made because it was felt that this would accomplish the main purpose of the benefit, while extension to age sixty-five would involve a greatly increased hazard and cost. The same fundamental reason caused the recommendation excluding any increase in disability benefit beyond 1 per cent. a month.

Total disability has been defined in accordance with the definition now in general use. The experience of most companies with a definition substantially similar to the one proposed has indicated that the courts will require this definition to be interpreted reasonably in a spirit of fairness to the insured and that, guided by such experience, the companies will be able to continue to apply this definition to actual cases without undue liberality on the one hand or undue strictness on the other. One company has recently received the approval of practically all of the Insurance Departments for a clause recognizing a 75% loss in earning capacity as total disability. The committees express no opinion as to whether such a provision would represent sound underwriting or would provide for benefits for partial rather than total disability, because they feel that this provision would render practically impossible the general uniformity which was

called for under a resolution of the National Convention of Insurance Commissioners.

Serious consideration was given to a proposed provision requiring a pro-rating of benefits in case these benefits appeared to be too large in proportion to earnings. Certain companies feel that some such clause is necessary and should be prescribed but the difficulties incident to drafting and administering a pro-rating clause are such that up to the present time only one company has deemed it expedient to adopt one. In view, however, of the strong feeling among certain companies that such a provision should be incorporated, the committees, while not prepared to recommend a pro-rating clause nor a specified provision therefor, have agreed that such a clause might be included with those provisions which may be necessary to the efficient administration of the coverage, and have covered the subject in general language under provision 15 (e). The committees feel it is important that in any event companies should apply corrective methods in underwriting practices to avoid over-insurance.

The committees endeavored to prepare provisions which would promote uniformity in principle with sufficient latitude for discretion on the part of Company executives.

While the adoption of these recommendations will require a change in practice and policy forms, this will be more than offset by the great advantage of uniformity which will in turn encourage the development of sound underwriting practice and permit the granting of substantial disability benefits with adequate premium rates and reserves based upon homogeneous experience.

The recommendations are in the form of standard provisions under three general classifications; prescribed, permitted and prohibited. The typical benefit in accordance with the proposed provisions would consist of waiver of premium and payment of an annuity of \$10 per month per thousand, with the first monthly payment of \$10 at the end of four months of continuous total disability provided disability occurred before age 60.

Adoption of these standard provisions will have among others the following results:

- (1) Elimination of the "professional man's policy", as the proposed provisions do not permit payment of benefits

when the insured is unable to perform the duties merely of his customary occupation.

- (2) Exclusion of the payment of any disability benefit for the first three months of disability—the maximum allowance permitting benefits to accrue after three months with the first payment at the end of four months if disability has continued so long.
- (3) Exclusion of any increasing disability benefit such as an increase of 10 per cent. a year or an increase of 50 per cent. after five years and 100 per cent. after ten years.
- (4) Prohibition of retroactive income payments for more than one year on delayed claims.
- (5) Requirement of retroactive waiver of premium for at least six months on delayed claims.
- (6) Requirement of recognition within certain limits of disability occurring within the grace period.
- (7) Exclusion of any disability benefit where disability occurs after age sixty except on certain endowments and deferred annuities.
- (8) Exclusion of disability claims when notice of claim is not submitted prior to death and prior to termination of disability.
- (9) Exclusion of benefits for partial disability.

The two committees unanimously favor departmental rulings instead of legislation for the purpose of putting the standard provisions into effect. This procedure would secure greater flexibility, thus permitting changes in the future to meet any new conditions that may arise.

The indications are that in a great majority of the States these requirements can be made effective under the existing law. The committees recommend that, in the few States in which such requirements cannot be made effective by departmental ruling, enabling legislation be sought.

The committees have not felt that it was wise to suggest a table of net premiums and reserves until the standard provisions governing the terms of the standard benefit were finally agreed upon. Since agreeing to recommend the provisions submitted herewith, the Company Actuaries have undertaken to study the

available data and hope to make definite recommendations with regard to net premiums and reserves before the end of the year.

The committees have not felt that the additional benefit in case of death from accidental means required any specific provisions but believe that this benefit should be limited to double indemnity, as should also accidental death benefits to travelers or on account of other specified causes.

Very truly yours,

Committee appointed by
National Convention of
Insurance Commissioners

Committee appointed by
Superintendent Beha

April 23, 1929.

STANDARD PROVISIONS FOR TOTAL AND PERMANENT DISABILITY
BENEFITS IN CONNECTION WITH LIFE POLICIES

Total and permanent disability provisions of contracts of life or endowment insurance (including term) and annuities (other than contracts of Industrial or Group Insurance or Group Annuities), whether contained in the policies or in supplementary contracts, issued after January 1, 1930, shall satisfy in substance the requirements set forth below.

In construing these requirements:

- (1) Waiver of premium includes refund of waived premiums if paid.
- (2) The term "income payments" means monthly payments made in addition to waiver of premium and all benefits otherwise provided by the contract.
- (3) The term "disability benefits" means waiver of premium, or both waiver of premium and income payments, whichever may be specified in the provision.
- (4) The term "maturity" means, in the case of endowment policies, the date at which the policy becomes payable as an endowment, and in the case of deferred annuities, the date on which payment of annuity commences.
- (5) The term "age 60" means either actual or rated age 60 of the insured or the policy anniversary nearest thereto as may be specified in the provision, and the term "age 65" shall be similarly construed.

I. The Following Provisions Are Prescribed.

- (1) That total disability is incapacity (resulting from bodily injury or disease) to engage in any occupation for remuneration or profit.
- (2) That total disability which has been continuous for a period specified in the provision (not less than four months nor more than one year) shall be presumed permanent.
- (3) That written notice of claim must be given to the company (a) during the lifetime of the insured, and (b) during the period of disability. Failure to give notice within the time provided in the policy shall not invalidate any claim if it shall be shown not to have been reasonably possible to give such notice and that notice was given as soon as was reasonably possible.
- (4) That if total and permanent disability is established as required by the provision, any premium or instalment thereof which fell due during such total continuous disability and during a period specified in the provision (not less than six months) immediately preceding notice of claim shall be waived.
- (5) That if total and permanent disability is established as required by the provision, which began after the due date of a premium or instalment thereof in default, but not later than the last day of grace, provided such due date was within a period specified in the provision (not less than six months) immediately preceding notice of claim, disability benefits shall be allowed as if the default had not occurred, but the insured shall be liable for the premium in default with interest thereon, if any.
- (6) That any dividends which would otherwise have become payable during disability shall be allowed as though the disability had not occurred, unless an annuity is provided as permitted by paragraph 12 hereof.
- (7) That upon recovery of the insured from total disability, disability benefits shall cease and premiums or instalments thereof becoming due after such recovery shall be payable.

II. The Following Provisions Are Permitted.

- (8) That the entire and irrecoverable loss of sight of both eyes or the severance of (or alternatively, the entire and irrecoverable loss of the use of) both hands or both feet, or of one hand and one foot, shall be deemed total disability.
- (9) That disability occasioned by certain risks or hazards specified in the provision shall be excluded from the coverage.
- (10) That the disability provision shall be canceled or modified in the event of the marriage of the insured if the insured is a female.
- (11) That disability benefits shall be payable either to the insured or to a beneficiary.
- (12) That in lieu of income payments, there shall be payable an annuity certain for a period of not more than ten years, the present value of which shall be equal to the amount of insurance, but that upon recovery such annuity shall cease, and the insurance shall be restored at a proportionate premium for an amount equal to the present value of the unpaid instalments.
- (13) That in the case of endowment policies or deferred annuities income payments shall be made during the continuance of disability after maturity, provided disability occurred prior to maturity and prior to the insured's attaining age sixty.
- (14) That in the case of endowment policies or deferred annuities maturing at an age not greater than sixty-five, disability benefits shall be allowed up to the date of maturity, in case the disability occurred after the insured attained age sixty, but before the date of maturity.
- (15) Any other provision not inconsistent with these requirements which may be necessary to the efficient administration of the coverage provided and the protection of the interests of the insurer or the insured.

The intention is to permit among others provisions such as the following:

- (a) That proof of disability shall be made at the time and in the form and manner as specified in the provision.
- (b) That the insurer may require proof of continuance of disability, including examination of the insured by the insurer at reasonable intervals.
- (c) That the insured may not convert the policy to a higher premium plan during continuance of disability.
- (d) That the insured may not change the mode of payment during the continuance of disability.
- (e) That a proportionate reduction of income payments, accompanied by return of premiums paid on the amount of such reduction, may be made in case the aggregate monthly amount payable to the insured on account of disability exceeds the percentage specified in the provision (not to exceed 100%) of monthly earned income at date of disability, or alternatively at date of application.

III. *The Following Provisions Are Prohibited.*

- (16) That disability benefits shall be allowed for disability other than that defined as total and permanent by paragraphs 1, 2 and 8 hereof.
- (17) That disability benefits shall be allowed for disability commencing after the insured has attained age sixty (except in accordance with paragraph 14 hereof).
- (18) That the face amount of insurance shall be reduced by the amount of any disability benefits (except in accordance with paragraph 12 hereof).
- (19) That in the case of life insurance policies the monthly income payment shall exceed 1 per cent. of the face amount of insurance (exclusive of additional accidental death or pure endowment benefits).
- (20) That in the case of deferred annuities with maturity at age 70 or earlier the monthly disability income payment

shall exceed one-twelfth of the annual annuity or in the case of deferred annuities with maturity at a later age a monthly income disability payment shall be allowed.

- (21) That income payments shall be made for any part of the first three months of total disability or for any fractional part of a month thereafter.
- (22) That income payments shall be made retroactive for a period of more than one year prior to written notice of claim of the company or in any case for any part of the first three months of total disability.*

* These proposed standard provisions were approved by the National Convention of Insurance Commissioners at Toronto, Canada in September, 1929 with the added recommendation that reserves be put on an adequate basis (see page 8).

APPENDIX D.

DEFINITION OF SYMBOLS

I. *Basic functions and probabilities.*

<i>Symbol</i>	<i>Definition</i>
l_x	The number of persons — whether active or disabled — who, according to a total or mixed life mortality table, attain age x .
l_x^{aa}	The number of active lives who, according to a combined mortality and disability table, attain age x .
l_x^{ii}	The number of disabled lives who, according to a combined mortality and disability table, attain age x . $l_x = l_x^{aa} + l_x^{ii}$.
l_x^i	The number of persons who, according to a disabled life mortality or continuance table, attain age x .
d_x	The number dying, whether active or disabled, between ages x and $x + 1$ out of l_x persons.
d_x^a	The number dying between ages x and $x + 1$ out of l_x^{aa} persons.
d_x^{aa}	The number dying, while active, according to a combined mortality and disability table, between ages x and $x + 1$, i.e., out of l_x^{aa} persons.
d_x^{ii}	The number dying, while disabled, according to a combined mortality and disability table, between ages x and $x + 1$, i.e., out of l_x^{ii} and i_x persons. $d_x = d_x^{aa} + d_x^{ii}$.
d_x^i	The number ceasing to be disabled between ages x and $x + 1$ out of l_x^i persons.

Symbol	Definition
i_x	The number of lives becoming disabled between ages x and $x+1$ out of l_x^{aa} persons.
p_x	The probability that a life aged x will survive one year. $p_x = \frac{l_{x+1}}{l_x}$.
${}_np_x$	The probability that a life aged x will survive n years. ${}_np_x = \frac{l_{x+n}}{l_x}$.
${}_np_x^a$	The probability that an active life aged x will survive n years. In a combined mortality and disability table, ${}_np_x^a = \frac{l_{x+n} - {}_np_x^d \cdot l_x^u}{l_x^{aa}}$.
${}_np_x^{aa}$	The probability that an active life aged x will be alive and active n years hence. ${}_np_x^{aa} = \frac{l_{x+n}^{aa}}{l_x^{aa}}$.
p_x^{at}	The probability that an active life aged x will be alive and disabled one year hence. $p_x^{at} = \frac{i_x (1 - \frac{1}{2} q_x^d)}{l_x^{aa}}$
${}_np_x^d$	The probability that a disabled life aged x will continue to be disabled for n years. ${}_np_x^d = \frac{l_{x+n}^d}{l_x^d}$.
q_x	The probability that a person aged x will die within one year. $q_x = \frac{d_x}{l_x}$.
q_x^a	The probability that an active life aged x will die within one year. In a combined mortality and disability table; $q_x^a = \frac{d_x^{aa} + \frac{1}{2} \cdot i_x \cdot q_x^d}{l_x^{aa}} = 1 - p_x^a$.
q_x^{aa}	The probability that an active life aged x will die, while active, within one year. $q_x^{aa} = \frac{d_x^{aa}}{l_x^{aa}}$.

*Symbol**Definition*

q_x^d The probability that a disabled life aged x will cease to be disabled within one year. $q_x^d = \frac{d_x^d}{l_x^d}$.

If no recoveries are assumed this is a disabled life mortality rate. If recoveries are assumed this is a disabled life termination rate.

${}_1q_x^d$ The probability that a disabled life aged x will cease to be disabled within $1/t$ th of a year.

$${}_1q_x^d = \frac{l_x^d - l_{x+\frac{1}{t}}^d}{l_x^d}.$$

r_x The probability that an active life aged x will become disabled within one year. $r_x = \frac{i_x}{l_x^{aa}}$.

r'_x The absolute annual rate of disability among active lives in the year following attained age x , i.e., the rate of disability which is independent of the mortality rate among active lives.

$$r'_x = \frac{i_x}{l_x^{aa} - \frac{1}{2} d_x^{aa}}.$$

II. Commutation Symbols.

$$\begin{aligned}
D_x &= v^x \cdot l_x & N_x &= \sum_{i=-x}^{i=0} D_i & S_x &= \sum_{i=-x}^{i=0} N_i \\
D_x^{aa} &= v^x \cdot l_x^{aa} & N_x^{aa} &= \sum_{i=-x}^{i=0} D_i^{aa} & S_x^{aa} &= \sum_{i=-x}^{i=0} N_i^{aa} \\
D_x^i &= v^x \cdot l_x^i & N_x^i &= \sum_{i=-x}^{i=0} D_i^i & S_x^i &= \sum_{i=-x}^{i=0} N_i^i \\
D_x^{ii} &= v^x \cdot l_x^{ii} & N_x^{ii} &= \sum_{i=-x}^{i=0} D_i^{ii} & S_x^{ii} &= \sum_{i=-x}^{i=0} N_i^{ii} \\
C_x &= v^{x+1} \cdot d_x & M_x &= \sum_{i=-x}^{i=0} C_i & R_x &= \sum_{i=-x}^{i=0} M_i \\
C_x^{aa} &= v^{x+1} \cdot d_x^{aa} & M_x^{aa} &= \sum_{i=-x}^{i=0} C_i^{aa} & R_x^{aa} &= \sum_{i=-x}^{i=0} M_i^{aa} \\
C_x^i &= v^{x+1} \cdot d_x^i & M_x^i &= \sum_{i=-x}^{i=0} C_i^i & R_x^i &= \sum_{i=-x}^{i=0} M_i^i \\
C_x^{ii} &= v^{x+1} \cdot d_x^{ii} & M_x^{ii} &= \sum_{i=-x}^{i=0} C_i^{ii} & R_x^{ii} &= \sum_{i=-x}^{i=0} M_i^{ii} \\
{}^r C_x^{at} &= v \cdot D_x^{aa} \cdot p_x^{at} & {}^r M_x^{at} &= \sum_{i=-x}^{i=0} {}^r C_i^{at} \\
&= v^{x+1} \cdot l_x^{aa} \cdot p_x^{at} \\
{}^{ra} C_x^{at} &= {}^r C_x^{at} \cdot a_{x+1}^i & {}^{ra} M_x^{at} &= \sum_{i=-x}^{i=0} {}^{ra} C_i^{at} \\
&= v \cdot D_x^{aa} \cdot p_x^{at} \cdot a_{x+1}^i \\
{}^{ra} C_x^{at} &= {}^r C_x^{at} \cdot a_{x+1}^i & {}^{ra} M_x^{at} &= \sum_{i=-x}^{i=0} {}^{ra} C_i^{at} \\
&= v \cdot D_x^{aa} \cdot p_x^{at} \cdot a_{x+1}^i \\
{}^{ra} C_x &= \frac{{}^r C_x^{at}}{D_{x+1}^i} & {}^{ra} M_x &= \sum_{i=-x}^{i=0} {}^{ra} C_i & {}^{ra} R_x &= \sum_{i=-x}^{i=0} {}^{ra} M_i \\
{}^{r1a} C_x^{at} &= {}^{ra} C_x \cdot S_{x+1}^i & {}^{r1a} M_x^{at} &= \sum_{i=-x}^{i=0} {}^{r1a} C_i^{at} \\
&= {}^r C_x^{at} \cdot \frac{S_{x+1}^i}{D_{x+1}^i} \\
{}^{r1a} C_x^{at} &= {}^{ra} C_x \cdot S_{x+2}^i & {}^{r1a} M_x^{at} &= \sum_{i=-x}^{i=0} {}^{r1a} C_i^{at} \\
&= {}^r C_x^{at} \cdot \frac{S_{x+2}^i}{D_{x+1}^i} \\
{}^{r1a} C_x^{at} &= {}^{ra} C_x (S_{x+1}^i - S_{x+k+1}^i) & {}^{r1a} M_x^{at} &= \sum_{i=-x}^{i=0} {}^{r1a} C_i^{at} \\
&= {}^r C_x^{at} \cdot \frac{S_{x+1}^i - S_{x+k+1}^i}{D_{x+1}^i}
\end{aligned}$$

[J. F. Little (T.A.S.A., XIV, 55) defined ${}^{ra} C_x (S_{x+1}^i - S_{x+k+1}^i)$ as ${}^{r1a} C_x$]

Commutation Symbols—(Continued)

$$\begin{aligned} {}^{rI}C_x^{at} &= {}^rC_x (S_{x+2}^t - S_{x+k+2}^t) \\ &= {}^rC_x^{at} \cdot \frac{S_{x+2}^t - S_{x+k+2}^t}{D_{x+1}^t} \end{aligned}$$

$$\begin{aligned} C_x^r &= v^{x+H} \cdot l_x^{aa} \cdot r_x \\ &= v^H \cdot D_x^{aa} \cdot r_x \end{aligned}$$

$$\begin{aligned} {}^u C_x^r &= C_x^r \cdot a_{[x+H]}^{(12)} \\ &= v^H \cdot D_x^{aa} \cdot r_x \cdot a_{[x+H]}^{(12)} \end{aligned}$$

$$\begin{aligned} {}^u C_x^r &= C_x^r \cdot a_{[x+H]:\overline{u-x-H}}^{(12)} \\ &= v^H \cdot D_x^{aa} \cdot r_x \cdot a_{[x+H]:\overline{u-x-H}}^{(12)} \end{aligned}$$

$${}^{rI}M_x^{at} = \sum_{z=x}^{z=\omega} {}^{rI}C_z^{at}$$

$$M_x^r = \sum_{z=x}^{z=\omega} C_z^r$$

${}_y M_x^r = \sum_{z=x}^{z=y-1} C_z^r$ where y is the limiting age of coverage.

$${}^u M_x^r = \sum_{z=x}^{z=\omega} {}^u C_z^r$$

${}_y M_x^r = \sum_{z=x}^{z=y-1} {}^u C_z^r$ where y is the limiting age of coverage.

$${}^u M_x^r = \sum_{z=x}^{z=u-1} {}^u C_z^r$$

${}_y M_x^r = \sum_{z=x}^{z=y-1} {}^u C_z^r$ where y is the limiting age of coverage and $u > y$.

III. *Annuities.*

<i>Symbol</i>	<i>Definition</i>
a_x	The present value of 1 per annum payable at the beginning of every year that a life now aged x enters upon. This type of annuity is commonly referred to as a life annuity-due or simply as an annuity-due. $a_x = \frac{N_x}{D_x}$.
$a_x : \overline{n} $	The present value of 1 per annum payable at the beginning of every year for n years provided a life now aged x is then alive. This type of annuity is commonly referred to as an n -year temporary annuity-due or simply as an n -year annuity-due. $a_x : \overline{n} = \frac{N_x - N_{x+n}}{D_x}$.
a_x^a	The value of a life annuity-due on an active life now aged x payable during survival, active or invalid. $a_x^a = a_x^{aa} + a_x^{di}$. In a combined mortality and disability table, $a_x^a = \frac{l_x \cdot a_x - l_x^{ii} \cdot a_x^i}{l_x^{aa}} = \frac{N_x - D_x^{ii} \cdot a_x^i}{D_x^{aa}}$
$a_x^a : \overline{n} $	The value of an n -year temporary annuity-due on an active life now aged x payable during survival, active or invalid. $a_x^a : \overline{n} = a_x^{aa} : \overline{n} + a_x^{di} : \overline{n} $. In a combined mortality and disability table, $a_x^a : \overline{n} = \frac{l_x \cdot a_x : \overline{n} - l_x^{ii} \cdot a_x^i : \overline{n} }{l_x^{aa}} \\ = \frac{N_x - N_{x+n} - D_x^{ii} \cdot a_x^i : \overline{n} }{D_x^{aa}}$
a_x^{aa}	The value of an annuity-due on an active life now aged x payable during activity. In a combined mortality and disability table, $a_x^{aa} = \frac{N_x^{aa}}{D_x^{aa}}$
$a_x^{aa} : \overline{n} $	The value of an n -year temporary annuity-due on an active life now aged x payable during activity. In a combined mortality and disability table, $a_x^{aa} : \overline{n} = \frac{N_x^{aa} - N_{x+n}^{aa}}{D_x^{aa}}$

*Symbol**Definition*

$a_{x:\overline{n}|:\overline{p-x}|}^{aa}$ The value of a temporary annuity-due on an active life now aged x payable for n years or for $y - x$ years, whichever is the shorter, and during activity.

In a combined mortality and disability table,

$$a_{x:\overline{n}|:\overline{y-x}|}^{aa} = \frac{N_x^{aa} - N_u^{aa}}{D_x^{aa}},$$

where $u = x + n$ or y , whichever is the less.

a_x^{at}

The value of an annuity payable at the end of each year provided an active life now aged x is then alive but disabled. $a_x^{at} = a_x^a - a_x^{aa}$.

In a combined mortality and disability table,

$$\begin{aligned} a_x^{at} &= \frac{l_x \cdot a_x - l_x^{tt} \cdot a_x^t}{l_x^{aa}} - a_x^{aa} \\ &= \frac{N_x^{tt} - D_x^{tt} \cdot a_x^t}{D_x^{aa}}. \end{aligned}$$

$a_{x:\overline{n-1}|}^{at}$

The value of a temporary annuity payable at the end of each year for the next $n - 1$ years provided an active life now aged x is then alive but disabled. $a_{x:\overline{n-1}|}^{at} = a_{x:\overline{n}|}^a - a_{x:\overline{n}|}^{aa}$.

In a combined mortality and disability table,

$$\begin{aligned} a_{x:\overline{n-1}|}^{at} &= \frac{l_x \cdot a_{x:\overline{n}|} - l_x^{tt} \cdot a_{x:\overline{n}|}^t}{l_x^{aa}} - a_{x:\overline{n}|}^{aa} \\ &= \frac{N_x^{tt} - N_{x+n}^{tt} - D_x^{tt} \cdot a_{x:\overline{n}|}^t}{D_x^{aa}}. \end{aligned}$$

$a_x^{a(t:\overline{n})}$

The value of an annuity payable at the end of each year provided an active life now aged x is then alive but disabled, and that the disability occurred prior to age $x + n$.

If the disability coverage ceases at age y so that the rate of disability for y and older ages is zero, this annuity is usually expressed by a_x^{at} if $x + n \leq y$.

$a_{x:\overline{t}|}^{a(t:\overline{n})}$

The value of a temporary annuity payable at the end of each year for the next t years provided an active life now aged x is then alive but

<i>Symbol</i>	<i>Definition</i>
	disabled and that the disability occurred prior to age $x + n$.
	If y is the limiting age of coverage, this annuity is usually expressed by $a_x^{a'; \bar{n}}$ if $x + n \leq y$ or $x + t$.
${}_n a_x^{a(t; \bar{n})}$	The value of a deferred temporary annuity payable at the beginning of each of t years, first payment deferred n years, provided an active life now aged x is then alive but disabled and that the disability occurred during the n years.
a_x^t	The value of an annuity-due on a disabled life now aged x payable until death or prior recovery. $a_x^t = \frac{N_x^t}{D_x^t}$
$a_x^t; \bar{n}$	The value of an n -year temporary annuity-due on a disabled life now aged x payable during the n years until death or prior recovery. $a_x^t; \bar{n} = \frac{N_x^t - N_{x+n}^t}{D_x^t}$
$a_{[x]}^{t(12)}$	The value of a select annuity-due of 1 per annum on a disabled life now aged x payable at the beginning of every month that the disabled life, which has qualified for disability benefits as of age x , continues alive and disabled.

IV. Valuation Factors.

$$\begin{aligned}
 k_x &= \frac{C_x}{D_{x+1}} & u_x &= \frac{D_x}{D_{x+1}} \\
 k_x^r &= \frac{C_x^r}{D_{x+1}^{aa}} & u_x^{aa} &= \frac{D_x^{aa}}{D_{x+1}^{aa}} \\
 {}^w k_x^r &= \frac{{}^w C_x^r}{D_{x+1}^{aa}} \\
 {}^u k_x^r &= \frac{{}^u C_x^r}{D_{x+1}^{aa}}
 \end{aligned}$$

APPENDIX E—BIBLIOGRAPHY

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